**Teaching & Learning Plan**

*Title:* The Binomial Distribution.

*Prior Knowledge:*

1. Combinations.
2. The Binomial Expansion.
3. Random Variables.
4. Probability distributions.

*Learning Outcomes:* Students will:

1. Know what a Bernoulli trial is.
2. Recognise when to use the Binomial distribution.
3. State any assumptions necessary to use the binomial distribution.
4. Apply the Binomial distribution to some everyday problems.

*Resources:*

1. PowerPoint presentation.
2. Dice or coin.
3. Autograph.
4. Activity Sheets

*New Vocabulary:* Bernoulli trial, Binomial Distribution.

*Middle Part:*

|  |  |  |  |
| --- | --- | --- | --- |
| **Student Learning Tasks:  Teacher Input** | **Student Activities: Possible and Expected Responses** | **Teacher’s Support and Actions** | **Checking Understanding** |
| *Introduction:*  *The story of Eastern Airlines Flight 855.*  *08:56. Plane take off from Miami.*  *09:14 54 miles from Nassua and 110 miles from Miami Engine 1 fails.*  *09:16 Engine 2 shows low pressure.*  *09:23 The pilot decided to return to Miami.*  *09:33 Engine 2 and 3 failed.*  *09:34 The pilot managed to start Engine 2 again.*  *09:46 Landed safely in Miami.*  *The probability of one of the independent jet engines on Flight 855 failing was .0001 or 1 in 10,000.*  *With the help of the Binomial distribution, we will answer the following important questions:*  *1. What was the probability of two   engines failing?*  *2. What was the probability of all three  engines failing?*  *3. And hence determine whether it was  likely that factors other than chance were  at play.*  *A Bernoulli trial is a trial with just two outcomes.*  *Examples*   1. *If you toss a coin once, then how many possible outcomes are there?* 2. *John has taken a cube and painted 4 sides red and 2 sides blue. John tosses the cube. How many possible outcomes are there?*   *When you toss a coin twice, what are the possible outcomes?*  ***Task 1****:*  *Complete the tree diagram on* [*Activity Sheet 1*](Activity%20Sheet%201.docx)*. Fill out the table of all possible outcomes.*  ***Question1****: From your results are some outcomes more likely than others?*  *For example, is it more likely to get exactly two heads than it is three heads?*  ***Question2****: What outcomes have the same chance or probability of occurring?*  ***Question3****: Why is a tree diagram unsuitable for a large number of trials?*  ***Question4:*** *How many branches are added for each extra trial?*  ***Task 2:***  *Complete* [*Activity Sheet 2*](Activity%20Sheet%202.docx) *now.*  ***Question1****: If you list all the possible outcomes of 10 Bernoulli trials, then how many of the outcomes will have 4 successes?*  ***Question2****: Generalise for n trials and r successes.*  ***Task 3****:*  [*Activity Sheet 3*](Activity%20Sheet%203.docx)  ***Question1****: In an experiment, with only two possible outcomes, the probability of a ‘success’ is p. If the experiment is repeated n times, what is the probability that there are exactly r successes? Give your answer in terms of p, r, and n.*    *Back to our original questions on the flight.*  *1. What was the probability of two   engines failing?*    *2. What was the probability of all three  engines failing?*    *3. And hence determine whether it was  likely that factors other than chance were  at play.*  *There was something more at play other than chance.*  ***Task 4****:*  [*Activity Sheet 4*](Activity%20Sheet%204.docx) | *HT and TH.*  *Yes*  *Yes*  *1H & 2H*  *3H & 0H*  *Grows too quickly*  *Twice the number added for previous trial.* | *Ensure all students have 2 outcomes.*  *Explain that they have performed two Bernoulli trials.*  *Explain to students that we are usually interested in the probabilities of just one of the outcomes, the outcome we label success. This choice is arbitrary. For example, we have chosen H in this activity to be ‘success’.* | *Have all students recorded two outcomes?* |

**Activity Sheet 1**

1. A coin is tossed three times. Complete the tree diagram for this experiment and complete the list of all possible outcomes.

H

H

H

T

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | H | H | H |
| 2 | H | H | T |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |
| 8 |  |  |  |

1. Toss a coin 3 times and note the outcome,

|  |  |  |
| --- | --- | --- |
|  |  |  |

1. Which one of the outcomes, 1 – 8 , in Q1 corresponds to your result,

**Activity Sheet 2**

1. A coin is tossed 4 times. List all ways of getting **exactly** 3 heads,

|  |  |  |  |
| --- | --- | --- | --- |
| H | H | H | T |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

Four boxes are colored red, blue, green and yellow. In how many different ways can you choose 3 boxes? Use your knowledge of combinations here.

1. How many different ways can I place three H’s in four boxes.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

Give your answer in the form

Say why this problem is equivalent to the problem posed in Question 2

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. A coin is tossed 4 times. Find the number of ways of getting

* exactly one head \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* exactly 2 heads \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* exactly 3 heads \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* 4 heads \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* No head \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Think again of filling four boxes.

Give your answers in the form

1. A coin is tossed *n* times. Find the number of ways of getting **exactly**,

* 1 tail \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* 5 tails \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* *r* tails \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* *n* tails \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Activity Sheet 3**

1. A coin is tossed 3 times. What is the probability of the following outcomes,

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (i) | H | T | T | : |  | X |  | X |  | = |  |
| (ii) | T | T | H | : |  | X |  | X |  | = |  |
| (iii) | T | H | T | : |  | X |  | X |  | = |  |

Therefore, what is the probability of getting exactly one head in 3 tosses \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. John has taken a cube and painted 4 sides red and 2 sides blue. John tosses the cube. If *p* is the probability of getting a red side in one throw and *q* is the probability of getting a blue, then write down the value of *p*, and the value of *q*.

p = \_\_\_\_\_ *q* = ­­­­­­­­­\_\_\_\_

1. If John tosses the cube 4 times, what is the probability of getting exactly three reds? Give your answer in the form

Note: Firstly you need to count the number of ways of getting exactly 3 reds, and multiply this by the probability of one favorable outcome (e.g. RRRB). This is just the Addition rule in probability.

**Activity Sheet 4**

1. John has told Tom that he will pay him €5 for every head that comes up in 4 tosses of a coin.
   1. What is the least amount of money Tom could win? \_\_\_\_\_\_\_\_\_\_\_\_\_
   2. What is the greatest amount Tom could win? \_\_\_\_\_\_\_\_\_\_\_\_\_\_
   3. What are the probabilities of the following outcomes,
      1. 0 heads,\_\_\_\_\_\_\_\_\_\_\_\_\_
      2. 1 head, \_\_\_\_\_\_\_\_\_\_\_\_\_
      3. 2 heads,\_\_\_\_\_\_\_\_\_\_\_\_\_
      4. 3 heads,\_\_\_\_\_\_\_\_\_\_\_\_\_
      5. 4 heads \_\_\_\_\_\_\_\_\_\_\_\_\_

Note: All possible outcomes as in c. above together with their probabilities are called a binomial distribution.

1. Graph the binomial distribution in 1 c. 