

Teaching & Learning Plans

The Multiplication of Fractions

Junior Certificate Syllabus



The Teaching & Learning Plans are structured as follows:



Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

- i. Student Learning Tasks Teacher Input: This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.
- ii. Student Activities Possible and Expected Responses: Gives details of possible student reactions and responses and possible misconceptions students may have.
- **iii. Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.
- iv. Checking Understanding: Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plans: The Multiplication of Fractions



Aims

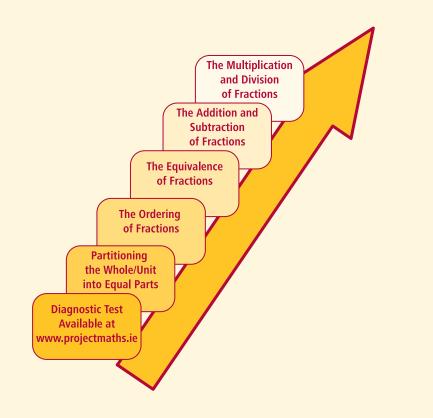
- To consolidate students' understanding of the multiplication of fractions
- To engage students with the everyday uses of fractions
- To engage students in activities that will help to reinforce the multiplication algorithm

Prior Knowledge

Students should have prior knowledge of some terms and ideas associated with fractions from the primary school curriculum, but the topic may need to be revisited to ensure that all students know the basics. Students may have certain 'misconceptions' based on intuition and personal experience. Students should be familiar with:

- the ordering of fractions
- the equivalence of fractions
- the addition and subtraction of fractions using fraction strips, fraction circles and symbols
- the multiplication of whole numbers as "groups of "
- the fraction wall (Appendix 1)

Your attention is drawn to "An Overview of Teaching & Learning Fractions" below:



Learning Outcomes

As a result of studying this topic, students will be able to

- multiply a whole number by a fraction and understand the procedure
- multiply a fraction by a whole number and understand the procedure
- multiply a fraction by a fraction and understand the procedure
- use the algorithm of multiplying numerators and denominators together and also be able to model what is happening
- appreciate that multiplication does not always make things bigger

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Some students may only be able to engage in activities which are relatively straightforward, while others may be able to engage in more open-ended and challenging activities. Selecting and assigning activities appropriate to a student's ability will cultivate and sustain his/ her interest in learning.

In this T & L Plan, students might be required to draw, provide a verbal explanation, compose a multiplication sentence, apply the algorithm. Teachers can provide students with various amounts and different styles of support during the class.

In interacting with the whole class, teachers can employ effective and inclusive questioning. Questions can be pitched at different levels and can move from basic questioning to ones which are of a higher order nature. In this T & L Plan, some students may be required to answer a question such as: **Aoife works during her summer holidays for 4 hours on Saturdays and earns €12 per hour. How much does she earn each Saturday?** A more challenging question can be reserved for others: **Can you explain in words how the multiplication algorithm works?** Sometimes students might be asked to devise a question themselves or to see if they could come up with an alternative question to one they might have been asked.

Besides whole-class teaching, teachers can consider different grouping strategies – such as group and pair work – to encourage student interaction, help students to verbalise their mathematical understanding and help to build student self-confidence and mathematical understanding. For example, in this T & L Plan students are asked to work in pairs to make up a question where the number line could be used for multiplication, do out a solution and swap their question with another pair; they can then be asked to compare answers.



Relationship to Junior Certificate Syllabus

Topic Number		Description of topic	Learning outcomes
		Students learn about	Students should be able to
3.1 Q:	Number Systems The set of rational numbers	The binary operations of addition, subtraction, multiplication and division and the relationships between these operations, beginning with whole numbers and integers. They explore some of the laws that govern these operations and use mathematical models to reinforce the algorithms they commonly use. Later, they revisit these operations in the context of rational numbers and irrational numbers (R/Q) and refine, revise and consolidate their ideas. Students learn strategies for computation that can be applied to any numbers; implicit in such computational methods are generalisations about numerical relationships with the operations being used. Students articulate the generalisation that underlies their strategy, firstly in the vernacular and then in symbolic language. Problems set in context, using diagrams to solve the problems so they can appreciate how the mathematical concepts are related to real life. Algorithms used to solve problems involving fractional amounts.	 investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers analyse solution strategies to problems

Project Maths Tionscadal Mata Development Team

Resources Required

Fraction strips, fraction circles, fraction stacks (optional), number lines, area model and fraction wall (Appendix1).

	Lesson Interaction				
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding		
 We are going to look at multiplying a whole number by a fraction. Firstly, we will look at whole 			 Can students give examples to show 		
number multiplication to examine what is happening in this number operation.			that they understand that " <i>a</i> x <i>b</i> " means		
 For example, Liz wants to give each of her 3 friends 4 bars. 			" a groups of b "?		
» How would you work out how many bars she needs?	• 3 x 4 bars = 12 bars	 Emphasise to students that multiplication is repeated addition. 			
» Can you draw a picture to model this situation?	Students may draw	» If they do this i.e.			
	• Students now draw	 ask them to think in terms of putting the bars into a bag for each friend and then draw a picture. » Draw this picture on the board "3 groups of 4 bars" 			
» Can you now put your picture into words?	• Liz needs 3 groups of 4 bars each for her friends.				



Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What does the notation 3 x 4 mean?	 It means 4 + 4 + 4. It means 3 times 4. 		
 Now we are going to take an example of multiplying a whole number by a fraction. 			
» Barry is having 4 of his friends over to his house for pizza. He is going to give them ² / ₃ of a pizza each.	 2³₃ + 2³₃ + 2³₃ + 2³₃ or 4x 2³₃ or 4 groups of 2³₃ each 		
» How is this like the last problem?			
» Can you model the situation using fraction circles?	· •	 Remind students that the amount in each group stays the same. 	
		 » Draw the picture of the pizzas on the board and write: "4 groups of ²/₃" 	





Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
 » Can you put your picture into words? » 4 groups of ²/₃ is equal to how many thirds? 	 4 groups of ²/₃ ⁸/₃ (there are 3 ways students can get this answerdraw, add or multiply) 	 » Allow students time to either draw, add or multiply to get their answer. » Circulate and check students' progress. 	 » Are students counting from their circles, adding 2⁴3 + 2⁴3 + 2⁴3 or multiplying 2⁴3 by 4? » Are students still relying on pictures or are they coming up with an algorithm?
		 » Pick 3 students who got the answer of % by 1. counting the shaded thirds in the picture 2. adding or 3. multiplying. » Ask these students to present their answers to 	
» How many pizzas do you think Barry would need for his friends?	• 2 ² ⁄ ₃ pizzas.	 the class. Allow students time to either draw, add or multiply to get their answer. Circulate. Pick 3 students who got the answer of 2 ²/₃ by counting the shaded thirds in the picture or adding or multiplying. 	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
	 One of the above students may draw the picture below on the board Another student may add ²/₃ + ²/₃ + ²/₃ + ²/₃ Another student may add ²/₃ + ²/₃ + ²/₃ = 2 ²/₃ 		
 So you have written how you would write 4 groups of ²/₃ as a multiplication sentence. 	 A third student may use a multiplication sentence: 4 x ²/₃ = ⁸/₃ = 2²/₃ 		
 Make a poster of another example like this one using words, pictures, multiplication sentences and a real world situation. 			» In pairs, can students come up with other examples like this one?
 » Could anyone come to the top of the class to discuss their poster, remembering to include: 1. words 2. pictures 3. multiplication sentences 4. real world situations ? 	 Students present their posters to the class. 	 Allow discussion and comments from the class as to why the example is a good one or, perhaps, why it may cause confusion. 	 » Did students come up with other (varied) suggestions, other than circles (e.g. cakes, apple tarts, etc.)?



Student Learning TasksTeacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If I ask you to multiply ² / ₃ by 4, what incorrect answer do you think I could get?	 (A possible incorrect answer could be) ²/₃ x 4 = ⁸/₁₂ 	 Allow time for students to think. 	
» Note: if for example ⁸ / ₁₂ is offered as an incorrect answer, facilitate the discussion.	 ⁸/₁₂ = ²/₃ and we multiplied ²/₃ by 4 to get ⁸/₁₂, so ⁸/₁₂ cannot be the answer to ²/₃ x 4. or 	 Accept all answers and write them on the board. 	 Can students see why 4 x ²/₃ = ⁸/₃ and not ⁸/₁₂ ? This is a common misconception?
» Why is this incorrect?	 We added up ²/₃ four times. We were adding up thirds-the size of the part didn't change, just the number of them changed. 	 Allow students to discuss why some of the answers are incorrect. 	 Students need to present more examples like this and verbalise why the answer is as it should be.
» Based on what has been done, write a question for the person sitting beside you, do out the solution, come up with an incorrect solution and be able to explain to your partner why this answer is incorrect.	» Students work on writing out a question, its solution and an incorrect solution and then pass the question to the student beside them.	 Circulate and make sure everyone is clear on what to do. 	
» Now swap your questions.	» Students check to see if they got the answer correct.		
» Explain the answer to each other.	got the answer correct.		
» Check to see if your answers agree.			
» Can you explain why the incorrect answer is incorrect?			



Student Learning Tasks:	Student Activities: Possible and	Teacher's Support and Actions	Checking
Teacher Input	Expected Responses		Understanding
 » Try this question: Sharon does ¾ of an hour 		» Allow students time to think	
of homework every night for 5 nights. How much		» Circulate	
homework does she do altogether? You are to use words, pictures and a multiplication sentence.		 As you check on students' progress, ask 3 students if they would come to the board and present their work; one to present it in words, one the picture and one the multiplication sentence. 	
	 Words: 5 groups of ³/₄ =¹⁵/₄ Picture: 	 Allow students time to write, draw and verbalise their answers to the class. (Although, many students will be able to do this without the 	
	$= \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{15}{4}$ (Repeated Addition) or $= 3\frac{3}{4}$	fraction strips or circles, using them helps to consolidate the concepts.)	
	• Multiplication sentence: $5 \times \frac{3}{4} = \frac{15}{4}$		

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding	Mathe Tionscadal Mata Development Tea
 We will now look at the fraction multiplication algorithm. Up to now, we haven't written the whole number as a fraction over 1. For a number to remain the same, what do we multiply or divide it by? So for the denominator to remain the same what do we multiply it by? How can we write 5 as a fraction? 	 1 Multiply it by 1 5⁄1 	 » Ask students for more examples of this and write them on the board e.g. 4x1=4 7x1=7 » Ask for more examples 		
 Could you rewrite the problem 5 x ³/₄ to show this? (Students would have been introduced to this concept in primary school.) 	• 5/1 x 3/4	 Ask for a few more examples from the students (highlighting this concept may be helpful). 		
 » Can you explain to each other how to multiply ⁵/₁ x ³/₄ using the multiplication algorithm. 				
» Can you explain in words how the multiplication algorithm works?	• You multiply the two numerators together and divide it by the two denominators multiplied together to get ¹⁵ / ₄	» Note: Delay providing the answer for as long as possible.	 Have the students understood the multiplication algorithm as: a/b x c/d = (a×c)/(b×d) Can students apply what they have learned about the multiplication algorithm to similar questions? Note: From now on questions will be done using words, pictures, multiplication sentences and algorithms. 	

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Student Learning Tasks:	Student Activities: Possible and	Teacher's Support and	Checking Understanding
Teacher Input	Expected Responses	Actions	
 » From Student Activity 1, complete Q1, 2 and 3. 		 » Distribute Student Activity 1 and ask students to answer Q1, 2, and 3. 	
		 » Circulate and check progress 	
		 Ask different students if they will present their work to the class. 	
	 Students present their work to the class. 	 Check questions on the board, questioning where necessary. 	 Can students verbalise their answers at the board.
 » Go to Student Activity 1, Q4 (i) and (ii). 		 » Circulate and check students' progress. 	
		 Ask a student to present their answer to part (i) using the number line. 	
» Let's look at Q4 (i).	» A student draws the following on the board: Time worked in hours		
	0 12 24 36 48		
» How much does she earn?	 4 hours at €12 per hour (4 groups of 12) = €48 	 Write on the board (4 hours x €12 = €12 + €12 + €12 + €12) 	 Can the students recognise that this is repeated addition? (4 hours x €12 = €12 + € 12 + €12 + €12)

Student Learning Tasks: Teacher Input	-		Checking Understanding	
» What does the picture show?	• A jump from 0 to the first tick represents an hour of work and I write in 12 at this tick mark to show how much is earned after 1 hour.		» Can students verbalise their picture?	
	 A jump from 1 to the second tick represents another hour of work and I write in 24 at this tick mark to show how much is earned after 2 hours, and so on for 3 and 4 until I get €48. 			
» What did you get for the multiplication sentences Q4 (ii)?		 Write the multiplication sentences on the board: 2 hours x €12 = €24 3 hours x €12 = €36 4 hours x €12 = €48 5 hours x €12 = €60 		
» Now, in pairs, make up a question where the number line could be use for multiplication?	d		 In pairs, can students make up other questions where the number line could be used for multiplication? 	
 » Do out a solution. » Swap your question with another pair when you have it completed and compare answers. 			» Did students come up with varied suggestions other than hours and money?	



Student Learning Tasks:	dent Learning Tasks: Student Activities: Possible		Checking Understanding	
Teacher Input	and Expected Responses	Actions		
 » Look at Q4 (iii). » Staying in your pairs, explain to each other how you would find out how much money Aoife will earn in 2½ hours? 		» As you check on students' progress, ask 3 students if they would come to the board to present their work; one to present using the picture, one the multiplication sentence and one using the multiplication algorithm.	» Can students verbalise their answer?	
	• Picture: Mark the number line halfway between 2 and 3. See how much money this represents	 » Emphasise to students that multiplication is repeated addition. » Note: While students may not convert 2½ to an improper fraction they should be able to work this out without the conversion. 	» Can students convert 2½ to an improper fraction?	



Student Learning Tasks:	Student Activities: Possible	Teacher's Support and	Checking Understanding
Teacher Input	and Expected Responses	Actions	
 » Let's discuss Student Activity 1,Q4 (iv). » How much money will Aoife earn in ¾ of an hour? 		» Give students time to draw or calculate this. (Some students will model this by drawing the number line and dividing the interval from 0 to 12 into quarters. They will see that each quarter of an hour Aoife earns € ¹² ⁄ ₄ and for ³ ⁄ ₄ she earns 3 x ¹² ⁄ ₄ . Others will use an algorithm.)	
		Note: Students should think of ¾ as three one quarters. Find the ¼ first and then get 3 times that. » Some students will be able to do this without	
		the picture (number line), but practice with an easy question will help them to use the number line for more complex problems.	
» Will it be bigger or smaller than €12?	 It will be less than €12. or 1 times 12 is 12 and as ¾ is less than 1, ¾ x 12 will be less than 12. 		» Can students appreciate why the answer is less than 12?



Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If you wish model this using the number line.	» Note: Students could draw the following: 0 12 24 3 6 9	 » Write on the board: Amount of money earned in euro is ³⁄₄ x 12 = €9 ³⁄₄ hour x €¹²⁄₁ = ³⁶⁄₄ = €9 	
 Write down the multiplication sentence, the algorithm and the answer. 	 ³⁄₄ x ¹²⁄₁ = ³⁶⁄₄ = €9 		
» Did multiplication by ¾ make €12 bigger?	 No, multiplication by a number less than 1 made €12 smaller. 	 Write on the board: Multiplication doesn't always make things bigger. 	
 Make up examples of other instances where multiplication doesn't make things bigger 	 Students verbalise other instances 	 Write students examples on the board 	» Did students come up with other (varied) suggestions?
» Ask a student to explain in words how you get ¾ x 12.	 You found how much she earned in ¼ of an hour so you divided 12 by 4, giving €3 and then for ¾ of an hour it was 3 times this which gives €9. Multiplying by ¾ is dividing by 4 and multiplying by 3. 		» Have students understood this concept that ¾ means dividing by 4 and multiplying by 3?
 » Would it matter in which order you do the question? 1. ¾ x 12 or 2. 12 x ¾ 	 ³/₄ x 12 = 9 12 x ³/₄ = ³⁶/₄ = 9 also. So No. 	 Allow students time to think and verbalise their answers to the class. Write on board: ³/₄ x 12 = 9 	
Try both.		$12 \times \frac{3}{4} = \frac{36}{4} = 9$	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Can you verbalise in terms of groups what each question means?	 One is ³/₄ of a group of 12 whereas the other is 12 groups of ³/₄. 		
 » Draw a picture to show each of these situations. 	 Students draw pictures to show that ¾ of a group of 12 is equivalent to 12 groups of ¾. 		
» Now complete Student Activity 1: Q5 & Q6.		 » Circulate and check progress. » Check questions on the 	
		board, questioning where necessary.	
 Now let's have a look at multiplying a fraction by a fraction. 			
» Cara had ^{2/5} of her "rectangular" birthday cake left over from her party. She ate ³ / ₄ of the leftover cake. How much of the full cake did she eat?			
» Think about what you are being asked?	• What is ¾ of ⅔ of a whole cake?		» Can students verbalise this question?





Student Learning Tasks:	Student Activities: Possible	Teacher's Support and Actions	Checking
Teacher Input	and Expected Responses		Understanding
 Answer this question using a picture, a multiplication sentence and the multiplication algorithm. 		» As you check on students' progress ask 2 students if they would come to the board and present their work; one to present using the picture, one the multiplication sentence/ multiplication algorithm.	 » Can students verbalise their answer? » Do students understand that fractions must
» Can you explain to me what you have drawn?	 I drew a rectangle to represent the whole cake. I divided it vertically into 5 equal parts and shaded in the amount left over after the party. 	 Students may be unsure of how to make the division but there will probably be someone who comes up with it. It is better not to tell the students how to do it but let them find it for themselves. Note: Students can also be given a rectangular strip of paper and they can fold the paper vertically first into fifths and then horizontally into quarters. 	be related to a referent whole i.e. the original rectangle/cake? (This is a very important point for students to understand)
» Can you explain to me	 I divided the whole cake 		» Can students show that the whole cake needs to be divided into quarters?
what this represents? » How many equal parts	horizontally into 4 equal parts.20		
are there now?			



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Student Learning Tasks:	Student Activities: Possible	Teacher's Support and	Checking Understanding				
Teacher Input	and Expected Responses	Actions					
 » Can you explain to me what this represents? » How many pieces are there in the shaded area? » What fraction of the whole are they? » Write the answer as a multiplication sentence/ algorithm. » Using your knowledge of equivalent fractions, can you simplify this fraction? » Now complete Student 	 I marked off ¾ of the previously shaded amount. I marked off ¾ of ⅔. This is the part eaten by Cara. 6 pieces 6 out of 20 ¾ x 2/5 = 6/20 %20 = ¾0 if we divide the numerator and denominator by 2. 	 » Some students see the algorithm straight away and others maybe not. It is better to have students point it out to the teacher than the other way around. » Circulate and check 	 Are students able to model multiplication of a fraction by a fraction using an area model? Can students see that multiplying tops and bottoms has a foundation in reality? The product ¾ of % means that if we divided the % of the remaining cake into 4 equal parts then ¾ of % is the amount given by 3 of those equal parts. 				
Activity 1: Q7.		progress.					
		 Check questions on the board, questioning where necessary. 					

Student Learning Tasks:	Student Activities: Possible	Teacher's Support and	Checking Understanding
Teacher Input	and Expected Responses	Actions	
Reflection: » Write down 3 things you learned about multiplying fractions today.	 How to: 1. Multiply a whole number by a fraction 2. Multiply a fraction by a whole number 3. Multiply a fraction by a 		
	 fraction 4. Pictures, multiplication sentences and algorithms can be very helpful when multiplying fractions. 5. Multiplication algorithm for multiplying two fractions can be 		
	generalised as: $a/b \times c/d = (a \times c)/(b \times d)$ 6. Multiplication does not always make things bigger.		
 Write down anything you found difficult today. 		 » Circulate and take note of any questions or difficulties students have noted and 	
 Write down any questions you may have. 		help them to answer them.	



Student Activity 1



1. The unit is
Picture
Words. The above diagrams showgroups of
Multiplication sentence:x=
Algorithm

2. Fill in the blank spaces below. When the answers are improper fractions, change them into proper fractions.

Multiplication Sentence	Words	Picture	Algorithm	Answer
4 x = ⁵ ⁄ ₆				
	3 groups of 5%			
		$ \bigotimes \bigotimes \bigotimes \bigotimes \bigotimes $		

3. Mentally picture the following problems before calculating the answers:

(i) 8 x ²/₃ =

(ii) 5 x ³/₁₁ =

(iii) 6 x ⁵⁄8 =

Student Activity 1

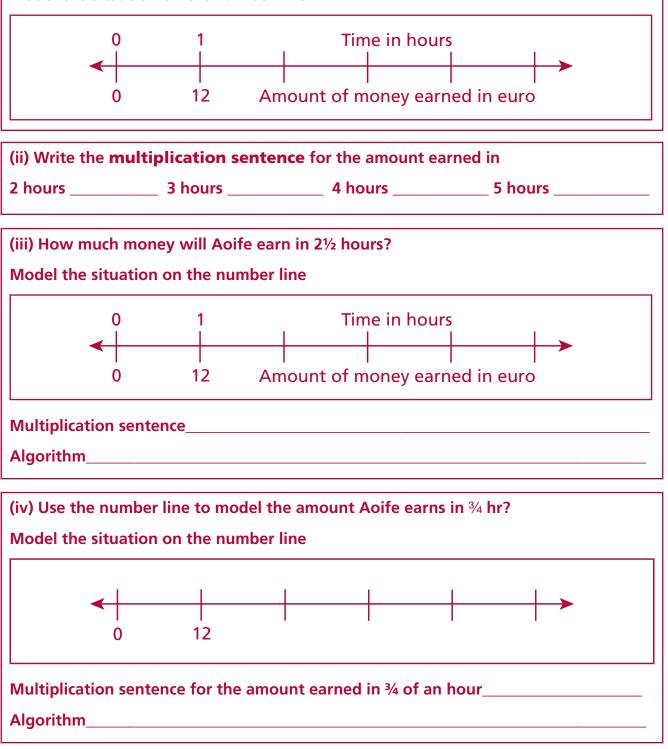


Aoife works during her Summer holidays for 4 hours on Saturdays and earns €12 per hour.

(i) How much does she earn each Saturday ?

Hint: If \in 12, the amount earned in 1 hour, is represented by the first tick mark after 0 on the number line below, mark in the amount earned after 2, 3, and 4 hours.

Model the situation on the number line

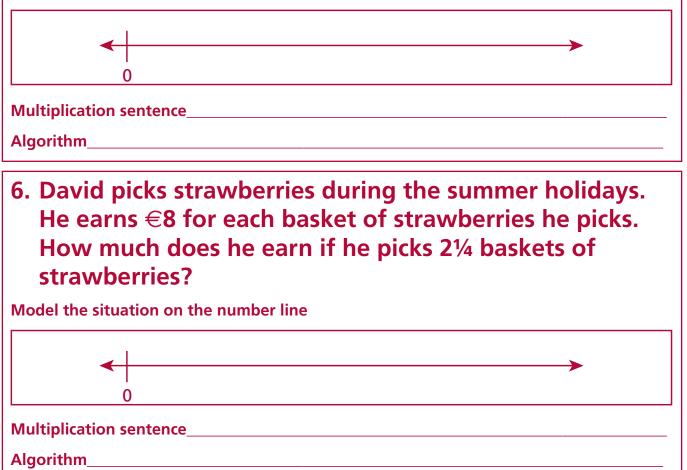


Student Activity 1



5. There are 24 students in Mary's class. ⁵/₆ of the class went on the school trip. How many students from Mary's class went on the school trip?

Model the situation on the number line



7. Tony tiled 4/5 of a bathroom wall. Next day he grouted 7/8 of the tiled section. What fraction of the bathroom wall had the tiles grouted?

Model this using an area model

Multiplication sentence_____

Algorithm_

Fraction Wall



1 unit or 1 "whole"																
	1/2					1/2										
	1⁄3					1	/3 1/3									
1,	/4			1/4	Ļ				1⁄4					1⁄4		
1⁄5)		1/5	5		1	/5			1/	, 5			1/	, 5	
1⁄6		1/	6		1∕6			1⁄6			1/	6		1	/6	
1⁄7		1⁄7		1⁄7		1,	/7		1⁄7			1⁄7			1⁄7	
1⁄8	1/8	, 8	1/8	, 3	1/3	, 8	1	/8		1⁄8		1	/8		1⁄8	3
1⁄9	1⁄9		1⁄9	1/	6	1	⁄9	1	/9	1	/9		1⁄9		1/9	, 9
1⁄10	1⁄10	1/-	10	1⁄10	1/	10	1/-	10	1⁄1	0	1/1	0	1⁄1	0	1/1	10
1⁄11 1	/11	1⁄11	1/-	11	11	1⁄	11	1/1	1	11	1	/11	1/	11	1/	, 11
1/12 1/	12 ¹	/12	1⁄12	1/1:	2 1	/12	1⁄1	2	/12	1/-	12	1⁄1:	2	/12	1/	/12
1⁄15 1⁄15	1⁄15	1⁄15	1⁄15	1⁄15	1⁄15	5 1/	, 15	1⁄15	¹ ⁄15	1⁄15	5	1/15	1⁄15	1⁄1	5	1⁄15
1⁄16 1⁄16	1⁄16	1⁄16	1⁄16	1⁄16	1⁄16	1⁄16	1⁄16	1⁄16	s 1⁄1	6 1	/16	¹ ⁄16	1⁄16	1⁄-	16	1⁄16
1⁄18 1⁄18	1/18 1/-	18 1/1	18 1/18	1⁄18	1⁄18	1⁄ ₁₈	¹ ⁄18	¹ ⁄18	1⁄18	1⁄18	1⁄1	8 1⁄-	, 18 ¹ /	18	/18	1⁄ ₁₈
1/20 1/20 1/	/20 1/20	1⁄20	1/20 1/	20 1/20	1⁄20	1⁄20	1⁄20	1⁄20	1⁄20	1/20	1⁄20	1⁄20	1⁄20	1⁄20	1⁄20	1⁄20
1/24 1/24 1/24	1/24 1/24	4 1⁄24	1/24 1/24	4 ¹ ⁄24 ¹ ⁄2	/24 1/24	4 1⁄24	1⁄24	/24 1/2	24 1/24	1⁄24	1⁄24	1/24 1	/24 1/2	4 1/24	1/24	1⁄24