

# Arithmetic Series

## Leaving Certificate Ordinary and Higher Level



### Relationship to Leaving Certificate syllabus Strand 3:

Students learn about	Students working at FL should be able to	In addition students working at OL should be able to	In addition students working at HL should be able to
3.1 Number systems		<ul style="list-style-type: none"> <li>– investigate patterns and find specific terms of a sequence.</li> <li>– generate rules/formulae from patterns</li> <li>– find the sum to <math>n</math> terms of an arithmetic series</li> </ul>	

### Misconceptions to watch out for during the whole plan:

- That the student recognizes the differences in a sequence and a series.
- That the student knows not all sequences and series are arithmetic.

**Note while this T&L is divided into sections, the speed at which a teacher progresses through the material will depend on the ability of the class and some sections may take more than 1 class.**

Student Learning tasks Teacher Input	Student Activities Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<b>Section 1. To find the sum of first n terms of an arithmetic series.</b>			
<p><b>Section 1 Stage 1</b> The teacher introduces the lesson with the following problem.. <b>"If Séan saves €40 per week for the first week and increases this amount by €5 per week each week after that, <u>how much will he save on the 10<sup>th</sup> week</u> and <u>how much in total will he have saved after the first 10 weeks?</u>"</b> Teacher then explains that the amount Séan saved on the 10<sup>th</sup> week was the 10th term of the sequence and the total amount he had saved in the first 10 weeks was the 10<sup>th</sup> term of the series.</p>	<p>Students can be asked to physically work out the answers to both these questions posed by Séan's savings pattern. Students may use calculators.</p>	<p>As the formula will not yet be introduced the emphasis at this stage is getting the students to understand the difference in <u>how much Séan will save on the 10<sup>th</sup> week</u> and <u>how much in total will he have saved after the first 10 weeks?</u></p>	<p>Do the students understand the difference in a sequence and a series?</p>
<p><b>Section 1 Stage 2</b> Teacher discusses the problem of "Tony who earned €20,000 in his first year of employment and got a €4000 a year increase each year</p>	<p>Care needs to be taken that students understand the purpose of this proof.</p>	<p><u>Note for a less able class teacher can select smaller numbers, but an average ability class will appreciate the difference more if working with larger figures.</u> In his 8th year of employment he will earn: 20,000+4,000+4,000+4,000+4,000+4,000+4,</p>	<p>Do the students see the difference in the 10<sup>th</sup> term and the sum of the first 10 terms? Do the students understand that "a",</p>

Student Learning tasks Teacher Input	Student Activities Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
after that, how much will he be earning in 8 years time and how much will his total earnings have been for the 8 years he was employed?" Teacher then introduces the formula for the sum of the first n terms of an arithmetic series. $S_n = \frac{n}{2} [2a + (n-1)d]$ without proof.		$000 + 4,000 = 48,000$ His total earnings will be $20,000 + 24,000 + 28,000 + 32,000 + 36,000 + 40,000 + 44,000 + 48,000 = 272,000$ Then explain how there is a formula for this $S_n$ $S_n = \frac{n}{2} [2a + (n-1)d]$ $S_n$ is the sum of the first n terms of an arithmetic series.  The proof of this formula is not required, but can be seen in Appendix A and it is recommended that a class with reasonable ability is shown where it is derived from.	"d" and "n" have the same meaning as they had for an arithmetic sequence?
<b>Section 1 Stage 3</b> Apply the formula to the problem posed in Stage 1 of this lesson and check if it worked.			Did students recognize that they should have got 272,000 for $S_n$ for Tony's problem at the top of the page?
<b>Section 1 Stage 4</b> Distribute Student Activity 1 and allow students to commence the problems.	Care needs to be taken that students understand the difference in $T_n$ and $S_n$ .	As students progress through this activity sheet the lesson can be stopped from time to time and discussions developed re the questions. The content of this discussion will depend on what the teacher seen in the students' answers.	Are students developing the formulas for $S_n$ ?
<b>Section 1 Wrap up and Homework</b> Teacher summaries the concept of $S_n$ and problems in			The Student Activity 1 sheets can be collected next day and corrected.

Student Learning tasks Teacher Input	Student Activities Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Student Activity 1 not already attempted by students given for homework.			
<b>Section 2. To further develop the concept of <math>S_n</math> of an arithmetic series</b>			
<b>Section 2 Stage 1</b> Students are asked to complete the interactive quiz called "Arithmetic Sequences and Series" on the student CD.	Students need to be encouraged to look at the explanations for those they get wrong.	If computers are not available, hard copies of these can be printed for the students.	Are students getting the majority of answers correct?
<b>Section 2 Stage 2</b> Teacher talks about the differences in $T_n$ and $S_n$ . Distribute Student Activity 2 sheet and ask student to do a selection of questions from this.	To identify the differences students can use the fact that $T$ =Term and $S$ =Sum.	The questions on this activity sheet are a mixture of knowledge students have already been given, but the teacher needs to monitor the responses the students (in particular the higher level students.) give as answers as this will determine if students are ready to move to a new topic.	
<b>Section 2 Wrap up</b> A discussion on the properties of arithmetic sequences and series to conclude this section of the course.		It is important that this section is concluded in a positive note and that students can see that arithmetic sequences and series have a lot of relevance in everyday life.	Are the students confident in their knowledge of this topic?

### Appendix A Proof of Formula for arithmetic series

(Students will not be required to prove this formula.)

$$S_n = T_1 + T_2 + T_3 + T_4 + \dots + T_{n-1} + T_n$$

$$S_n = a + a + d + a + 2d + a + 3d + \dots + a + (n-2)d + a + (n-1)d \quad (1)$$

Writing  $S_n$  in reverse:

$$S_n = a + (n-1)d + a + (n-2)d + \dots + a + 3d + a + 2d + a + d + a \quad (2)$$

Adding (1) and (2)

$$S_n = a + (a + d) + (a + 2d) + (a + 3d) + \dots + (a + (n-2)d) + (a + (n-1)d) \quad (1)$$

$$S_n = (a + (n-1)d) + (a + (n-2)d) + \dots + (a + 3d) + (a + 2d) + (a + d) + a \quad (2)$$

$$2 S_n = \{2a+(n-1)d\} + \{2a+(n-1)d\} + \{2a+(n-1)d\} + \dots + \{2a+(n-1)d\} + \{2a+(n-1)d\}$$

$$2 S_n = n\{2a+(n-1)d\}$$

$$S_n = \frac{n}{2} \{2a+(n-1)d\} \text{ Formula as per tables but note}$$

$$S_n = \frac{n}{2} \{a+a+(n-1)d\} = \frac{n}{2} \{\text{first term} + \text{last term}\}$$

## Student Activity 1

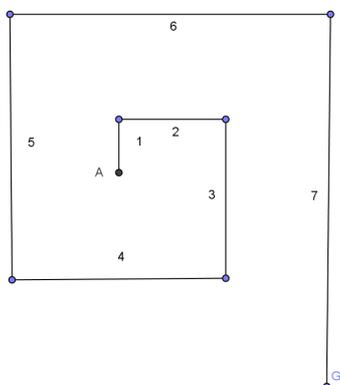
**(Calculations must be shown in all cases.)**

- 1) A factory produced 10, 13, 16 and 19 items respectively in its first four weeks of business. If this pattern continues how many items will this factory produce in the first year of business?
  
  
  
  
  
  
  
  
  
  
- 2) If James saves €40 per week for the first week and increases this amount by €5 per week every week for the next 10 weeks, how much will he have saved after the 10 weeks?
  
  
  
  
  
  
  
  
  
  
- 3) A woman has a starting salary of €20,000 and gets an increase of €2,000 per year for every year worked. How much will this lady earn in her working life, if she retires after 40 years?
  
  
  
  
  
  
  
  
  
  
- 4) If you're new employer offered you choice of 2 salary packages. Package A had a starting salary of €12,000 per year with an increase of €2,000 per year for every consecutive year you stayed with the firm and Package B had a starting salary of €20,000 and increase of €1,500 per year for every consecutive year you stayed with the firm. Assuming you plan to remain in the firm for 10 years which is the best package and by how much?
  
  
  
  
  
  
  
  
  
  
- 5) If a water tank containing 377 litres of water has developed a leak, on the first day the tank leaks 5 litres of water and every day after that as the leak develops it leaks a further 4 litres more than the previous day. Apply  $S_n$  formula and show calculations to determine how many days it is until the tank is empty.

6) In a cinema, there are 140 seats in the front row, 135 in the second and 130 in the third row and this pattern continues until the last row. If the last row has 45 seats, how many rows are there in the cinema and calculate the total number of seats in the cinema.

7) A craftsman uses 100 beads the first day of business and this amount increases by 15 each day after that. If he works 24 days in the month, how many beads will he need to order in advance in order to have a month's supply?

8) Find the total amount of metal required to continue this shape with 20 sides.



9) Emer purchases a new car every year. If the first car she purchases costs €20,000 and each year after that the car costs €3,000 more than the previous one, how much will she spend on cars in the next 10 years?

- 10) Emer purchases a new car every second year. If the first car she purchases costs €20,000 and each time she changes the car after that it costs an extra €6,000 more than the previous one, how much will she spend on cars in the next 10 years?

Compare the answers in questions 9 and 10 and state how you would advice Emer.

- 11) Find  $S_n$  for the arithmetic Series 2, 4, 6, 8,...

- 12) How many terms of the arithmetic series  $1 + 3 + 5 + \dots$ , is required to give a sum greater than 600? Write a story that this could represent.

- 13) If Kayla got her new mobile phone on the first of April, she sent 1 text that day and every day after that she sent an extra 2 texts per day. How many texts will she sent on the 30<sup>th</sup> April and how many texts will she sent in the month of April that year? (April has 30 days.)

- 14) Is it possible for an arithmetic series to have a first term and a common difference that are both non-zero and yet for the sum of the terms in the series to be zero? If so, give an example and explain the circumstance that causes this to happen. If not, explain why not.

## Student Activity 2

**(Calculations must be shown in all cases.)**

1) Jonathan has some initial savings and increases the amount he saves each month by a regular amount. If the total amount he saved in 8 years is €1690 and he saved €220 in the 5<sup>th</sup> year. Find how much his initial savings were and how much did he increase his savings by each year.

2) If  $S_4 = 26$  and  $S_6 = 57$ , find  $T_n$ .

3)

The first term of an arithmetic series is  $-2$  and the second term is 4.

**(i)** Find  $d$ , the common difference.

**(ii)** Find  $T_{10}$ , the tenth term of the series.

**(iii)** The  $k$ th term of the series is 292. Find  $k$ .

**(iv)** Find  $S_{20}$ , the sum of the first 20 terms of the series.

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i)

ii)

iii)

iv)

4)

The sum of the first  $n$  terms of an arithmetic series is given by  $S_n = n^2 - 16n$ .

**(i)** Use  $S_1$  and  $S_2$  to find the first term and the common difference.

**(ii)** Find  $T_n$ , the  $n$ th term of the series.

**(iii)** Find the values of  $n \in \mathbf{N}$  for which  $S_n = -63$ .

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i)

ii)

iii)

5)

Three consecutive terms of an arithmetic series are  $4x+11$ ,  $2x+11$ , and  $3x+17$ .  
Find the value of  $x$ .

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6) A household found uses 100 grams of sugar every day, how much will it use in a year (365 days)?

- 7) If a household with a very sweet tooth found it was using 1 Kilogram of sugar per week and decided to reduce this amount by 100 grams per week. How much sugar per week would it be using at the end of the year (52 weeks in the year)? What is the total amount of sugar this household would use this year?

- 8) Prove that the formula for the sum of the sum n Natural numbers is  $S_n = \frac{n(n+1)}{2}$ .