Teaching & Learning Plan

Inferential Statistics for Means

Leaving Certificate Syllabus

The Teaching & Learning Plans are structured as follows:

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.
ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.
iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.
iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plans:

TITLE

Aims[1]

The aim of this series of lessons is to enable students to:

- To understand why sampling is important.
- To understand there is a link between statistics and probability.
- To understand the phrase “inferential statistics”.
- Understand the link between 95% confidence and the Empirical Rule.
- To recognise how sampling variability influences the use of sample information to make statements about the population.
- To understand what factors we must keep in mind when we use sample information to make statements about the population.
- To understand the idea of a confidence interval.
- To understand that a sample mean might not be the same as the population mean.
- To understand the idea of a hypothesis test.
- To understand how to conduct a hypothesis test on a population mean.
- To apply knowledge and skills relating to statistics to solve problems.
- To use mathematical language, both written and spoken, to communicate understanding effectively.

Prior Knowledge

Students have prior knowledge of:

- Quantifying probabilities from Teaching and Learning Plan 1: Introduction to Probability
- Task on Household sizes from page 2 of the Workshop 10 booklet on www.projectmaths.ie
- The Empirical Rule
- Sampling Variability
- The difference between a population and a sample.
- Simple random sampling
- Describing the shape, centre and spread of distributions
- The Data Handling Cycle

[1] This Teaching & Learning Plan illustrates a number of strategies to support the implementation of *Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (Department of Education & Skills 2011). Attention to the recommended strategies will be noted at intervals within the Lesson Interaction Section of this Teaching and Learning Plan.

Learning Outcomes

As a result of studying this topic, students will be able to:

- Understand the effect of sampling variability on our ability to use the mean of a single sample to make a statement about the mean of a population.
- Understand the existence of the distribution of sample means.
- Describe the shape of the distribution of sample means for a given population, including calculating the centre of this distribution and its standard deviation.
- Construct a 95% confidence interval for the mean of a single sample using z-scores.
- Construct a 95% confidence interval for the mean of a population using z-scores.
- Conduct a hypothesis test on the population mean using a 95% confidence interval.
- Conduct a hypothesis test on the population mean using p-values.
- Understand how inferential statistics might be used in a range of every-day applications.

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open-ended and challenging activities.

In interacting with the whole class, teachers can make adjustments to meet the needs of all of the students.

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to articulate their mathematics openly and to share their learning.

Relationship to Leaving Certificate Syllabus

Sub-Topic	**Learning Outcomes**	
Students learn about	**Students working at OL should be able to**	**In addition students working at HL should be able to**
1.3 Outcomes of random processes		-use simulations to explore the variability of sample statistics from a known population, to construct sampling distributions and to draw conclusions about the sampling distribution of the mean – solve problems involving reading probabilities from the normal distribution tables
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	– discuss populations and samples – decide to what extent conclusions can be generalised	
1.7 Analysing, interpreting and drawing inferences from data	– recognise how sampling variability influences the use of sample information to make statements about the population – use appropriate tools to describe variability drawing inferences about the population from the sample – interpret the analysis and relate the interpretation to the original question – interpret a histogram in terms of distribution of data – make decisions based on the empirical rule – recognise the concept of a hypothesis test	– construct 95% confidence intervals for the population mean from a large sample and for the population proportion, in both cases using z tables – use sampling distributions as the basis for informal inference – perform univariate large sample tests of the population mean (two-tailed z-test only) – use and interpret p-values

Resources Required

Formulae and Tables Booklet, Whiteboards, rulers, *GeoGebra*, calculators and a pack of 200 schoolbag weights (see appendix D)

Lesson Interaction			
Student Learning Tasks: **Teacher Input**	**Student Activities: Possible and Expected Responses**	**Teacher's Supports and Actions**	**Checking Understanding**
Section A – Introducing sampling variability and its impact on statistical inference			
• In today's lesson we are going to carry out a statistical investigation. • You should recall that when we do a statistical investigation we follow the data-handling cycle. • We would like to answer the following question: "What is the average weight of an Irish post-primary student's school bag?"	•	• Have a poster of the data-handling cycle ready under **Section A** of the board plan. • Beside Stage 1 of the data-handling cycle write the question "What is the average weight of an Irish post-primary student's schoolbag? • Identify this as Stage 1 of the data-handling cycle.	• Do students recall that we follow the data-handling cycle when carrying out a statistical investigation? • Do students recognise that Stage 1 of the data-handling cycle requires a question to be asked?
• When we say "Irish post primary students" how many students do we mean?	• A lot. • 50,000 students. • 360,000 students. • All of them. • All post-primary students in Ireland.		• Do students understand that when we say "Irish post-primary students" we mean all of them?
• In statistics when we refer to "all" or "everybody", what	• The population.	• Write the word "population" in the word bank on the board. • Encourage students to write their own description of the term in their copybooks.	• Do students recall that, in statistics, the complete set of elements, e.g. people/items/units is known as the population?

name do we give to this group? • So we would like to know the average school-bag weight of the population of Irish post-primary students. • In other words, we are interested in answering a question about a population.			• Do students understand that when we say “Irish post-primary students” we mean the population of Irish post-primary students or all Irish post-primary students?
• When we say “average”, what do we mean? • For the purposes of our investigation we are going to use the mean as our measure of average.	• The typical value of a set of data. • The value that the data is centred on. • The mean. • The mode. • The mean. • The central tendency of a set of data.	•	• Do students understand that there are three measures of average? • Do students understand that from now on when we talk about the average we mean the mean?
• Can anybody suggest how we might find the mean weight of an Irish post-primary student’s school bag?	• We need to survey some people. • We need some data. • We could ask everybody here in the room.	• Highlight the second stage of the data-handling cycle “Collect Data” on the board.	• Do students recognise that, to answer the question, we need some data?
• If we were to gather the data ourselves,	• We could ask them all. • We could ask some students.	• Beside Stage 2 of the data-handling cycle add the terms “census” and “sample”.	• Do students understand that, in general, when gathering data you

how many students could we ask?	• We could take a sample of students. • 100. • 1,000. • All the students in our school.		have two choices - you can survey the entire population or a subset of the population?
• You have suggested two different approaches - one where you ask all students and one where you ask some students. Can you explain why you might choose one approach over the other?	• Asking everybody should provide a more accurate answer. • Asking everybody is expensive and takes a long time. • It wouldn't be possible to ask every second-level student. • Sampling is faster and cheaper. • If you sample you mightn't get an accurate answer. • When you sample you have to be careful to make sure the sample is representative.	• Add some of the important advantages and disadvantages of sampling vs. taking a census to data-handling cycle.	• Do students recognise that there are advantages and disadvantages to both ways of collecting data? • Can students identify the advantages and disadvantages of each way of collecting data?
• For many reasons you have just discussed, when answering a question in statistics, we usually use data from a single sample instead of data from the entire population. Note: There are times when the entire population is included when gathering data.	•	• Circle the word "sample" beside stage 2 of the data-handling cycle.	• Do students recognise that sampling is used in the majority of statistical investigations? • Do students understand why sampling is used in the majority of statistical investigation? • Do students understand that the use of sampling raises the question of how accurate the results of a statistical investigation are?

The census is one such case.			
• You pointed out one of the disadvantages to sampling – that of accuracy. • Because of this, we are going to investigate if it's possible to use the result from a single sample to accurately answer a question about a population. • To investigate this, I have created (simulated) my own population of students and we are going to see if we can use a single random sample from this population to find the mean of the population. • I have used pieces of card to simulate a population of students or more accurately to simulate my population of schoolbag weights. There are 200 pieces of card (or 200	•	• Circle the term "accurate" beside Stage 2 of the data-handling cycle. • Show students container of cards. • Show students the envelope with the population mean sealed in it. Pin it to the board. • Under **Section A** of the board plan, write the heading "Population". Underneath it write "No. of students in population = 200" and "Mean schoolbag weight = ?". • Encourage each group of students to replicate what's written on the board on their own miniature whiteboard.	• Do students understand that I have created a population so that I can investigate how reliable a sample is for answering a question about a population? • Do students understand that each unit of my population is represented by a piece of card? • Do students understand that the number on each card is the weight of one student's schoolbag? • Do students understand that we are going to use the simulated population to see if it's possible to use a single sample to determine the mean of a population?

schoolbag weights) in a population. Each card has the weight (in kilograms) of one student's schoolbag. • I have determined the mean school bag weight for the population and I've written it in this envelope. • We are now going to see if, by choosing a random sample from my population, I can find out what the mean schoolbag weight of the population is. • That is we are going to see if it's possible to use a single random sample to answer a question about a population.			
• In turns, I would like each group to choose a simple random sample of 36 schoolbag weights (cards) from the container and calculate the mean	• Students choose a simple random sample of 36 cards by drawing cards from the container.	• Under **Section A** of the board plan add the number of students in the population and in a single sample under the headings of "Population" and "Sample" • Under **Section A** of the board plan, write "Calculate the mean schoolbag weight" beside Stage 3 of the data-handling cycle. • Under the headings of "Population" and "Sample" add the following statement "Mean schoolbag weight = "	• Can students choose a simple random sample?

weight of the 36 schoolbags. • How do we choose a simple random sample? • This is Stage 3 of the data-handling cycle – analyse the data. • How do we calculate the mean schoolbag weight?	• By assigning a number to each card and using a random-number generator to identify which card to choose. • By shuffling the cards in the container and drawing without looking. • Add up all the schoolbag weights in our sample and divide by 36. • Enter the data in the calculator in statistics mode and find the mean. • Students calculate the mean of their samples using their calculators.	<table><tr><td>Population No. of students in the population = 200 Mean schoolbag weight in the population =</td><td>Sample No. of students in my sample = 36 Mean schoolbag weight in the sample =</td></tr></table>• Encourage each group of students to replicate what's written on the board in their own copybooks. • Distribute a copy of the instructions for using a calculator to calculate the mean and standard deviation of a dataset (see **Appendix A** for details). • Circulate to make sure students are completing the task correctly. • Encourage students to write their results in the appropriate space on their whiteboard.	• Do students know how to use the STAT mode on their calculator to calculate the mean of a dataset?
• I also want you to calculate the standard deviation of your sample. • Can anybody suggest why we might want to calculate the standard deviation of our sample?	• To measure the variation in our sample. • To understand how spread out the values in our sample are.	• Encourage students to take a note of their standard deviation.	• Do students understand what standard deviation tells us about a set of data?

	• Because when you use the mean as your average you use the standard deviation as your spread. • To understand how close together the values in our sample are. • Students calculate the standard deviation of their samples using their calculators.		• Can students use their calculator to calculate the standard deviation of their sample?
• Group 1, could you tell me the mean schoolbag weight for students in your sample?	• 5.5 kg. • *Note: This is a mean from one particular sample.*	• Under **Section A** of the board plan, write Group 1's result under the heading "Sample". Population: No. of students in the population = 200; Mean schoolbag weight in the population = 5.5 kg Sample: No. of students in my sample = 36; Mean schoolbag weight in the sample = 5.5 kg	• Is Group 1's result reasonable?
• I am now going to use the result from Group 1's sample to do what we set out to do – to make a statement about the entire population using a single sample. • The mean weight of schoolbags belonging to students in the population is 5.5 kg. • This is the final stage in the data-handling cycle – interpret the results.		• Under **Section A** of the board plan, add Group 1's result to the appropriate location under the heading "Population". • Beside Stage 4 of the data-handling cycle add in the statement "The mean weight of schoolbags belonging to students in the population is 5.5 kg".	

• So we've completed the four stages of our investigation. Are you happy with the answer to our original question?	• Yes. • No I got a different answer. • No – our group got a different mean. • No – if we used our group's result we'd have a different conclusion. • We all got different answers. • Why are we using Group 1's answer? • How do we know Group 1's result is the correct one?	• Under **Section A** of the board plan, write each group's sample mean in the correct location under the heading "Sample". Population No. of students in the population = 200 Mean schoolbag weight in the population = 5.5 kg Sample No. of students in my sample = 36 Mean schoolbag weight in the sample = 5.5 kg, 4.8 kg, 4.9 kg, 5.2 kg, 4.6 kg, 5.3 kg, 5.1 kg, 4.7 kg	• Do students recognise that each group got a different value for the mean weight? • Do students recognise the issue this raises in using the mean of a single sample to make a statement about a population?
• The fact that we all get different means when we analyse a sample is known as sampling variability. • Can you explain why we all get different means i.e. can you explain why sampling variability occurs?	• Our samples were randomly chosen. • We all chose different samples from the container. • We chose our samples randomly so you wouldn't expect the answers to be the same. • Every group's sample is made up of different values.	• Encourage students to discuss with each other what sampling variability means and to write the term and its description into their copybooks.	• Can students explain what sampling variability is? • Do students understand why sampling variability occurs? • Can students explain why sampling variability occurs?
• The aim of this activity was to see if we can use a single sample to determine the mean schoolbag weight of students in a population. • Because I simulated the population, I know what the <u>population mean</u> is – remember		• Encourage students to explain their reasoning.	

it's written in the envelope on the board. • Given what we've just discovered – how confident would you be that Group 1's mean is the same as the population mean?	• Not very confident. • I'd say it's around the right answer. • Reasonably confident. • I don't think it's likely to be the same.		• Do students recognise the difficulty in using a single sample to make a statement about a population? • Do students understand that sampling variability presents us with a problem when we try to use a single sample to make a statement about a population?
• Can you explain to me why you're not very confident in Group 1's result?	• Well, it's just one of the possible results we could get. • Because of sampling variability. • Other groups got values different to Group 1. • There's nothing special about Group 1's result. • Maybe our result is the correct one.	• Encourage students to discuss their ideas with each other. • Encourage each group to share their thinking with the other groups in the classroom.	• Do students understand that Group 1's result is only one of the possible answers we can get when we sample?
• You said Group 1's result was just one result you could get by choosing a sample of size 36 from a population of 200. • Across the whole class we took 8 different samples. How many different samples could we take?	• A lot. • Thousands. • $\binom{200}{36}$ • ${}_{200}C_{36}$ • $\approx 4.5 \times 10^{46}$	• If students are having difficulty answering this question, review combinations, through the use of examples.	• Can students apply their knowledge of combinations to calculate the number of combinations which may be selected from a population of 200?

• That is, how many unique samples of size 36 could you choose from a population of 200?	*Note: This is not the correct number of unique combinations as there are a number of repeated weights in the population.*		
• Given this, what is the chance of the mean of a single sample being equal to the population mean? • Can you explain why this is so?	• Very small. • Almost zero. • Highly unlikely. • $\approx \frac{1}{4.5\times10^{46}}$ (Not correct[†]). • $\approx 2.2 \times 10^{-47}$(Not correct[†]). • It's not likely. • If all means are equally likely then the chance is small. • That'll depend on the number of different sample means you can calculate when choosing samples of size 36 from a population of 200. • There are so many possible answers – the chance of mine being correct is tiny. • There are so many different sample means you could get – the chance of landing on a value equal to the population mean is really small. • There's a good chance that you'll get a sample mean which is not the same as the population mean.	• Encourage students to explain their reasoning.	• Do students understand that because there are so many different sample means, it is unlikely our sample mean will be the population mean?

	[†]*This is not the correct number of unique combinations as there are a number of repeated weights in the population and different combinations will still yield the same mean.*		
• Let's see if our suspicions are correct. • The mean weight of schoolbags in the population of students is 5.1 kg. Does group 1's result match this value? • Did any group's sample get this result? • We have shorthand notation to represent the mean of a population. We use the Greek letter μ to represent population mean. • We also have shorthand notation to represent the mean of	• No. • No. • No group got a mean of 5.1 kg but most of our answers were close. • One group got the same as the population mean but we'd have no way of knowing this without knowing the population mean.	• Open the envelope with the population mean written in it and show it to students. • Amend the information under **Section A** of the board plan under the heading "Population" and beside Stage 4 of the data-handling cycle so that the correct population mean is now included. • Amend the information under **Section A** of the board plan to include the correct notation for the population mean and for the sample mean. **Population** No. of students in the population = 200 Mean schoolbag weight in the population = 5.5 kg $\mu = 5.5$ kg **Sample** No. of students in my sample = 36 Mean schoolbag weight in the sample = $\overline{x_1} = 5.5$ kg $\overline{x_2} = 4.8$ kg $\overline{x_3} = 4.9$ kg $\overline{x_4} = 5.2$ kg $\overline{x_5} = 4.6$ kg $\overline{x_6} = 5.3$ kg $\overline{x_7} = 5.1$ kg $\overline{x_8} = 4.7$ kg	• Do students understand that there is little chance that the mean they obtained from their sample is the same as the population mean? • Do students understand that μ means the mean of a population? • Do students understand that $\bar{x}$ means the mean of a single sample?

a single sample. We use $\bar{x}$ to represent the mean of a single sample. • In a real statistical investigation would we know if our sample mean was "the correct value"?	• No. • No – the only way we'd know that is if we knew the population mean and that's what we're trying to find out in the first place.		• Do students understand that, even if their result was the same as the population mean, in a real statistical investigation (where the population mean is unknown) they'd have no way of knowing this?
• So we have a problem. We want to use a single sample to answer a question about the population but, because of sampling variability, the chance of the mean of the sample being equal to the population mean is minimal. • For this reason we cannot say that the mean of the population is equal to our sample mean.		• Under **Section A** of the board plan, write the statement "Because of sampling variability, we cannot say that $\mu = \bar{x}$". • Sketch a number line, mark in the population mean and use a different colour to mark in the sample mean	• Do students understand that they cannot assume that their sample mean is the same as the population mean? • Do students understand that the chance of their sample mean being equal to the population mean is low? • Do students understand that there is no way to know if the mean of

• Is it possible, then, to use a single sample to make a reliable statement about a population? • From our limited number of sample means it appears as though the mean of a single sample tends to be close to the mean of the population. • It seems then that we could use a single sample to make a reasonable statement, to the effect of "The	• No. • We can't say what the population mean is exactly. • Maybe. • All our answers are different but most of them are all around the population mean so we can estimate what the population mean is. • We could say that the mean schoolbag weight for students in the population is around the mean of our single sample.	• Under **Section A** of the board plan, write the statement "The mean schoolbag weight of students in the population is close to 5.5 kg".	their sample is the same as the mean of the population? • Do students understand that this presents a major problem if we want to use a single sample when answering a question about a population? • Do students understand that we can use our sample mean to approximate the population mean?

population mean is close to the value of our sample mean". • I want each group to use their sample mean to make this type of statement about the population mean. • Are all the statements freasonable? Are all the statements consistent with each other? Are all the statements consistent with the true mean of the population? • So it is possible to use the mean of a single sample to approximate the mean of a population.	• Students use their sample mean to make a statement about the population mean which is of the form "The population mean is around….". • Yes. • Well they're more reasonable than our original statements. • Yes because the population mean is in and around each group's sample mean. • Yes because each group's sample mean is close to every other group's sample mean. • Most of the statements are consistent but one group's result is not close to any of the other groups' or to the population mean. • Yes but we have only looked at 8 samples. Will every sample mean be in and around the population mean?	*Note: Each group should replace the 5.5 kg with the mean weight obtained from their sample.* • Under **Section A** of the board plan write the following "It seems reasonable to say $\mu \simeq \bar{x}$".	• Do students understand that we can make this type of statement because the sample means tend to be close to the population mean? • Do students understand that (most of) the statements about the population mean are fair and consistent with each other?
SECTION B – Understanding the distribution of sample means			
• We have seen that sampling variability means that we cannot		• Refer to **Section A** of the board plan as you summarise what has been learned.	• Do students understand that, because of sampling variability, we can only use the mean of a single

equate the mean of a single sample with the mean of a population, however it appears that we can approximate a population mean using the mean of a single sample. • A couple of problems exist with the way in which we achieved this. • Firstly we do not know if this approximation is valid since we based our understanding on a very small number of samples. • It would be prudent then to look at a larger group of sample means and see if they exhibit similar behaviour. • Secondly, when we say “The population mean is *approximately* equal to the sample mean” or “The population mean is *close to* the sample mean” what exactly		• Under **Section B** of the board plan write the following: *Problem 1: Our conclusion to **Section A** was based on a small number of sample means ⇒ we should check this conclusion using more sample means.* • Under **Section B** of the board plan write the following: *Problem 2: The language we used in our conclusion to **Section A** is too subjective ⇒ we need a mathematical way of describing the variation between the mean of the population and the mean of a single sample.*	sample to approximate the mean of a population? • Do students understand that this result was based on a very small number of sample means? • Do students recognise that it would be wise to look at a larger group of sample means to see if this behaviour (the sample means lying close to the population mean) persists? • Do students recognise the subjectivity of using language such as “approximately” or “around” and the problem that this presents for mathematics?

do we mean by *approximately* or *close to*? • One person's interpretation of this language will be different to another's. • This is a feature of mathematics i.e. there is no room for ambiguity in the language that's used to describe outcomes • Stating it another way we need some mathematical way to precisely describe the variation between a population mean and the mean of a single sample. • To address this we're going to go back to our simulated population of schoolbag weights and investigate it in more detail. • Let's start our investigation by looking at our simulated population.	• Students answer **Section B: Student Activity 1**	• Distribute **Section B: Student Activity 1** to all students. • Move around the room to make sure students understand the task. • Encourage students do discuss the questions with each other. • Open the *GeoGebra* file "**TheDistributionOfSampleMeans.ggb**". The dark-blue	

• In groups, I want you to answer Question 1 of **Section B: Student Activity 1**. • What shape is the distribution?	• It's roughly normal. • It's skewed to the right – slightly. • It's roughly symmetric.	histogram shows the population distribution. Click on the button "Normal population".	• Can students describe the shape of the distribution? • Do students recognise the population distribution as normal?
• How do we determine the centre of a dataset?	• Using mean, median or mode.		• Can students recall the different ways to locate the centre of a dataset?
• What is the centre of the distribution?	• It's centred around 5. • Its mode is 5. • Its mean is 5.1. That was the value written in the envelope.	• Demonstrate the range of the population on the population distribution on the board.	
• How do we measure the spread of a dataset?	• Using standard deviation, interquartile range or range.		• Can students recall the different ways to measure the spread of a dataset?
• What is the spread of the distribution?	• Its values range from just above 0 to just below 10. • I need to know the values in the dataset to calculate the standard deviation. • The interquartile range is ≈ 3.		
• Using *GeoGebra*, I can quickly calculate the standard deviation of		• In the *GeoGebra* file "**TheDistributionOfSampleMeans.ggb**" click on the button "Show Parameters" to reveal the population mean and the population standard deviation.	• Do students recall that σ represents the standard deviation of a population?

the distribution. It is 2.57. • There is shorthand notation to represent the standard deviation of a population and that is the Greek letter σ. • What name could you give to this distribution?	• The distribution of all schoolbag weights. • The population distribution.	• Write the term "Population Distribution" in the word-bank.	• Do students recognise that this is the population distribution?
• In **Section A** we concluded that, while the means of individual samples are not equal to the population mean, they do approximate the population mean. Let's use the sample means calculated by each group to construct a histogram of the sample means. This will provide a visual of the distribution of sample means. • Does the distribution of the each group's sample means back up what we already know	• Yes – we know that different samples give different means and the distribution demonstrates this. • Yes – all the answers are different. • Yes – the distribution demonstrates sampling variability.	• Highlight the results from each group on the board. • In the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", type each group's mean and standard deviation into the input boxes, clicking "Submit your Sample" each time. • Explain that this is similar to the number-line plot we used in **Section A**, only this time we are visualising them using a histogram. • Point out that the sample means calculated by the class are close to the population mean.	• Do students recognise that the distribution of sample means demonstrates sampling variability? • Do students recognise that the sample means are close to the population mean?

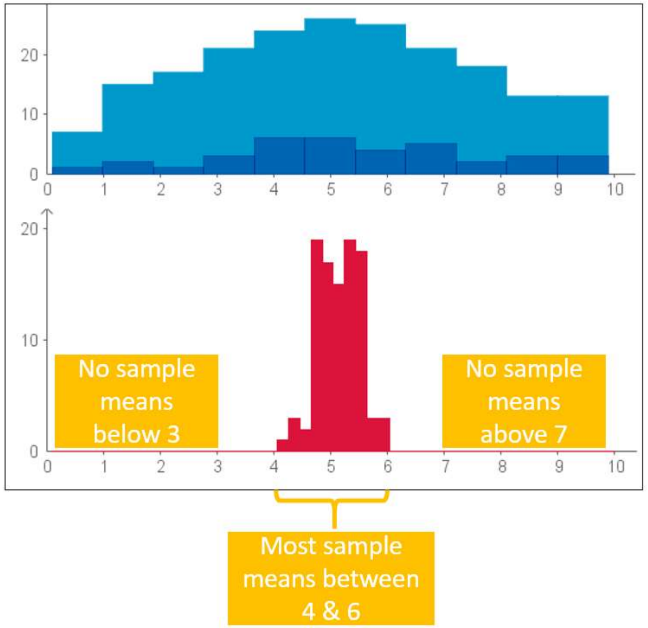
about sampling? Explain.	• Yes – all the answers are close to each other. • Yes, while the answers all vary, they're all close to the answer we're looking for - the population mean of 5.1.		
• To create this distribution we used each group's sample mean. • We called the first distribution the population distribution. What should we call the second distribution?	• The mean distribution. • The sample distribution. • The distribution of the sample means. • The sample-means distribution.		• Do students understand that the second distribution is made up of sample means? • Do students understand why the second distribution is known as the distribution of sample means?
• It is known as the distribution of the sample means because it is made up of the means of different samples.		• Write the different names given to the distribution of sample means in the word-bank. Encourage students to write each term in their journal with an appropriate description.	
• It is often called the sampling distribution of the mean or the	• Flat. • It's hard to tell.	• Use a marker to "throw a rope" over the distribution of sample means which is projected onto the board.	• Do students recognise that it is difficult to discern a shape to the

sampling distribution for short. • What shape is the distribution of the sample means?	• There aren't enough points in it to tell its shape. • We need more points to tell. • It doesn't have a shape.		distribution given the small number of data items?
• It is difficult to discern the shape of the distribution since we only have the few values we calculated in class to visualise it. • We have also stated that all the sample means are close to the population mean. This is a dangerous assumption since we have only looked at a small number of sample means. • The only reason we have so few sample means in our distribution is due to limited amount of time available. We only had time to take a few samples and calculate their means. Using ICT can get around this problem.	•		• Do students understand that we are only looking at a tiny fraction of all the possible means from samples of size 36? • Do students recognise that taking more samples will give me a better understanding of the distribution of sample means? • Do students understand that our assumption that sample means are close to the population mean is based on a very small number of sample means and so is unreliable? • Do students understand that GeoGebra is being used to create more samples?

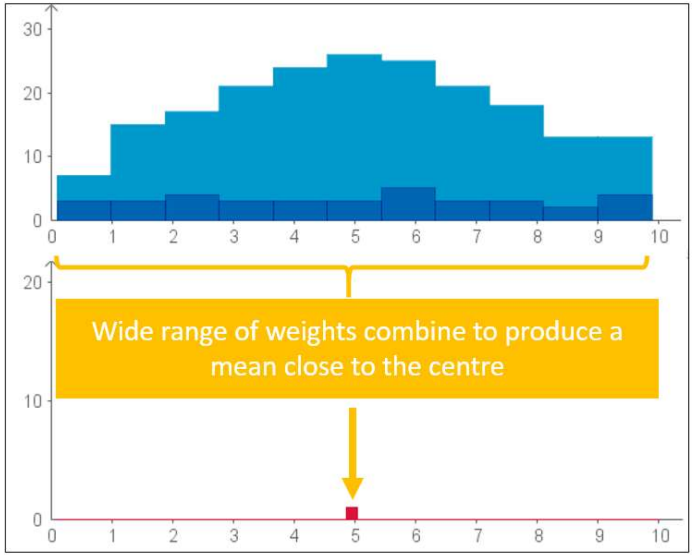
• I am going to use GeoGebra to take more samples of size 36, calculate each sample's mean and include each mean in our histogram. I will continue to do this until our histogram is made up of 100 values so that we might obtain a more complete picture of the distribution of sample means.			• Do students understand that GeoGebra is creating new samples of size 36, calculating the mean of each sample and adding each mean to our histogram?
• Before I do, I would like you to answer Question 2 and Question 3 of **Section B: Student Activity 1**.	• Students work on answering Question 2 and Question 3 of **Section B: Student Activity 1**.	• Move around the room and encourage students to explain the reasoning behind their answers. • Use suitable questioning to aid student understanding. • Encourage students to answer Question 2 and Question 3 by ticking the appropriate option and by marking in their answer on the probability line.	
• How likely it is that I will get a sample mean with a value less than 3? Put another way, how many of the 100 sample means would you expect to have a value less than 3? Explain your reasoning.	• Unlikely. • Fairly unlikely. • No chance. • There's a good few of the population with a value less than 3 so there's a reasonable chance. • Approximately 45 of the population have weights less than 3 kg, so the probability of getting a mean less than 3 is $\approx \frac{45}{200}$ (incorrect).		• Can students correctly predict the likelihood of getting a sample mean less than 3? • Can students explain their reasoning?

	• I'd expect approximately 200 of my samples to have values less than 3. • Lots of the samples should have means less than 3. • When we sampled no one got a sample mean less than 3, so it's unlikely. • There is a very small chance, as to produce a sample mean of less than 3 you would need most of your sample to have values less than 3. If the sample is randomly chosen from the given population this is extremely unlikely.		
• How likely is it that I will get a sample mean with a value of between 4 and 6? Put another way, how many of the 100 sample means would you expect to have a value between 4 and 6?	• There's a really good chance of this happening. • There's a high probability. • Many of the population weights lie in this range so there is a high probability of getting a sample with a mean between 4 & 6. • Approximately $\frac{1}{3}$ of the population lies in this range so I'd expect 33 of my 100 samples to have means in this range. • It is very likely as all of the groups in class got sample means between 4 & 6.		• Can students correctly predict the likelihood of getting a sample mean between 4 and 6? • Can students explain their reasoning?

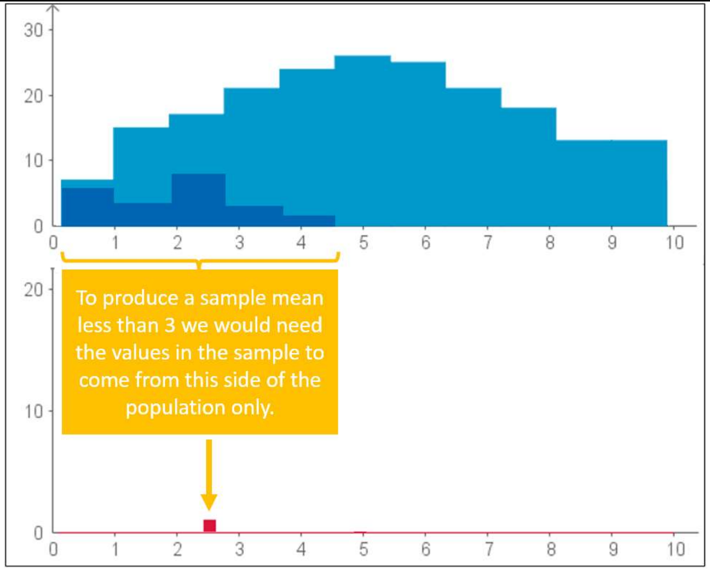
	• It is very likely as in a large enough sample even extreme weights should combine to produce a mean close to 5. • I'd expect some of the sample means to be within this range.		
• Let's see if your predictions are correct. • Based on our simulation, what is the probability of getting a sample mean less than 3? • Is this more or is it less than you predicted? • Based on our simulation, what is the chance of getting a sample mean with a	• Zero. • There's no chance. • It's extremely unlikely. • It's possible but very unlikely. • In 100 samples, none had a mean less than 3. • It's much less than I predicted. • I thought there'd be a 20% chance but it's much smaller than that. • It's as I predicted. • It's almost certain. • It's very likely. • Over 90%. • Over 95%.	• On the *GeoGebra* File "**DistributionOfSampleMeans.ggb**", click on the "Generate 1 Sample" button. • Continue to click on the "Generate 1 Sample" button until the distribution of sample means has 100 values. • Encourage students to compare their predictions from Question 2 and Question 3 with what the simulation demonstrates as you continue to click on the "Generate 1 Sample" button. • Encourage students to use the histogram of the sample means to estimate the probability of getting a sample mean less than 3 and to estimate the probability of getting a sample mean between 4 and 6. • On the distribution of sample means on the board, highlight the lack of sample means in the region below 3.	• Do students recognise that the probability of getting a sample mean with a value less than 3 is virtually zero? • Are students' predictions in agreement with the simulation? • Do students recognise that the probability of getting a sample mean with a value between 4 and 6 is very high?

value between 4 and 6?	• Almost all the sample means are between 4 and 6.		
• Is this more or is it less than you predicted?	• It's much more. • I thought there was a 36% chance of a sample mean between 4 and 6 but the probability is much higher than this. • It's as I predicted.		• Are students' predictions in agreement with the simulation?
• Given that $\approx 20\%$ of the population weights are less than 3, why is it so unlikely to get a sample-mean less than 3?	• I don't know. • To produce a sample mean of less than 3, you would need most of your sample values to be less than 3. The chance of this happening is very small when you take a simple random sample from the population. • It's very unlikely to get 36 values in your sample which will result in a mean of less than 3.	• On the *distribution of sample means* on the board, highlight the presence of almost all sample means in the region between 4 & 6. • On the *GeoGebra* File "**DistributionOfSampleMeans.ggb**", click the "Reset" button. • On the *GeoGebra* File "**DistributionOfSampleMeans.ggb**", click the "Generate 1 Sample" button. • Encourage students to look at the population values which make up the sample (these are shown in the	• Do students understand that to generate a mean of less than 3, you would need the vast majority of the values in your sample to have a value less than 3? • Do students understand that it is highly unlikely that most of the values in a sample of 36 which is drawn from this population will have a value less than 3?

dark blue histogram superimposed on the light-blue population histogram) and how these combine to produce the resulting sample mean.



- Encourage students to then think about the 36 weights which would be needed to produce a mean of 3 or less.
- Encourage students to write out a list of weights (maybe not as many as 36) which would give a mean weight less than 3.
- If required, write a list of weights on the board which have a mean less than 3.
- If required, sketch what a sample with a mean value less than 3 looks like.

• Given that $\approx 36\%$ of the population weights are between 4 and 6, why is the probability of getting a sample-mean with a value between 4 and 6 so high?	• There are loads of ways to produce a sample mean with a value between 4 and 6. • It's not just the values between 4 and 6 which can produce a mean between 4 and 6. For example a weight of 2 and a weight of 8 produce a mean weight of 5. • It is likely that our sample will contain values below 4 and above 6 but when these values combine they produce a mean between 4 and 6. • Even if our sample contains a value below 4 it is likely, due to our sample being random, that there will be a value in our sample greater than 6. These two "extreme" values combine to produce a mean value between 4 and 6. • Even if our sample contains values outside this range it is likely that some will be above the range and some below. Overall these pairs of values are likely to combine to produce a mean value inside this range.	 • On the *GeoGebra* File "**DistributionOfSampleMeans.ggb**", click the "Reset" button. • On the *GeoGebra* File "**DistributionOfSampleMeans.ggb**", click the "Generate 1 Sample" button. • Encourage students to look at the population values which combine to produce a sample mean between 4 and 6 (these are shown in the dark blue histogram superimposed on the population histogram). • Highlight the presence of extreme values in the sample and how there are similar numbers of extreme values on either side of the mean. • Encourage students to then think about the 36 weights which would be needed to produce a mean of between 4 and 6.	• Do students understand that to produce a mean weight between 4 and 6, you don't need all 36 weights to be between 4 and 6? • Do students understand that this property of means results in a really high probability of getting a sample mean between 4 and 6? • Can students verbalise why the probability of a sample mean between 4 and 6 is so high?

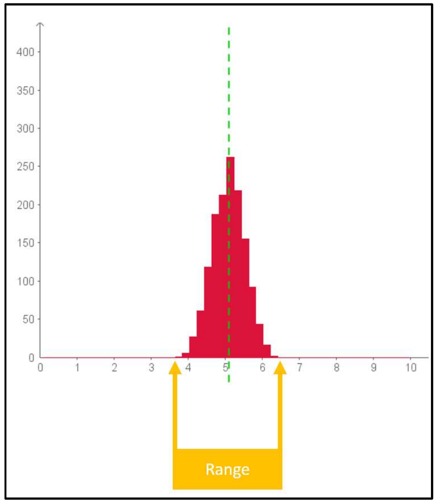
		• Encourage students to write out a list of weights (maybe not as many as 36) which would give a mean weight between 4 and 6. • If required, write a list of weights on the board which have a mean between 4 and 6.	
• 100 sample means is still a small number. Let's increase this to 1000 sample means. • Does our enlarged distribution reinforce what we've just discovered about the number of sample means below 3 and between 4 and 6? • While our distribution of 1000 sample means is just a small fraction of all the possible samples from this population, it reveals some really important facts. To help us identify this information I would like you to answer Question 4, Question 5 and Question 6 of	• Yes. • Students work on answering Question 4 and Question 5 of **Section B: Student Activity 1**.	• On the *GeoGebra* File "**DistributionOfSampleMeans.ggb**", click the "Generate 100 samples" button nine times to produce a distribution consisting of 1000 sample means. • Move around the room to ensure students are on task. • Use suitable questioning to help students complete the task.	• Do students recognise that the enlarged distribution also demonstrates how unlikely it is to get a sample mean with a value less than 3? • Do students recognise that the enlarged distribution also demonstrates how likely it is to get a sample mean with a value between 4 and 6?

Section B: Student Activity 1. • What shape is the distribution of sample means?	• It's normal. • It's symmetric. • It's Gaussian. • It has a single mode. • Bell-shaped.	• Use a marker to "throw a rope" over the distribution of sample means. • Under **Section B** of the board plan add the following: "The distribution of sample means is normal".	• Do students recognise that the distribution of sample means is normal?
• Where is the centre of the distribution of sample means?	• Around 5. • Its mean is approximately 5. • Just above 5. • The same as the population mean. • It has a mode around 5. • It's centred on a value just above 5.	• Point to the centre of the distribution of the sample means on the *GeoGebra* file "**DistributionOfSampleMeans.ggb**".	• Can students estimate the centre of the distribution of the sample means?
• Instead of estimating, could we actually calculate the centre of the distribution of the sample means?	• Yes. • We could find the mode of the sample means. • We could find the median of the means. • We could calculate the mean of all the sample means. • We could add up all the sample means and divide by the number of sample means. • We could find the average of the sample means. • The mean of the means. • The mean of the sample means.	• Write the term "mean of the sample means" in the word-bank on the board.	• Can students describe how to find the centre of the distribution of sample means?

• We will choose the mean as our measure of the distribution's centre. Given that we already have a population mean (μ) and a sample mean ($\bar{x}$), what should I call this mean? • We call the centre of the distribution of the sample means the **mean of the sample means**. It is denoted by the symbol $\mu_{\bar{x}}$. • Can you explain why $\mu_{\bar{x}}$ is used to represent the mean of the sample means? • Using *GeoGebra*, I can calculate that $\mu_{\bar{x}} \cong 5.12$. This tells me that the centre of the distribution of sample means is 5.12. • Is there a relationship between the centre of the population distribution (μ) and	• The mean of all the sample means. • Yes – μ means mean and $\bar{x}$ means the mean of a sample so $\mu_{\bar{x}}$ means the mean of the sample means. • Yes. • Yes. They're approximately equal. • They're the same.	• Add the symbol $\mu_{\bar{x}}$ to the appropriate place in the word-bank. • On the *GeoGebra* file "**DistributionOfSampleMeans.ggb**" click on the button "Show Statistics" to reveal the mean of the distribution of the sample means. • On the *GeoGebra* file "**DistributionOfSampleMeans.ggb**" click on the button "Align Means" to visualise the fact that $\mu_{\bar{x}} = \mu$.	• Can students come up with the term "mean of the sample means" themselves? • Do students understand that the mean of the sample means is the centre of the distribution of sample means? • Do students understand that $\mu_{\bar{x}}$ means the mean of the sample means and the centre of the distribution of the sample means? • Can students explain why $\mu_{\bar{x}}$ is used to denote the mean of the sample means? • Do students recognise that the mean of the sample means is equal to the population mean?

the centre of the distribution of the sample means ($\mu_{\bar{x}}$)? Explain • Does this tell me anything useful in relation to the sampling process? Explain. • So it seems that when we sample, even	• The mean of the sample means equals the mean of the population. • $\mu_{\bar{x}} = \mu$. • Not really. • Yes. • Yes – it tells me that the means I get when I sample tend to be close to the population mean. • Yes it tells me that while different samples give me different sample means, the values tend to be centred on the population mean. • Yes it confirms what we suspected from our own sample means: that the sample means are roughly the same as the population mean. • Yes it tells me that most of the time the mean I get from a sample will be in or around the population mean. • We're trying to use a single sample mean to make a statement about a population mean and this result suggests we might be able to.	• Under **Section B** of the board plan write the following: "The mean of the sample means equals the mean of the population $\mu_{\bar{x}} = \mu$ (while different samples produce different means, these means tend to be close to the population mean)"	• Can students explain what $\mu_{\bar{x}} = \mu$ means in terms of sampling variability? • Do students understand that $\mu_{\bar{x}} = \mu$ says that, while sampling

though different samples give different means, all of these means are centred on the mean of the population. • Another way of saying this is that the mean of the sample means is equal to the population mean. • While we have only demonstrated that $\mu_{\bar{x}} \cong \mu$, using 1000 samples, there is a theorem in mathematics which says that $\mu_{\bar{x}} \equiv \mu$, provided certain conditions are satisfied. (**The central-limit theorem**) • This is confirmation that while sample means do not necessarily equal the population mean they are close to it. • With this confirmed, let's turn our attention to understanding how		• Encourage students to add this to the notes in their own copybooks. • Using the *GeoGebra* file "**DistributionOfSampleMeans.ggb**" highlight the fact that, while there are lots of different sample means, they all seem to hang around the population mean.	variability produces different means from different samples, these sample means hang around the population mean? • Do students understand the importance of this result in terms of our original question of using a single sample to make a statement about a population? •

much a sample mean is likely to vary from the population mean.			
• Is there a way in which we could measure how much the different sample means vary from the population mean?	• Yes we could measure the spread of the sample means. • We could use the range of the sample means. • We could calculate the standard deviation of all the sample means we got. • We could use standard deviation as the sample means are distributed normally.	• Use the GeoGebra file "**DistributionOfSampleMeans.ggb**" to highlight the range of the data. 	• Can students describe how to measure the amount of variation in a distribution?
• What is the spread on the distribution of sample means?	• Its range is from ≈ 3.6 to ≈ 6.4. • Its spread is small compared to the population. • It has a small standard deviation. • It's much less than the standard deviation of the population.		• Can students estimate the range of the sample means?

• We will use standard deviation as our measure of spread since we are dealing with a symmetric distribution and standard deviation is a good measure of spread in such cases. • If I wanted to calculate the standard deviation of the distribution of sample means using my calculator, what values would I need to input? • I can do this calculation using *GeoGebra* and when I do I get a standard deviation of 0.43. Given that we already have the standard deviation for our population (σ) and the standard deviation for a single sample (s),	• The sample means. • The means of all the samples we took. • All the sample means. • The standard deviation. • The standard deviation of means. • The standard deviation of the sample means.	• Write the term "standard deviation of the sample means" in the word-bank on the board. • Highlight the different notations already on the board. • Add the notation $\sigma_{\bar{x}}$ at the appropriate location in the word-bank.	• Can students recognise that they don't have enough information to calculate the standard deviation of the distribution of the sample means? • Can students describe the information they would need to calculate the standard deviation of the distribution of the sample means? • Can students come up with the term "standard deviation of the sample means" by themselves?

<table>
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<td>what should we call this standard deviation?</td>
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- Could you suggest suitable notation to represent the standard deviation of the sample means? Explain your choice.

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- $\sigma_{\bar{x}}$.
- $\sigma_{\bar{x}}$ - because σ means standard deviation and $\bar{x}$ means the mean of a sample.
- $\sigma_{\bar{x}}$ – the standard deviation of the sample means.

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- On the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", highlight the value of the population standard deviation and the standard deviation of the sample means.

<table>
<tr><td></td><td>Population</td></tr>
<tr><td>N</td><td>300</td></tr>
<tr><td>μ</td><td>5.12</td></tr>
<tr><td>σ</td><td>2.57</td></tr>
</table>
<table>
<tr><td>Sample means</td><td></td></tr>
<tr><td>Sample size</td><td>36</td></tr>
<tr><td>No. Samples</td><td>1000</td></tr>
<tr><td>$\mu_{\bar{x}}$</td><td>5.12</td></tr>
<tr><td>$\sigma_{\bar{x}}$</td><td>0.43</td></tr>
</table>

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- Can students come up with the correct notation to denote the standard deviation of the distribution of the sample means?
- Do students understand that $\sigma_{\bar{x}}$ is the standard deviation of the sample means?

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- Previously we saw that there is a relationship between the population mean and the mean of the sample means. Is there an equivalent relationship between $\sigma_{\bar{x}}$ and σ? Explain.

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- No.
- They're definitely not equal.
- There doesn't seem to be.
- The standard deviation of the sample means is a lot less than the standard deviation of the population.

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- Use the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", to make the connection between the value of $\sigma_{\bar{x}}$ and the width of the distribution of sample means and between the value of σ and the width of the population distribution.

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- Do students recognise that $\sigma \neq \sigma_{\bar{x}}$?
- Do students recognise that $\sigma > \sigma_{\bar{x}}$?

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- Why is $\sigma_{\bar{x}}$ less than σ?

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- The distribution of sample means is much narrower than the population distribution.
- Because when we take a large enough sample from the population, the chance of getting a sample mean far

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- Use the GeoGebra file "**DistributionOfSampleMeans.ggb**" to remind students why the sample means are clustered so closely together. To do so click the "Generate 1 sample" button and highlight how extreme values from the population still produce a sample mean close in value to the population mean.

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- Can students explain why $\sigma_{\bar{x}}$ is less than σ?
- Do students understand why the majority of $\bar{x}$ values are close to one another?
- Do students understand that when an $\bar{x}$ value is calculated from a

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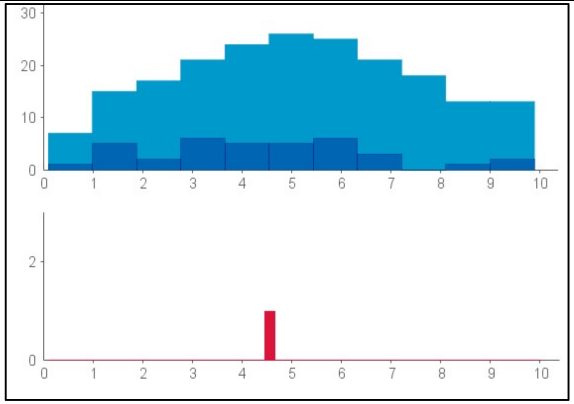
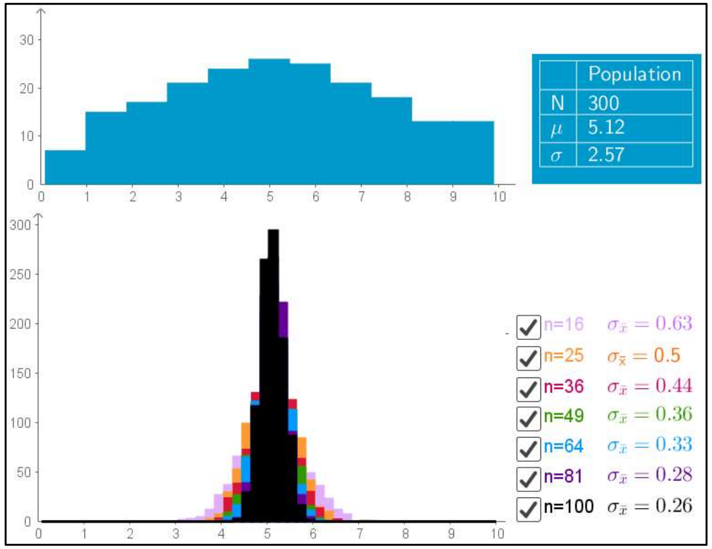
	away from the population mean is small. • Because when we take a simple random sample from the population the extreme values (on either side of the mean) in our sample tend to combine to produce a mean close to the centre of the distribution. Because of this our sample means tend not to vary too much. • There is much less variation in the sample means compared to the values in the population. • The sample means vary much less because each value is an average of several population values.		large-enough sample, extreme values tend to combine to generate an $\bar{x}$ value close to $\mu_{\bar{x}}$?
• So $\sigma_{\bar{x}}$ is a fraction of the size of σ – can we quantify this? Let's look at the ratio of $\sigma : \sigma_{\bar{x}}$. • To the nearest integer, what is the ratio of $\sigma : \sigma_{\bar{x}}$?		• Encourage every student to calculate the ratio $\sigma : \sigma_{\bar{x}}$. • Encourage students to verbalise the relationship between $\sigma\ and\ \sigma_{\bar{x}}$.	• Can students calculate the ratio $\sigma : \sigma_{\bar{x}}$ to the nearest integer?
• Can you express this relationship in words?	• 6. • $\sigma_{\bar{x}}$ is six times smaller than σ. • $\sigma_{\bar{x}}$ is one sixth the size of σ.		• Can students describe what this result means for the relationship between $\sigma\ and\ \sigma_{\bar{x}}$?

• I asked you to calculate the ratio of to the nearest integer because our value is only an estimate of the true ratio since we only used 1000 samples. If we were to use all the possible samples of size 36 from the population of 200 we would find that the ratio of $\sigma : \sigma_{\bar{x}}$ is exactly 6 – without rounding. (As predicted by the central-limit theorem).	• σ is six times bigger than $\sigma_{\bar{x}}$. • If I knew σ then to find $\sigma_{\bar{x}}$ all I'd need to do is divide by 6.		• Do students appreciate that, while $\sigma : \sigma_{\bar{x}} \cong 6$ this is only an approximation based on 1000 sample means, in reality $\sigma : \sigma_{\bar{x}}$ is equal to 6 for this case. • Do students understand that our approximation is due to the fact that we're only examining a small portion of the samples in the distribution of the sample means?
• The ratio of $\sigma : \sigma_{\bar{x}}$ is 6 but will it always be 6 or does the ratio change. In other words is there a way to predict the relationship between σ and $\sigma_{\bar{x}}$ without having to calculate $\sigma : \sigma_{\bar{x}}$?	• I don't know. • I'm not sure.		
• A little more GeoGebra might help us to fully understand how $\sigma_{\bar{x}}$ and σ are			

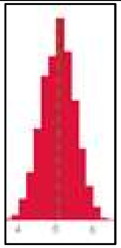
related. Along with my 1000 samples of size 36 I am going to simulate 1000 samples of size 16, 1000 samples of size 25, 1000 samples of size 49, 1000 samples of size 64, 1000 samples of size 81 and 1000 samples of size 100. • As I do so I want you to record the corresponding values of σ and $\sigma_{\bar{x}}$ and then calculate the ratio of $\sigma : \sigma_{\bar{x}}$. Fill your results into the table in Question 1 of **Section B: Student Activity 2**.	• Students fill in the table in **Section B: Student Activity 2** calculating the ratio as they do so.	• Distribute **Section B: Student Activity 2** to students. • In the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", click on the button "Compare Distributions", then click off the check box "n=36" to hide the distribution of samples means for size n=36. • In the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", click on each checkbox in sequence starting with n=16 and ending with n=81. • Highlight the values of σ and $\sigma_{\bar{x}}$ which students should record in their table. • Encourage students to complete each row of the table as each checkbox is clicked. 	• Can students fill in the table correctly? • Can students calculate $\sigma : \sigma_{\bar{x}}$ correctly for each example?
• Now, working in pairs, I want you to answer Question 2 – Question 7 of **Section B: Student Activity 2**.	• Students answer Question 2 – Question 7 of **Section B: Student Activity 2**.		• Can students answer Question 2 – Question 7 of **Section A: Student Activity 2** correctly?

• Can you describe what is happening to the value of σ as the size of our 1000 samples increases?	• It’s the same. • It’s not changing. • It’s constant.	• Encourage students to discuss their answers to Question 2 – Question 7 of **Section B: Student Activity 2**. • Circulate to ensure students are on task. • Use suitable questioning to help students progress their thinking.	• Do students understand why the value of σ is constant?
• Why is this so?	• Because we’re dealing with the same population all the time. • We’re only changing the sampling but the population is the same. • The population is the same.	• In the *GeoGebra* file “**DistributionOfSampleMeans.ggb**”, click off each of the check boxes. Click on the n=16 checkbox and then the n=81 checkbox and highlight the difference in the spread of each distribution.	
• Can you describe what is happening to $\sigma_{\bar{x}}$ as the size of our 1000 samples increases?	• It’s decreasing. • It’s getting smaller.		• Do students recognise that the value of $\sigma_{\bar{x}}$ is decreasing as we move down the table?
• Can you describe the relationship between $\sigma_{\bar{x}}$ and sample size?	• As one gets bigger the other gets smaller. • As sample size increases, $\sigma_{\bar{x}}$ decreases.		• Can students describe the relationship between $\sigma_{\bar{x}}$ and sample size? • Do students recognise that increasing sample size reduces $\sigma_{\bar{x}}$?
• Do the graphs of the distributions back up this relationship? Explain.	• Yes.	• Encourage students to think about the effect of an outlier on a small sample compared to its effect on a large sample.	• Can students relate changes in $\sigma_{\bar{x}}$ to physical changes in the distribution of the sample means?

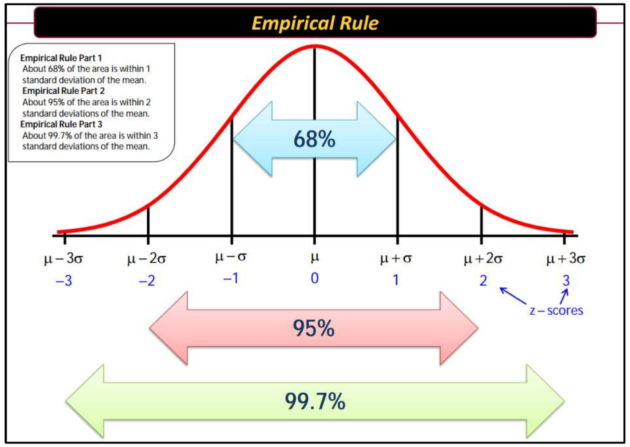
• Why does $\sigma_{\bar{x}}$ get smaller as sample size gets bigger?	• Yes. As sample size increases the width of the distribution decreases. • Yes – as the sample size increases the sample means are closer together. • I'm not sure. • As sample size gets bigger there should be less variation in the mean of the sample. • The more values you have in a sample the less influence a few extreme values will have on the mean. This means you'd expect all the means to be closer together. As a result standard deviation should be smaller. • In a larger sample, individual extreme values will have less influence on the mean.	• In the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", click off the n=16 checkbox to leave the n=81 checkbox clicked. Ask students to predict what will happen when the n=100 checkbox is clicked.	• Can students explain why an increase in sample size results in a decrease in $\sigma_{\bar{x}}$? • Do students understand that in larger samples a small number of extreme values contribute less to the calculation of the mean than they would in a small sample? • Do students understand that the larger a sample is the less influence a single outlier will have on its mean?
• If I were to take 1000 samples of size of 100, what would you expect the distribution of the sample means to look like? Explain your reasoning.	• I would expect it to be narrower than the distributions of smaller sample-size. • I would expect the distribution of sample means to be less spread out.	• In the *GeoGebra* file "**DistributionOfSampleMeans.ggb**", click on the n=100 checkbox. Encourage students to reflect on whether the distribution is as they predicted. • Show the working out of the ratio of $\sigma : \sigma_{\bar{x}}$ on the board.	• Can students predict what the distribution of the sample means will look like for a sample size of 100? • Do students recognise that a large sample size will mean a distribution of sample means which has a smaller spread?

• For a sample size of 100 what will the ratio of $\sigma : \sigma_{\bar{x}}$ be? • Let's see if your prediction is correct. Use the values of σ and $\sigma_{\bar{x}}$ to calculate the value of $\sigma : \sigma_{\bar{x}}$. • Can you now describe the general relationship between σ, $\sigma_{\bar{x}}$ and n? Fill your answer into Question 8 of **Section B: Student Activity 2**. • Can you explain the significance of this	• I would expect the sample means to be closer to the centre of the distribution. • I'd expect a smaller standard deviation as outliers will have less influence on the mean. • I don't know. • 10 • $\sqrt{100}$ • $\sigma = \sqrt{n} \times \sigma_{\bar{x}}$ • $\sigma : \sigma_{\bar{x}} = \sqrt{n}$ • $\frac{\sigma}{\sigma_{\bar{x}}} = \sqrt{n}$ • $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ • The standard deviation of the population is $\sqrt{n}$ times the standard deviation of all the sample means. • The ratio of $\sigma : \sigma_{\bar{x}}$ is $\sqrt{n}$. • The standard deviation of the sample means is $\sqrt{n}$ times smaller than the standard deviation of the population.	• Encourage students to use their table as a guide for writing down the relationship. • If students struggle with writing down the relationship it may be useful to rephrase the question in the following way: "If you knew the value of σ and the sample size (n), how would you calculate the value of $\sigma_{\bar{x}}$? • Encourage students to verbalise the relationship and to write it using mathematical notation. • Write the following under **Section B** of the board plan: "The standard deviation of the sample means is less than the standard deviation of the population and depends on sample size $\sigma_{\bar{x}} = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{n}}}$ $\Rightarrow$ the bigger the sample we use, the closer its mean is likely to be to the population mean. " • Relate this relationship back to the physical properties of the distribution of sample means. • Encourage students to discuss what this relationship means in terms of our ability to use the mean of a single sample to make a statement about the population mean. • Use the distributions for n=16, n=25,…in the *GeoGebra* file "**DistributionOfSampleMeans.ggb**" to relate the relationship $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ back to the ability to use a single sample to make a statement about the population, highlighting the fact that a larger sample size means a	• Can students predict the correct ratio of : $\sigma_{\bar{x}}$? • Can students write down the general relationship between σ, $\sigma_{\bar{x}}$ and n?

relationship in terms of using a single sample to make a statement about a population?	• No. • Using a large sample is a good idea. • If we use a large sample, the mean we get is likely to be close to the population mean. • While the answers from different samples give different sample means, when you use a larger sample it is more likely your sample mean will be close to the population mean. • When sample size is larger, there is less variation between the means from different samples. • Means calculated from different samples will be different but with larger samples this difference will be smaller. • Your sample mean will not equal your population mean but it is more likely to be close to it if you use a larger sample.	greater chance of getting a sample mean closer to the population mean.	• Do students understand that by using a larger sample our sample mean is more likely to be close to the population mean? • Do students understand how this relationship is important when using a single sample to make a statement about the population?
• At the start of this section we set out to		• Use **Section B** of the board plan to highlight the facts we've discovered so far:	• Do students recognise the main facts that we have discovered

address two problems which arose during **Section A**. • Over the course of this section we have confirmed that the means of individual samples differ but that these means are centred on the mean of the population from which they are drawn. • We understand much more about how sample means vary from one sample to the next in that they do so normally around the population mean. • We've also seen that the spread of the sample means is much less than the spread of the population and depends on the spread of the population and the size of our sample.		 ○ The distribution of sample means is normal. ○ $\mu_{\bar{x}} = \mu$. The sample means are centred on the population mean. ○ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. The sample means vary about the population mean. The level of variation depends on the variation in the population and the size of the sample.	about the distribution of sample means?
SECTION C – Building a 95% confidence interval for the mean of a single sample			
• When measuring sampling variability we have, up to now, used			• Do students understand that we have used standard deviation to

standard deviation as our preferred measure. • There is another way to describe the variation in a set of normally-distributed data which is equally important to understand and can sometimes prove more useful.			measure the level of sampling variability? • Do students understand that we are now going to use another measure to quantify the level of sampling variability?
• To understand this approach, I would like you to answer Question 1 – Question 3 of **Section C: Student Activity 1**.	• Students work to answer Question 1 – Question 3 of **Section C: Student Activity 1**.	• Distribute copies of **Section C: Student Activity 1** to all students. • Circulate around the room to make sure students understand what they are meant to do.	
• What shape will the distribution of sample means be? • How do you know this?	• I don't know. • Normal. • Bell-shaped. • Gaussian. • We've already seen that it is normal. • Well any of the distributions of sample means that we've seen have been normal in shape. • I didn't have to. It's the same as the population mean.	• Use the GeoGebra file "**TheDistributionOfSampleMeans.ggb**" to model the sampling in this question. Click "RESET" to clear all previous work. Change the "Default Sample Size" to 100. Generate 1000 samples by clicking on the button "Generate 100 samples" ten times. Rescale the y-axis so that the entire distribution fits on screen. • Highlight the normal shape of the distribution of sample means. • Refer to **Section B** of the board plan to remind students that $\mu_{\bar{x}} = \mu$.	• Do students understand that the distribution of sample means of size 1000 is normal?

• How did you calculate the mean of the distribution of sample means?	• I just took the value of the population mean as they're equal. • We know it's the same as the population mean. • $\mu_{\bar{x}} = \mu$.		• Can students use $\mu_{\bar{x}} = \mu$ to calculate the mean of the sample means?
• How did you calculate the standard deviation of the sample means?	• I used my formula. • I divided the population standard deviation by the square root of the sample size. • $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.	• Refer to **Section B** of the board plan to remind students that $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. • Write up the calculation of the standard deviation of the sample means on the board.	• Can students use $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ to calculate the standard deviation of the sample means?
• To complete the statements in parts (i) – (iii) of **Section C: Student Activity 1**, what prior knowledge of statistics did you use?	• I just estimated by looking at my distribution. • The Empirical Rule. • I used z-scores.	• Add a poster summarising The Empirical Rule to **Section C** of the board plan.  • Write the z-score formula: $z = \frac{\lvert\bar{x}-\mu\rvert}{\sigma}$ under **Section C** of the board plan.	• Do students recall The Empirical Rule and the use of z-scores to make statements about a normal distribution?
• 68% of the sample means should lie	• 2.55 – 7.69 (incorrect). • 4.863 – 5.377. • $\mu \pm \frac{\sigma}{\sqrt{n}}$.	• Encourage students to use the histogram of the distribution of samples means from the GeoGebra file	• Can students apply The Empirical Rule and/or z-scores to calculate the interval which contains 68% of the sample means?

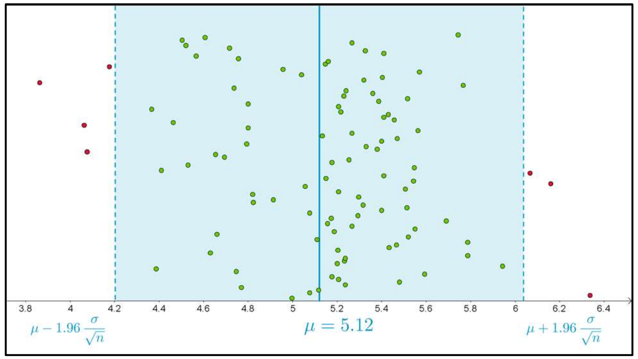
between what two values? • We have disagreement about the result. Which value is correct? How might we check? • So the answer of 4.863 – 5.377 is correct. Can you explain how somebody might get an incorrect answer or 2.55 – 7.69? • So we need to be careful when answering questions about sample means. If we are using **The Empirical Rule** or z-	• 4.863 – 5.377 is correct. • If we look at our sketch of the distribution of sample means we can see that almost 100% of the sample means lie between 2.55 – 7.69 so this cannot be the correct answer. • By using the wrong standard deviation. • By using the standard deviation of the population instead of the standard deviation of the sample means. • By looking at the population instead of the distribution of sample means. • We're asked about the percentage of sample means and not the percentage of the population. So we need to use $\sigma_{\bar{x}}$ in our calculation and not σ.	"**TheDistributionOfSampleMeans.ggb**" to decide which answer is correct. • If no student can describe how the incorrect answer was obtained, it may be useful to call on a student who got the incorrect answer to describe their calculation. • Highlight the distribution of sample means on the board so that students understand that we are answering questions about it and not the population distribution. • Explain that although the formula for calculating a z-score is $z = \frac{\lvert\bar{x}-\mu\rvert}{\sigma}$, the standard deviation here is the standard deviation of the distribution of sample means i.e. $\sigma_{\bar{x}}$.	• Can students use the image of the distribution of sample means to identify which interval is correct? • Can students identify the misconception which leads to an incorrect interval of 2.55 – 7.69? • Do students understand that when asked to make statements about sample means they must use the mean and standard deviation of the sample means and not those of the population distribution?

scores to answer questions about sample means we must make sure that we focus on the distribution of sample means and not the population distribution. Accordingly we must also be careful to use the standard deviation of the sample means ($\sigma_{\bar{x}}$) and not the standard deviation of the population (σ).		• On the poster of The Empirical Rule add in the exact values for the 95% interval and explain that ± 2 only gives an approximation to 95% of the data. • Check that students understand that they are to use z-scores for their calculations and not The Empirical Rule.	• Do students understand that The Empirical Rule is an approximation of z-scores? • Do students understand that the bound on the 95% interval is ± 1.96 and not 2?
• Some of you used The Empirical Rule and some of you used z-scores to get your answers. Which is better?	• The Empirical Rule is quicker. • z-scores. • z-scores – The Empirical Rule is just an approximation. • z-scores are more accurate. • They only differ when looking at 95%.	• Check that students are using the standard deviation of the sample means and not the standard deviation of the population in their calculations. • Check that students are using $\pm 1.96\sigma$ and not $\pm 2\sigma$ in their calculation.	
• For accuracy, from here on we will use z-scores unless otherwise asked.		• Check that students are sketching the distribution of sample means correctly.	• Do students use $\frac{\sigma}{\sqrt{n}}$ as the standard deviation in their calculation of the 95% interval? • Do students use $\pm 1.96\frac{\sigma}{\sqrt{n}}$ as the ends of the 95% interval?
• In light of our discussions, I would like you to go back and	• Students work on revising their answers to Question 2	• Remind students that The Empirical Rule and z-scores may only be used with data which is normally distributed.	

check your answers for Question 2 and Question 3 of **Section C: Student Activity 1**.	and Question 3 **of Section C: Student Activity 1**.		
• Each question in **Section C: Student Activity 1** begins by asking what shape the distribution of sample means is. Why do you think this question was asked each time?	• So we'd know what shape to draw. • Because it's really important. • Everything else we're asked to do is based on the fact that the distribution of sample means is normal. • You can only use z-scores and The Empirical Rule with a normal distribution.	• Write up the calculation of the 95% Confidence Interval on the board. $5.12 \pm 1.96\frac{2.57}{\sqrt{50}}$ $= 5.12 \pm 0.712$ $= 4.408 - 5.832$	• Do students recognise that the construction of our intervals is dependent on the fact that the distribution of sample means is normal?
• In Question 2 (d) (ii) of **Section C: Student Activity 1** you are asked to complete the statement "Before I choose a random sample of size 30…..". What did you write down for this?	• The same as in part (i). • 4.408 – 5.832.		
• Can you explain what this statement means? What does it mean to be 95% confident?	• I don't know. • Before I choose a sample I know there will be a 95% chance that it will have a mean between these two values.		• Do students recognise that we can use the fact that 95% of the sample means are within a certain interval to make a probabilistic statement about a single sample mean?

• In Question 2 (d) (iii) of **Section C: Student Activity 1** you are asked to write down the 95% confidence interval for the sample mean. What did you write down for this? • Given this, can you explain what the "95% confidence interval for a sample mean" means? • It's important to highlight what we've just done in	• Since 95% of all the possible sample means fall between these two values, I can be 95% confident that a single sample will have a mean between the same two values. • The same as in the other two parts of 2 (d). • It's the same as the last part. • 4.408 – 5.832. • It's the range where most of the sample means will lie. • 95% of all the possible sample means will be inside this interval. • Before I choose a random sample of size 50, I know there's a 95% chance it will have a value inside this interval. • When samples are chosen from a population they can all have different means but 95% of them will have means within this interval.	• Write the term "95% confidence interval for a sample mean" under **Section C** of the board plan. • Write the following explanation of the 95% confidence interval under **Section C** of the board plan: "I can be 95% confident that the mean of a single sample will fall within this interval" • Add in "I can be 95% confident that: $\mu - 1.96\frac{\sigma}{\sqrt{n}} \leq \bar{x} \leq \mu + 1.96\frac{\sigma}{\sqrt{n}}$" • Under **Section C** of the board plan write the following statement: "A confidence interval provides a mathematical way to predict a single sample mean using the mean of a population**.**"	• Are students comfortable using inequalities to represent an interval? • Can students explain what a 95% confidence interval means? • Do students understand that we now have a mathematical way to

constructing a 95% confidence interval. Prior to this when we spoke about the relationship between a sample mean and the population mean we used ambiguous language such as “The sample mean is around the population mean” or “The population mean is roughly the same as the sample mean”. By constructing a confidence interval we are now providing a measure of how close a sample mean is likely to be to the population mean. • This was the second of the problems we identified at the start of **Section B**. • It may be useful to visualise the 95% confidence interval in another way. To do so		• Refer to the two problems identified at the start of **Section B**. • Open the GeoGebra file “**The95%ConfidenceInterval.ggb**” • Set the confidence level 95%. • Set the sample size to 100. • Click on the “Show C.I. for a Sample Mean“ button.	relate the mean of a single sample to the mean of a population?

I am going to use GeoGebra. • The shaded area shows the 95% confidence interval for a sample mean. I am going to generate 100 samples of size 100 and plot their means.		• Click on the “Generate 1 sample” button 100 times to generate 100 sample means. 	• Can students predict how many sample means will fall within the 95% confidence interval?
• How many of these samples will have means which fall within the 95% confidence interval?	• Most of them. • 95% of them. • Approximately 95% of them.	• Click on the “Generate 1 sample” button in the GeoGebra file “**The95%ConfidenceInterval.ggb**” to generate a single sample and to see if the sample mean falls within the 95% confidence interval.	• Do students understand that it is unlikely that exactly 95% of the sample means will fall within the 95% confidence interval?
• Why don’t exactly 95% of the sample means fall within the 95% confidence interval?	• We’re only looking at a small number of all the possible sample means. • If we looked at all the possible sample means it would be 95%.	• Refer to The Empirical Rule under **Section C** of the board plan to remind students that it is possible to construct other confidence intervals. • In the GeoGebra file “**The95%ConfidenceInterval.ggb**” change the confidence interval from 95% to 68%. • Click on the button “Show C.I. for a Sample Mean”. • Generate 100 samples by repeatedly clicking the “Generate 1 sample” button or by clicking the “Generate 100 samples” button.	• Can students relate the 95% confidence interval to the probability of a single sample having a mean within the interval? • Do students understand that it’s possible to construct intervals with a different confidence level?
• Before I click the “Generate 1 sample button” in the GeoGebra file, are you certain that its mean will fall inside our interval?	• Yes. • No. • It’s very likely but not certain. • I’m 95% confident. • There’s a 95% chance that it will.		

• We have focused on constructing a 95% confidence interval for a sample mean. Do you think it's possible to construct other types of confidence interval? Explain. • It is possible to construct any confidence interval you like. • Why then have we focused on the 95% confidence interval? • We focus on the 95% confidence interval because it is the most-commonly-used confidence interval in statistical investigations. • For Question 3 of **Section C: Student Activity 1**, what values did you get for your 95% confidence interval?	• Yes. • Yes we could construct a 68% confidence interval or a 99.7% confidence interval. • We could use z-scores to construct any confidence interval we like. • It's less work. • The other confidence intervals are too hard. • It's more important. • It's the one which is most used. • 4.2 - 6.04.	• Highlight the fact that we have focused on the 95% confidence interval under **Section C** of the board plan.	• Do students understand the difference between a 68%, 95% and 99.7% confidence interval in terms of how confident we can be in predicting the value of a sample mean? • Do students understand that we focus on a 95% confidence interval because it is the most commonly-used interval in statistical studies but that it is possible to use z-scores to construct any confidence interval?

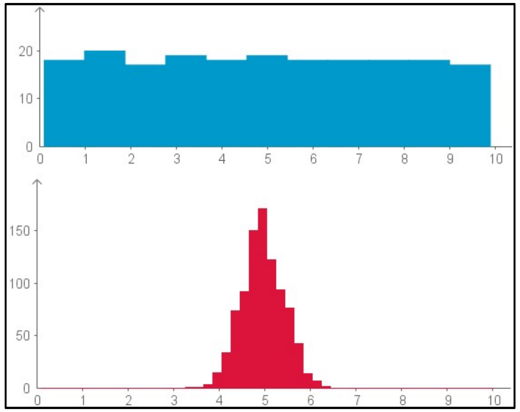
• Is this 95% confidence interval the same as the 95% confidence interval you calculated in Question 2 of **Section C: Student Activity 1**?	• No. • No – it's wider. • This 95% confidence interval is wider.		• Do students understand how sample size affects the width of a confidence interval?
• How can this be, given that we are dealing with the same population? How can the 95% confidence interval be different?	• We're using a smaller sample. • The population is the same but the distribution of sample means is not. • We have a wider distribution of sample means because we are using a smaller sample size.	• Remind students of the calculation of the 95% confidence interval and of the relationship between $\sigma_{\bar{x}}$ and sample size. • Use the GeoGebra file "**TheDistributionOfSampleMeans.ggb**" to generate 100 sample means with a sample size of 50 and repeat with a sample size of 30. Highlight the wider distribution of sample means for a sample size of 30. • Under **Section C** of the board plan, write the following: "As sample size increases the width of a confidence interval decreases".	
• Why does sample size affect the 95% confidence interval?	• The standard deviation of the distribution of sample means is bigger because sample size is smaller. • The confidence interval is constructed using the standard deviation of the sample means. We know that the sample size affects this. • The confidence interval depends on $\sigma_{\bar{x}}$ and we know $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.	• Under **Section C** of the board plan write the following: "Larger sample size gives us a greater ability to predict sample-mean values".	• Do students understand why sample size affects the width of a confidence interval?

• Can you explain why statisticians usually prefer to use as large a sample size as possible?	• In smaller samples there will be greater variation in the means. • Extreme values in a sample will have more of an effect if the sample is small which causes greater variation in the sample means. • Smaller sample size produces a wider distribution of sample means. This increases the width of the 95% interval. • Because the sample means will not vary as much. • The sample means are more likely to be closer to the population mean. • For a single sample chosen from the population I can be more confident that its mean will be closer to the population mean. • 95% of the possible sample means will fall within a shorter distance from the population mean. • Your 95% confidence interval will be smaller so your sample mean is more likely to be closer to the population mean.		• Do students understand that a larger sample size means a narrower confidence interval and a greater accuracy in predicting a single sample mean?
• We looked briefly at three different confidence intervals (68%, 95% and 99.7%).		• Open the GeoGebra file "**The95%ConfidenceInterval.ggb**". • Change the confidence interval to 68%. • Click on the button "Show C.I. for a Sample Mean" and note the range of this interval. • Change the confidence interval to 95%, click on the button "Show C.I. for a Sample Mean" and note the range of this interval. • Change the confidence interval to 99.7%, click on the button "Show C.I. for a Sample Mean" and note the range of this interval. • Under **Section C** of the board plan write the following statement: "The greater the level of confidence, the wider the confidence interval".	• Do students understand that higher confidence means a wider interval and why this is so?

• Looking at these again in Q1 (d) of **Section C: Student Activity 1**, do you notice anything about the widths of the different intervals? • Why does the width of the confidence interval increase as the level of confidence increases? • So we've now discovered another way to describe how the sample means vary about the population mean. • We've seen that we can use a confidence interval to predict the likely value of a single sample mean. • More specifically we've seen that we can use z-scores to construct an interval,	• You can predict a sample mean with greater accuracy. • They're different. • The confidence intervals get wider as the level of confidence increases. • Because of sample size (incorrect). • If you want the interval to capture a higher percentage of the sample means it will need to be wider. • If you want to be more confident that the interval will capture a given sample mean, the interval will need to be wider.	• As you summarise what we've learned about confidence intervals, refer to the various information under **Section C** of the board plan.	• Can students explain why a higher level of confidence requires a wider interval? • Do students have a general sense of what a confidence interval tells us about the mean of a single sample? • Do students understand that a confidence interval provides a way of describing the variation in the means of different samples using the mean of the population?

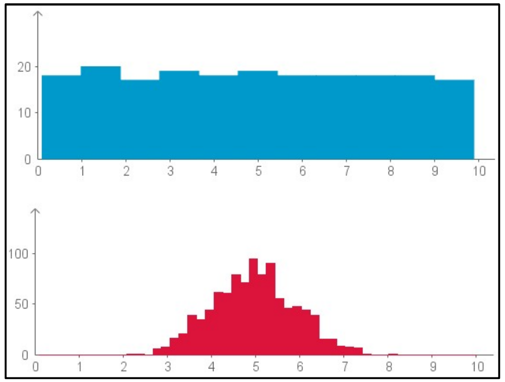
within which we can be 95% confident a single sample mean will reside. This is an improvement on the indefinite language of “close to” and “approximately” we had been using to describe the relationship between a sample mean and the population mean. • We’ve seen that a larger sample size reduces the width of a confidence interval, thereby giving us a greater power of prediction for a single sample mean. • We’ve also seen that as we increase our confidence level the width of our confidence interval must increase.			
• We’ve seen that we can use z-statistics or confidence intervals as another way to describe the variability of sample means and			• Do students recognise that our ability to construct a confidence interval is predicated on the fact that the distribution of sample means is normal?

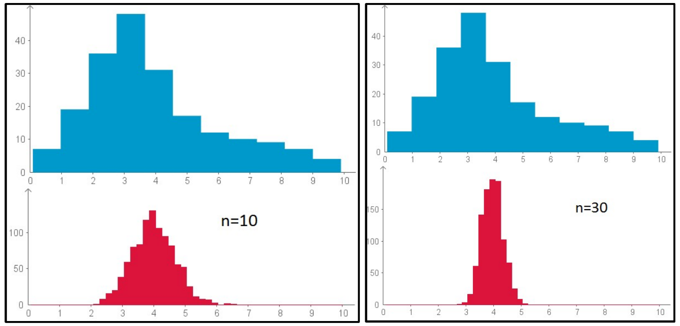
to describe the relationship between the mean of a single sample and the mean of a population. • The ability to construct a 95% confidence interval is based on the fact that the distribution of sample means is normal. If it were not normal we would not have been able to do this in the way we did (using z-scores).			
• It's important to ask the question: why is the distribution of sample means normal? Is it always normal?	• I don't know. • Because our population is normal. • The population from which the samples are drawn is approximately normal so maybe that's why.	• Use the various distributions on the board plan to highlight the fact that all the sampling distributions we have constructed for this population are normal.	• Do students recognise that we don't yet know if the distribution of sample means is always normal or if it's only normal under certain circumstances?
• Is it reasonable to suggest that the reason the distribution of sample means is normal is because the population is normal. Let's investigate.			

• I'm going to change my population distribution from one which is approximately normal to one which is uniform. What shape do you think the corresponding distribution of sample means will be? I'm using a sample size of 36.	• Flat. • Uniform • Normal.	• Open the GeoGebra file "**TheDistributionOfSampleMeans.ggb**". • Click the "Uniform Distribution" button. • Make sure the default sample size is 36.	• Do students understand that a different population may produce a distribution of sample means which is not normal?
• Is the distribution of sample means for our uniform population as you might expect?	• No. • No – it's normal. • No – I suspected it would be uniform.	• Click on the "Generate 100 samples" button ten times to create a distribution of sample means consisting of 1000 values.  	• Do students recognise that the distribution of sample means for a uniform population is not uniform? • Do students recognise that the distribution of sample means for a uniform population is normal? • Do students realise that this is a surprising result?
• How can a uniform population produce a distribution of sample means which is normal?	• I don't know. • It must have something to do with how the values in the samples combine to generate a sample mean. • The different sample means you can get from the population must be normal.		
• In fact, there is a theorem in mathematics which			• Do students understand that we've demonstrated that this distribution of sample means from a uniform population is normal

proves that a population can have any shape but its distribution of sample means will be normal. This is only true provided the sample size is large enough (This is the Central Limit Theorem).			but have not proved that the distribution of sample means is always normal? • Do students understand what the Central Limit Theorem says?
• Why do you think the distribution of sample means is normal only for samples big enough?	• You need a certain number of values in your sample so that one extreme value won’t skew the sample means. • Smaller samples will be more affected by an extreme value which will result in a distribution of sample means which is not normal. • You need enough pairs of values to combine to produce a sample mean close to the population mean.		• Can students offer suggestions as to why the distribution of sample means is normal only if the sample size is large enough? • Do students have a feel for the minimum sample size needed for the distribution of sample means to be normal?
• How big does a sample need to be to produce a distribution of sample means which is normal?	• I don’t know. • It works for 36 so a value close to that.		• Do students understand that a sample size of at least 30 is needed for a population distribution which has any shape to have a distribution of sample means which is normal?
• In fact, for a population distribution of any		• Click the “Reset” button and change the default sample size to 10.	

<table>
<tr>
<td>shape, statisticians choose 30 as the minimum size a sample can be so that the distribution of sample means is normal.

• While this is true for any population, if a population distribution is normal, the corresponding distribution of sample means will be normal</td>
<td></td>
<td>• Click on the “Generate 100 samples” button ten times to create a distribution of sample means consisting of 1000 values.

• Use a marker to “throw a rope” over the distribution of sample means, thereby highlighting its asymmetry and the fact that it is not normal.

• Repeat the above steps using a skewed population to reinforce the fact that irrespective of the population distribution, once you choose a large enough sample, the distribution of sample means will be normal.


• Click the “RESET” button on the GeoGebra file.
• Change the population distribution to “NORMAL”.
• Change the sample size to “10”.</td>
<td>• Do students understand that the distribution of sample means of a normal population distribution will be normal for sample sizes less than 30?</td>
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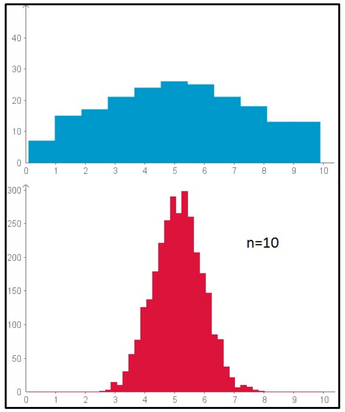
for sample sizes much smaller than 30. • In light of our new knowledge I would like you to answer Question 4 of **Section**	• Students complete Q4 of **Section C: Student Activity 1**.	• Click on the “Generate 100 samples” 30 times to produce a distribution of samples means with 2000 values in it. • Highlight the normal shape of the distribution of sample means with a sample size of only 10.  • Under **Section C** of the board plan write the following statements: “The distribution of sample means is normal for any population, provided the sample size is 30 or more”. “The distribution of sample means is normal for sample sizes less than 30 if the population is normal” • Encourage students to make their own notes summarising the board plan. • Move around the classroom to ensure that all students understand what they are meant to be doing.	• Do students understand that even though the population distribution is positively skewed, because the sample size exceeds 30, the

C: Student Activity 1. You may work in pairs. • What shape will the distribution of sample means be? Why is this? • Why won't John's distribution of sample means be normal? • What range of values did you get for the 95% confidence interval? • What does the 95% confidence interval tell us?	• Normal. • Bell-shaped. • Normal because our sample size is large enough. • Normal, because the distribution of sample means will always be normal once the sample size is above 30. • Because his sample size is too small. • Because his sample size is 20, which is less than the cut-off for normality. • 3.39 – 4.55. • It tells us where the mean of a sample is most likely to be. • It tells us where 95% of all the sample means will lie. • It tells us where 95% of all the sample means from samples of size 50 will lie. • It tells us that before we take a sample there is a 95% probability that its mean will fall within this interval. • You can only work out a 95% confidence interval for a	• Write up the calculation of the 95% confidence interval on the board. • If student need reminding of the meaning of a 95% confidence interval, use the GeoGebra file "**The95%ConfidenceInterval.ggb**" to do so.	corresponding distribution of sample means is normal? • Do students understand that because the sample size is less than 30 and the population is not normal, the corresponding distribution of sample means will not be normal? • Can students calculate the 95% confidence interval for a sample size of 50? • Can students explain the meaning of the 95% confidence interval?

• Why do you think you weren't asked to work out the 95% confidence interval for John's sample?	normal distribution and John's distribution of sample means is not normal. • John's sample size means that his distribution of sample means will not be normal. So we cannot use z-scores to construct a confidence interval.		• Do students understand that you cannot use z-scores to construct a 95% confidence interval if the distribution of sample means is not normal?
• We have discovered a lot of new statistics which are very important. • Let's take some time to summarise what we've learned. • I want you to work in pairs to answer **Section C: Student Activity 2**. • I want you to now check that you matched up each set correctly. • What information did you use to match the	• Students work on answering **Section C: Student Activity 2**.	• Distribute copies of **Section C: Student Activity 2** to all students. • Walk around the room to check that all students understand what they are meant to be doing. • If students are having difficulties, use suitable questioning to help them. • Use the board plan to highlight information which is important for answering **Section C: Student Activity 2**. • Display the solutions to the matching activity on the board. • Use the board plan to highlight the specific facts that allow students to correctly match up each set.	• Can students use their newly-acquired statistical information to correctly match one element from each set? • Can students use logic to sort their way through the matching activity? • Can students verbalise how they are matching each element to one another?

distributions of sample means (set B) to the population distributions (set A)?	• The fact that both distributions are centred on the same value. • $\mu_{\bar{x}} = \mu$. • The mean of the sample means is equal to the population mean.		• Can students identify the relationships which allow them to match Set B to set A? • Do students recognise that there may be different relationships which allow them to match elements of one set to elements of another?
• How did you match the population mean and population standard deviation (set C) to sets A & B?	• By inspecting where the centre of each distribution is. • By comparing the population mean to the centre of each distribution.		• Can students identify the relationships which allow them to match Set C to the already-matched sets?
• What information did you use to match the mean of the sample means and the standard deviation of the sample means (set D) with sets A, B and C?	• I compared the means of the sample means to the centre of the sampling distributions. • I compared the means of the sample means to the population means. • If there were different sampling distributions with the same mean I used the sample size to decide one was the correct match.		• Can students identify the relationships which allow them to match Set D to the already-matched sets?
• How did you match set E to the already-matched sets?	• In the case of sampling distributions with the same mean but different standard deviations I matched the narrower distribution with the standard deviation which used the larger sample size.		• Can students identify the relationships which allow them to match Set E to the already-matched sets?

	• I matched the ones with the same value for $\mu_{\bar{x}}$.		
	• In the case of two elements of set E having the same value for $\mu_{\bar{x}}$ I worked out the standard deviations of set D and chose the one with the matching value of $\sigma_{\bar{x}}$.		
	• In the case of two elements of set E having the same value for $\mu_{\bar{x}}$ I decided which to use by matching the wider sampling distribution to the higher value for $\sigma_{\bar{x}}$.		
• How did you match set F to the already-matched sets?	• I worked out the 95% confidence interval using the values of $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ already chosen. • I estimated the 95% confidence interval by inspecting the distribution of sample means. • I matched the element of set F which said it was not possible to construct a 95% confidence interval with the set which had a non-normal distribution of sample means.		• Can students identify the relationships which allow them to match Set F to the already-matched sets?
• How did you match the sample size (set G) to the already-matched sets?	• Sample size appears in the calculation of $\sigma_{\bar{x}}$ so I matched the correct sample size with the correct element of set D.		• Can students identify the relationships which allow them to match Set G to the already-matched sets?

• What information did you use to match set H with the already-matched sets?	• Since sample size affects the width of the distribution of sample means I matched smaller sample sizes to wider sampling distributions. • Since sample size affects the width of the 95% confidence interval I matched smaller sample sizes with wider confidence intervals. • Set H shows the calculation of the 95% confidence interval so I worked out each one and matched them to the correct set by comparing them to set F. • The value of $\sigma_{\bar{x}}$ appears in the calculation of the 95% confidence interval so I compared set H to set D to make the correct maths.		• Can students identify the relationships which allow them to match Set H to the already-matched sets? • Do students see the many relationships which exist between the distributions of sample means and a population distribution?
SECTION D – Building a 95% confidence interval for a population mean			
• We started this lesson with a simple question: "What is the average weight of an Irish-secondary-school student's schoolbag? " • As we answered this question we progressed through	•	• Refer to the relevant information in **Section A** of the board plan.	• Do students recall what the original aim of our statistical investigation was? • Do students recall the problem of sampling variability?

the data-handling cycle but discovered a problem with using a single sample to answer a question about a population, namely that of sampling variability. • We saw that, because of sampling variability, it is not correct to infer that the mean of a population is equal to the mean of a sample. • We saw that we could get around this problem somewhat by using indefinite language such as "The population mean is roughly the same as the sample mean" but agreed that this is too subjective. • In an attempt to come up with a more precise way to describe the variation in the sample means, we carried out an in-depth investigation of the different sample		• Refer to the relevant information in **Section B** of the board plan.	• Can students describe in their own words what sampling variability is, why it happens and its implications for statistical investigation? • Do students understand $\mu \neq \bar{x}$? • Do students understand that using the language of approximation we can still relate μ and $\bar{x}$ but that we need to do this mathematically? • Do students recall the key relationships which exist between a population and its distribution of sample means?

means that one can get from a population. In doing so we discovered many useful facts about the distribution of sample means. • Ultimately we learned that, while different samples will produce different means and we cannot predict the values of these, we can construct an interval in which a sample mean is very likely to reside. • The interval we chose to construct is a 95% confidence interval. That is, while different samples will produce different means we can be 95% confident that any sample will have a mean that falls within this interval. • So we used a confidence interval to move from a subjective statement "The sample mean is **roughly the same** as		• Refer to the relevant information in **Section C** of the board plan.	• Do students understand that a 95% confidence interval gives us a mathematical way to relate the mean of a single sample to the mean of a population? • Do students understand that we can create any confidence interval we like but that we choose to focus on a 95% confidence interval?

the population mean" to a mathematical statement "I am **95% confident** that a sample mean lies within $\pm\mathbf{1.96}\boldsymbol{\sigma_{\bar{x}}}$ **of the population mean**". • This is a statement relating the mean of a single sample to the mean of a population. We are now going to adapt it to do what we originally set out to do – to make a fair statement about the mean of a population using the mean of a single sample. • In other words we are going to learn how to write the relationship we developed in **Section C** in reverse.			• Do students recognise that our 95% confidence interval relates the mean of a single sample to the mean of a population whereas we want to relate the mean of a population to the mean of a single sample? • Do students understand that we need to reverse our relationship in order to accomplish what we first set out to do?
• Let's return to our original investigation. If you recall we had a simulated population of schoolbag weights with a specific mean and standard			• Do students recall the original investigation?

deviation. ($\mu = 5.12, \sigma = 2.57$). • I would now like you to construct a 95% confidence interval for the mean of a single sample of size 36, drawn from this population. • What range of values did you get for the 95% confidence interval? • What does this interval mean? • During **Section A** of this lesson each group calculated their own sample mean. When we plot these we can see that, as expected, most of them fall within our 95%	• Students complete the calculation for a 95% confidence interval for a sample of size 36. • $4.28 - 5.96$ • $4.28 \leq \bar{x} \leq 5.96$ • 5.12 ± 0.428 • $5.12 \pm 1.96 \frac{2.57}{\sqrt{36}}$ • 95% of all possible sample means of size 36 will fall within this interval. • Before I choose a single sample of size 36, I can be 95% confident its mean will fall within this interval. • I can be 95% confident that the mean of a single sample will fall within 5.12 ± 0.428.	• Under **Section D** of the board plan, write up the 95% confidence interval for a sample of size 36. • Open the GeoGebra file "**The95%ConfidenceInterval.ggb**" • Set the confidence interval to 95%. • Set the sample size to 36. • Click on the "Show C.I. for a Sample Mean" button. • Refer to the list of sample means written under **Section A** of the board plan. • Use the "Input Sample" button in the GeoGebra file "**The95%onfidenceInterval.ggb**" to input each group's sample mean and sample standard deviation. **Do not close the GeGebra file**. • Remind students that we are only looking at a tiny number of samples and that if we had more samples, we would clearly see that $\sim$95% fall within our confidence interval.	• Can students calculate the 95% confidence interval for the mean of a sample of size 36? • Can students explain what the 95% confidence interval for a sample mean means? • Do students understand that the 95% confidence interval for a sample mean does what it's supposed to?

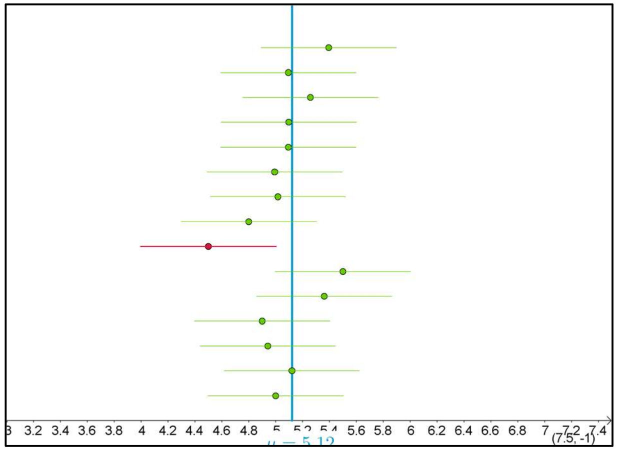
confidence interval for a sample mean. • Remember this confidence interval is a statement about the mean of a single sample. • Let's now change this into a statement about the mean of the population. • To do so I want you to think of the 95% C.I. for a sample mean as a distance either side of the population mean as opposed to an interval between two end values. In other words we can think of the 95% confidence interval for a sample mean as a distance of ± 0.428 either side of 5.12" • More generally we can say that the 95% confidence interval is between $\pm 1.96\sigma_{\bar{x}}$ of the population mean.		• Under **Section D** of the board plan write the 95% confidence interval for the mean of a single sample, labelling it as such. • Add an additional label of "A statement about the mean of a single sample based on the mean of the population. • Under **Section D** of the board plan write the following: "I am 95% confident that $\bar{x}$ lies within ± 0.428 of 5.12" • Under **Section D** of the board plan write the following: "I am 95% confident that a single sample mean will lie within $\pm 1.96\sigma_{\bar{x}}$ of the population mean".	• Do students understand that we are dealing with a statement about a sample mean? • Do students understand that we want a statement about a population mean? • Do students understand that the 95% confidence interval for a sample mean may be interpreted as a distance either side of the population mean?

• If I'm 95% confident that the mean of a single sample will lie within $\pm 1.96\sigma_{\bar{x}}$ of the population mean then is it fair to say that I am also 95% confident that the population mean will lie within $\pm 1.96\sigma_{\bar{x}}$ of the mean of a single sample. • With this simple step we have changed our statement about a sample mean to a statement about a population mean. • Whereas before we had a 95% confidence interval for the mean of a single sample, we now have a 95% confidence interval for the mean of a population. • We have now achieved what we set out to achieve way back at the start of **Section A**. We have a	• I don't know. • Yes it is. • Yes, if A is a certain distance from B then B must be the same distance from A.	• If needed, draw a diagram on the board showing how if A is a certain distance from B that B is the same distance from A. • Under **Section D** of the board plan write the following: "I am also 95% confident that the population mean will lie within $\pm 1.96\sigma_{\bar{x}}$ of a single sample mean." • Under **Section D** of the board plan write the following heading: "95% C.I. for a Population Mean". • Under **Section D** of the board plan write the following: "The 95% C.I. for a population mean provides a mathematical way to make a statement about the population mean using the mean of a single sample".	• Do students understand that if I'm 95% confident that a single sample mean will lie within a certain distance of the population mean then I am also 95% confident that the population mean will lie within an equal distance of a sample mean? • Do students understand that we have changed a statement about a sample mean into a statement about a population mean? • Do students understand we have constructed a new type of 95% confidence interval? • Do students understand that this change allows us to complete our original investigation; that is to make a statement about the mean

mathematical way to make a statement about the mean of a population using the mean of a single sample.		• Under **Section D** of the board plan write the following: "The 95% C.I. for a population mean is derived from the 95% C.I. for a sample mean".	weight of the population of schoolbags using a single sample?
• How did we construct this new 95% confidence interval, that is, the 95% confidence interval for a population mean?	• Using the formula $\bar{x} - 1.96\sigma_{\bar{x}} \leq \mu \leq \bar{x} + 1.96\sigma_{\bar{x}}$ • Using the 95% C.I. for a sample mean.	• Under **Section D** of the board plan write the following "The 95% C.I. for a population mean: I can be 95% confident that the population mean will fall within this interval".	• Do students understand that the 95% C.I. for a population mean is based on the 95% C.I. for a sample mean? • Do students understand what the 95% C.I. for a population mean does?
• What does this 95% confidence interval allow us to do?	• It tells us the range of values over which the population mean is most likely to reside. • It allows us to predict the population mean using the mean of a single sample. • It relates μ to $\bar{x}$ with 95% confidence. • It lets us make a mathematically-sound statement about a population mean using a single sample.	• Use the descriptions of the 95% C.I. for a sample mean from **Section C** of the board plan and the 95% C.I. for a population mean from **Section D** of the board plan to compare and contrast the two types of C.I.	
• We now have two different 95% confidence intervals – the 95% C.I. for a sample mean and the 95% C.I. for a	• One allows us to predict what the mean of a single sample is likely to be while the other allows us to predict what the		• Do students understand that there are two different types of 95% C.I.? • Do students understand the difference between the two types of 95% C.I. we have constructed?

population mean. Can you explain the difference between them?	mean of a population is likely to be. • The 95% C.I. for a sample mean uses a population mean to predict the likely value of the mean of a single sample. The 95% C.I. for a population mean uses a sample mean to predict a likely value for the population mean.		
• Let's return to our original schoolbags problem and put our new 95% confidence interval to work. • You all have your own sample means from **Section A** of the lesson. I want each group to now use its own sample mean to complete the calculation of your own 95% confidence interval for the population mean. • When you do this I want you to complete the statistical investigation we started in **Section A** by making a concluding statement about the	• Students work in groups to complete the calculation of their own 95% confidence interval.	• Move around the room to ensure students understand what they are supposed to do.	• Can students complete the calculation of their own 95% confidence interval for the population mean? • Can students make a suitable concluding statement about the mean schoolbag weight of the population?

mean schoolbag weight of the population.			
• What did you get for your 95% confidence interval for the population mean?	• Each group should respond with a different 95% confidence interval.	• Add the various 95% confidence intervals for the mean of the population to **Section D** of the board plan.	
• Do you all have the same range of values for your confidence interval for the mean of a population?	• No. • No but our intervals overlap quite a bit.		• Do students recognise that there exist many 95% confidence-intervals for the mean of a population?
• Why is this?	• Because the 95% confidence interval for a population mean depends on the sample mean. • It depends on $\bar{x}$ and each group has a different $\bar{x}$ value.		• Do students understand why there are many different 95% confidence-interval for the mean of a population?
• What statement did you make about the mean schoolbag weight of the population?	• Each group should respond with a statement based on their 95% confidence interval of the form "I am 95% confident that the mean schoolbag weight for the population is between …."	• Add each group's concluding statement to **Section D** of the board plan.	
• So we all have different answers for the 95% confidence interval and different	• No – we can't all have different conclusions. • Yes because our 95% confidence intervals overlap	• Highlight the different 95% C.I.'s and the different concluding statements under **Section D** of the board plan.	• Do students understand that the fact the 95% confidence intervals

statements about the mean schoolbag weight of the population. Is this acceptable? • Isn't this the same problem we encountered when we tried to complete our investigation in **Section A** of the lesson? • It may be helpful to visualise our conclusions to understand if it is acceptable to have different conclusions here. • What do you notice about the 95% confidence intervals? • We see that, although every group's 95% C.I. and concluding statement are different, most of	so our conclusions are actually very similar. • Yes because we're not saying that the mean weight is definitely within our interval, rather we're attaching a level of confidence to our conclusion. • It's not the same problem because we're using a range of values in our conclusion as opposed to a single value. • It's not the same. We're using an interval and many of our intervals overlap. • They're all different. • They're different but they overlap a lot. • Most of them capture the population mean.	• Return to the GeoGebra file "**The95%ConfidenceInterval.ggb**". • Click on the button "Show C.I.'s for the Population Mean" to show each group's sample mean and their corresponding 95% confidence interval.  • Highlight the fact that most of the 95% C.I.'s capture the population mean and so are acceptable statements.	don't all agree is not the same problem as the individual sample means not agreeing? • Can students see that the 95% confidence intervals for the population mean tend to capture the population mean?

them capture the population mean. • This means they are acceptable statements about the population mean.			• Can students understand that, although each group's concluding statements are different, because they capture the population mean they are all acceptable?
• Do all 95% C.I.'s capture the population mean?	• No. • Most of them do.		• Do students recognise that not all 95% C.I.'s and not all concluding statements capture the population mean?
• Is this okay? Explain.	• No. The ones which don't capture the population mean are not valid statements (incorrect). • Yes because we're not certain that our intervals capture the population mean. • Yes because we're only 95% confident that our interval captures the population mean.		• Do students understand that, because we say we are only 95% confident, that this is okay?
• How many of the 95% C.I.'s would you expect to capture the population mean?	• Most of them. • 95% of them. • Approximately 95% of them.	• Remind students that we are only looking at a small number of 95% C.I.'s and that if we were to look at a much larger set, we would see that ~95% of them would capture the population mean.	• Do students understand that only 95% of the 95% C.I.'s will capture the population mean?
• So do we all agree that we now have a fair		• Under **Section D** of the board plan write the following: "A 95% confidence interval gives us a fair way to make a	

and mathematical way to use the mean of a single sample to make a statement about the mean of a population? • How do we make such a statement? • There is one remaining problem with this approach – in calculating the 95% confidence intervals we need the standard deviation of the sample means ($\sigma_{\bar{x}}$). We learned in **Section B** of this lesson that we can calculate this value using the standard deviation of the population ($\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$). The problem is we usually don't know σ when carrying out a statistical investigation.	• Yes. • Yes – but I honestly thought we'd never get there. • Using a confidence interval. • Using the 95% confidence interval to a population mean.	statement about the mean of a population using the mean of a single sample". • Under **Section B** of the board plan, highlight the formula relating the standard deviation of the sample means to the standard deviation of the population ($\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$). • Under **Section D** of the board plan highlight how the construction of a 95% confidence interval depends on the value of $\sigma_{\bar{x}}$.	• Do students understand that we've finally achieved what we set out to do – that is to use the mean of a single sample to make a fair and accurate statement about the mean of a population? • Do students understand that a 95% confidence interval is the tool for making a fair statement about the mean of a population using the mean of a single sample? • Do students understand that in our construction of a 95% confidence interval we use the standard deviation of the population and that we won't always have this information?

• How can we overcome this problem?	• I don't know. • Guess the standard deviation. • Use the standard deviation of our sample (s).		
• Let's see if it's reasonable to replace σ with s.			
• What is the value of our population standard deviation (σ)?	• 2.57		
• What are the values of the standard deviation of our various samples?	• Each group responds with the standard deviation of their sample.	• Under **Section D** of the board plan, write the value of the population standard deviation and the standard deviations of each group's sample.	• Can students see that the standard deviation of a single sample is a decent estimator of the standard deviation of the population?
• So s is a decent approximation for σ. In cases where we don't know the value of σ we use s as our best estimate.		• Under **Section D** of the board plan, write the following: "When constructing a 95% C.I. for a population mean use σ if you know it. Otherwise use s as your best estimate of σ.	• Do students understand that if we don't know the value for σ, we should use s as our best estimate?
• Let's assume now that we don't know anything about our population of schoolbag weights. All we have is our sample of 36 schoolbag weights and we want	• Students work on calculating a 95% confidence interval for		• Do students understand that in most statistical investigations the only information you will have is about your single sample, namely

to make a fair and mathematical statement about the mean weight of the population of schoolbags.	the population mean using their single sample.		its mean, standard deviation and size?
• I want you to use your own sample mean and standard deviation to make a statement about the population mean.		• Remind students that this is the information they are likely to have in a typical statistical investigation.	• Can students use their single sample to construct a 95% confidence interval for the mean of the population? • Do students understand that they must use the standard deviation of their sample in calculating the 95% confidence interval?
• What statement did you make about the mean schoolbag weight of the population of secondary-school students?	• I am 95% confident that the mean schoolbag weight lies between ……. • I can be 95% confident that the mean of the population of schoolbag weights is ………	• Return to the GeoGebra file "**The95%ConfidenceInterval.ggb**". • Click on the button "Show interval around $\bar{x}$" to construct each group's 95% confidence interval.	• Can students make a fair statement about the population mean using their single sample?
• Are these fair statements? Explain.	• Yes. • Yes – they are all different but each one is an equally-correct 95% confidence interval.	• Uncheck the checkbox "Use σ" so that each group's 95% C.I. is based on the standard deviation of their sample(s) as opposed to the standard deviation of the population(σ).	• Do students understand that, although each group's interval is different, each interval is equally valid?

• The 95% confidence intervals are slightly different widths. We haven't seen this happen before. Why are the 95% confidence intervals different widths?	• Yes as most of the confidence intervals will hold the population mean. • Yes – a small number of the confidence intervals will not capture the population mean but this is okay as we're only saying we're 95% confident. • Because we're using s as an approximation for σ and each group has a different value for s. • The width of the 95% confidence interval depends on the standard deviation of the population. We are all using our own s values to approximate this and so we all get different widths. • We are all using different values for s to approximate σ.	• Highlight the fact that using s instead of σ has a small effect on the 95% C.I.'s for the population mean and that most of the confidence intervals still capture the population mean. Checking and unchecking the checkbox "Use σ" may help to show this difference. • Highlight the difference in width of each group's 95% confidence interval.	• Do students understand why each group's confidence interval has a different width?
• In making this statement, we used z-scores to construct our confidence interval. Doing so assumes that we're dealing with a normal distribution. Can we assume this? Remember we know nothing about the population this time.	• Yes – the distribution of sample means is always normal once our sample size is over 30. Ours is 36. • It doesn't matter what shape the population distribution is – it's the distribution of sample means we're interested in and this is normal.	• Use **Section B** of the board plan to remind students that the distribution of sample means is normal if the population distribution is normal or if we're dealing with a sample of size ≥ 30.	• Do students understand that it is the normality of the distribution of sample means which is important when constructing a 95% confidence interval for the mean? • Can students recall the conditions which guarantee normality?

	• Even though we know nothing about the population distribution it doesn't matter, as our approach relies on the distribution of sample means being normal and it is since we have a sample size ≥ 30.		
• The aim of our statistical investigation was to make a statement about the mean of the population of schoolbag weights. In doing so we needed the statement to be mathematical and fair. • I would now like you to take a few minutes to reflect on how we do so. • As you reflect, I would like you to consider how to answer **Section D: Students Activity 1**.	➢ Students work on answering **Section D: Student Activity 1**.	• Hand out a copy of **Section D: Student Activity 1** to every student. • Encourage students to discuss the main concepts on the board plan as they answer **Section D: Student Activity 1**.	• Can students identify the key concepts which underpin the construction of a 95% confidence interval?
• Would it be correct for the GPA to state the following: "The mean amount of time spent training each week by all Gaelic players in	➢ No. ➢ No because this is the result of a single sample. ➢ No. This is the mean of a single sample. If we took a different sample we'd	• Refer to **Section A** of the board plan to remind students of the problem of sampling variability.	• Can students recall the problem of sampling variability and what it says about our ability to use a single sample to make a statement about a population?

Ireland is 7.2 hours"? Explain.	probably get a different result. ➢ No – we cannot equate the mean of a sample with the mean of the entire population. ➢ No because of sampling variability.		• Do students understand that we can't equate the population mean with the mean of a single sample?
• Is it possible, then, to use the mean of the sample of 100 players to make a statement about the population of GAA club players?	➢ Yes. ➢ Yes – but we'll need to use a range of values. ➢ Yes we can use a confidence interval. ➢ Yes – by using a 95% confidence interval.	• Refer to **Section D** of the board plan to remind students of the use of a confidence interval to relate the mean of a sample to the mean of a population.	• Do students understand that a confidence interval is the key to relating the mean of a sample to the mean of a population?
• To use z-scores to construct a 95% confidence interval we must first know that the sample means are distributed normally. Do we know that this is the case in this investigation?	➢ Yes – because if the sample size is ≥ 30 we are guaranteed that the distribution of sample means is normal. We're using a sample of 100 so the distribution is normal. ➢ Yes, the distribution of sample means is normal because we are dealing with a large sample size.	• Refer to **Section B** of the board plan to remind students that we need a distribution of sample means which is normal.	• Do students understand that it is the shape of the distribution of sample means which is important for constructing our 95% confidence interval? • Do students understand the conditions needed to guarantee normality of the distribution of sample means?
• If the sample size was less than 30 could we construct a 95%	➢ No. ➢ Not necessarily.	• Refer to **Section C** of the board plan to remind students of the conditions which guarantee normality of the distribution of sample means.	

confidence interval in the same way? • What statement should the GPA make about the training regime of all Gaelic club players in Ireland?	➢ Only if we know that the population distribution is normal. ➢ Even for sample sizes less than 30, the distribution of sample means can still be normal, provided the underlying population is normal. So if we knew the population was normal we could construct the 95% confidence interval. ➢ We can be 95% confident that the mean amount of time spent training every week is between 6.96 hours and 7.44 hours. ➢ The 95% confidence interval for the mean amount of time spent training every week by Gaelic club players is 6.96 hours – 7.44 hours.		• Can students apply their knowledge to make a suitable statement about the population?
SECTION E: Hypothesis Testing			
• We are now going to look at how we use statistics to test a claim. • Can anybody explain to me what I mean when I say "claim"?	➢ When somebody looks for some money. ➢ When somebody says something belongs to them.		

	➢ When somebody claims something is true. ➢ When something is said to be true.		
• When I say "claim" I mean something very specific – it is a statement about a population, usually without any proof or evidence. Can anybody give me an example of a claim they've seen or heard?	➢ 40% of people want a change in Taoiseach. ➢ Duracell batteries last three times longer than other batteries. ➢ The mean lifetime of an iPhone battery is 9.5 hours. ➢ The mean industrial wage is €35,500. ➢ 80% of women use a skincare product every day.	• Under **Section E** of the board plan write the following: "Claim: A statement about a population without proof".	• Do students understand what the word "claim" means in the context of statistics?
• In this lesson we are going to look at a specific type of claim – that involving a mean.			
• Scientists make claims all the time and then carry out experiments to test their claims. Scientists use a different word to "claim". Does anybody know what the word is?	➢ A theory (incorrect). ➢ A hypothesis.		• Do students understand the meaning of the word "hypothesis" in the context of statistics?
• So we are interested in testing a claim or a		• Under **Section E** of the board plan, add "Hypothesis" as an alternative word to "Claim".	

hypothesis. The way in which we do so is called a hypothesis test. • Can anybody explain why it is important to be able to test a hypothesis or claim?	 ➢ If you want to know if a claim is true. ➢ If you can't check a claim then people can claim anything they want. ➢ In science, the only way to know if something is true is to test it. ➢ If somebody's developing a new drug for treating illness you need to be able to test if it works. ➢ Companies claim stuff all the time in adverts. We need to be able to tell what's true and what's false.	• Title **Section E** of the board plan as "Hypothesis Testing on Means".	• Do students recognise the need to investigate a claim or to test a hypothesis? • Do students understand that hypothesis testing provides a mathematical way to check if a claim is true or not and how important this is?
• Let's look at one such claim: "An EU survey reports that the mean height of adult male Europeans is 181 cm with a standard deviation of 7 cm. An Irish newspaper carries out some research on this. They sample 100 adult Irish males and find that		• Distribute copies of **Section E: Student Activity 1** to all students. • Encourage students to read through the claim in part (i) of the activity.	

the mean of the sample is 179 cm. Based on this sample, are Irish males different in height to other Europeans (use a 5% significance level)?		• Under **Section E** of the board plan write the following: “The null hypothesis is a statement about **the population**”.	
• When we carry out a hypothesis test we follow the data-handling cycle as normal. • We first state what is being claimed. This is the “ask a question” stage of the data-handling cycle. We call our statement of the claim the null hypothesis. The null hypothesis is a claim about the population.			• Do students understand that the null hypothesis is a statement of the claim being made? • Do students understand that the null hypothesis is a statement about the population?
• Can you state the null hypothesis in this test?	➢ Irish men are different in height to other Europeans. ➢ The mean height of adult Irish males is not the same as that of other Europeans. ➢ Irish men are the same size as other European men. ➢ Irish men are taller than other Europeans.	• Under **Section E** of the board plan, write up the different statements of the null hypothesis as given by students.	• Can students correctly state the null hypothesis for this claim?

• We need to be very careful when we state the null hypothesis. The word "null" is short for "null effect" or "no change". When we state a null hypothesis it should always be a statement of "no change". • Are all the null hypotheses statements of no change? • We also said that the null hypothesis is always a statement about a population. What is our population here?	➢ Irish men are smaller than other Europeans. ➢ No. ➢ No – only one of them is. ➢ The only one that is a null statement is "Irish men are the same size as other European men". ➢ European men (wrong). ➢ Irish men. ➢ Adult Irish males. ➢ The height of Irish men is the same as European men.	• Add the following to the description of the null hypothesis on the board plan: "A null hypothesis is a statement of **no change**." • Go through each statement of the null hypothesis on the board and mark those which do and do not meet the criteria for the statement of a null hypothesis. Explain why a given statement is or is not an acceptable null hypothesis. • Encourage students to identify why some statements of the null hypothesis are acceptable and why others are not. • Add students' new statements of the null hypothesis to the board plan. Check that the new statements are acceptable.	• Do students understand that the null hypothesis must be a statement of "no change"? • Can students identify null hypotheses which are valid for the given claim and those which are not? • Can students explain why some null hypotheses are valid and others are not? • Can students identify the population?

• Since the null hypothesis should be a statement of no change and about a population can we rethink what it should be in this case? • All of these are fine ways to state the null hypothesis for this case, however when you state the null hypothesis I would like you to try to have a measurement in your statement if possible – just like the statement which says "Adult Irish males have a mean height of 181 cm". It will make the subsequent steps in a hypothesis easier to understand if the null hypothesis is stated in this way. • Could we also use mathematical notation to state the null hypothesis?	➢ Adult Irish males have the same height as adult European males. ➢ Adult Irish males have a mean height of 181 cm. ➢ Yes: $\mu = 181\ cm$ where μ is the mean height of the population of adult Irish males.	➢ Add the statement "Adult Irish males have a mean height of 181 cm" to the list of null hypotheses and highlight it. ➢ Add the mathematical statement of the null hypothesis ($\mu = 181\ cm$) to the list of null hypotheses on the board. ➢ Add the notation for the null hypothesis (H_0: $\mu = 181\ cm$) to the mathematical statements of the null hypothesis on the board.	• Do students understand that there are many valid ways to form a null hypotheses but that we would like ours to include a measurement if possible? • Can students write the null hypothesis using a measurement? • Can students use their knowledge of statistical notation to state the null hypothesis as a mathematical equation? • Do students recognise that H_0 is the notation used to represent the null hypothesis?

• We use the symbol H_0 to represent the null hypothesis. The "0" is intended to represent "no change" or a "null effect". • So the null hypothesis can be stated in words or using a mathematical equation. • Learning to correctly state the null hypothesis is a skill to master with practice. With this in mind, I would like you to read each claim in part (ii) of **Section E: Student Activity 1** and write a suitable null hypothesis for each one. • For claim A of **Section E: Student Activity 1**, what did you write for your null hypothesis?	➢ Students work on filling in the first column of part (ii) of **Section E: Student Activity 1**. ➢ Leaving Certificate results in maths have not changed since 2014. ➢ The mean mark for Leaving Cert. maths in 2016 is 385. ➢ Leaving Certificate results in maths are the same in 2016 as they were in 2014.	➢ Move around the room to ensure that students understand what they are meant to be doing. ➢ Encourage students to check that their null hypothesis is a statement of the claim, is a statement about the population and is a statement of no change. ➢ Encourage students to use words and symbols to state their null hypotheses. ➢ Encourage students to have a measurement in their statement of the null hypothesis.	• Do students understand that the "0" stands for "null effect" or "zero change"? • Do students understand how to take a claim and form a null hypothesis? • Can students form a valid null hypothesis?

	➢ $H_0: \mu = 385$ where μ is the mean mark obtained in Leaving Cert. maths in 2016.	➢ Go through each statement and check that it satisfies the necessary criteria.	
• Are all of these statements about the population?	➢ Yes.		• Do students recognise that the null hypothesis is a statement about the population?
• What is our population?	➢ Leaving Cert. students in 2016. ➢ All 2016 Leaving Cert. students.	➢ Highlight the statement of the null hypothesis as an equation.	
• Are all of these statements of no change?	➢ Yes.		• Do students recognise the importance of the null hypothesis being a statement of no change? • Can students write the null hypothesis as a mathematical equation?
• For claim B of **Section E: Student Activity 1**, what is the null hypothesis?	➢ 1. The mean annual income has changed since 2012 (incorrect). ➢ 2. The mean annual income is different in 2016 to 2012 (incorrect). ➢ 3. The mean annual income has increased since 2012 (incorrect). ➢ 4. The mean annual income in 2016 is €38,280 (incorrect)		• Can students form a valid null hypothesis? • Can students form the null hypothesis as a statement about the population? • Can students form the null hypothesis as a statement of no change? • Can students write the null hypothesis as a mathematical equation?

• We have a wide variety of statements of the null hypothesis. To help us sort and check them I want you to group similar statements together? Give a reason for your choice of grouping.	➢ 5. The mean annual income four-years later is the same as it was in 2012. ➢ 6. The mean annual income has not changed since 2012. ➢ 7. The mean annual income in 2016 is €39,400. ➢ 8. $\mu = €39{,}400$ where μ is the mean annual income in 2016. ➢ Statements 1 – 4 are similar because they all talk about the mean annual income changing. ➢ Statements 5- 8 are similar because they all talk about the mean annual income in 2016 being the same as it was in 2012.	➢ Use a suitable method to group like statements.	• Can students identify statements which are similar in form?
• Can you identify any statements which are not null hypotheses? Explain your reasoning.	➢ Statements 1 – 4 are not null statements. Nulls statements are meant to be statements of no change, however these are statements of change.	➢ Highlight the statements which are acceptable as a null hypothesis.	• Can students identify which statements do not meet the criteria for a null hypothesis?
• Can you identify statements which are acceptable null hypotheses? Explain your reasoning.	➢ Statements 5 – 8 are acceptable. They all describe no change from the original situation.	➢ Encourage students to ask for help if they are unsure if their statements are acceptable or not.	• Can students identify which statements meet the criteria for a null hypothesis?
• Working in pairs I want you to now			• Can students identify which statements are correct null hypotheses?

check that all your null-hypothesis statements in **Section E: Student Activity 1** are statements of the claim, are statements about the population and are statements of no change. • Whenever we state a claim we also state the counter claim. If the claim is known as the null hypothesis then the counter claim is the alternative hypothesis. • The alternative hypothesis is a statement which is contrary to the null hypothesis. The alternative hypothesis states that the null hypothesis is not true. • Looking at this in another way we can think of the alternative hypothesis	➢ Students work in pairs to check that they have constructed correct null hypotheses.	➢ Under **Section E** of the board plan, write the following description of the alternative hypothesis: "Alternative hypothesis: a statement of the counter claim". ➢ Under **Section E** of the board plan add the following: "Alternative Hypothesis: A statement of change". ➢ Beside the description of the alternative hypothesis on the board, add the symbols H_1 and H_A. ➢ Write up the different student statements of the alternative hypothesis.	• Can students modify their incorrect null hypotheses so that they are suitable? • Do students understand that for every null hypothesis there is a corresponding alternative hypothesis? • Do students understand that the alternative hypothesis is a statement which contradicts the alternative hypothesis? • Do students understand that the alternative hypothesis is a statement of change?

as a statement of change. • We use H_1 or H_A to represent the alternative hypothesis. • If we look at our original claim in part (i) of **Section E: Student Activity 1**, how would you state the alternative hypothesis. • We need to be careful when stating the alternative hypothesis to make sure it is a proper contrary statement of the null hypothesis.	➢ Adult Irish males are taller than adult European males (incorrect). ➢ Adult Irish males are smaller than adult European males (incorrect). ➢ Adult Irish males are not the same height as adult European males. ➢ The population of adult, Irish males does not have a mean height of 181 cm. ➢ $\mu \neq 181\ cm$ where μ is the mean height of adult Irish males.	➢ Go through each statement and explain why it is an acceptable or unacceptable statement of the alternative hypothesis. ➢ Highlight the statement of the alternative hypothesis which has the following form: $\mu \neq 181\ cm$.	• Do students understand that H_1 or H_A is used to represent the alternative hypothesis? • Can students correctly state the alternative hypothesis?

• If the null hypothesis says that a quantity has a specific value, the alternative hypothesis should state that it doesn't have this value. • Writing the null hypothesis as an equation using mathematical symbols can be helpful in identifying the alternative hypothesis. If $H_0: \mu = a$ then the alternative hypothesis is simply $H_0: \mu \neq a$. • I now want you to return to **Section E: Student Activity 1** and fill in the alternative hypothesis for each claim. • What did you write as the alternative hypothesis in Claim A of **Section E: Student Activity 1**?	➢ Students work on completing **Section E: Student Activity 1**. ➢ Maths results have changed since 2014. ➢ The mean mark obtained by Leaving Cert. students in maths is not 385. ➢ $\mu \neq 385$ where μ is the mean mark obtained by Leaving Cert. students in maths in 2016. ➢ $\mu = 396$ (incorrect).	➢ Circulate around the room and observe students' attempts to write down alternative hypotheses. Take note of any statements that are worth discussing with the class as a whole. ➢ Go through each statement and explain why it is an acceptable or unacceptable statement of the alternative hypothesis.	• Do students understand that writing the null hypothesis as an equation can be helpful in forming a correct alternative hypothesis? • Can students write a correct alternative hypothesis for each claim?

• Why is it not acceptable to write the alternative hypothesis as $\mu = 396$? • For Claim B of **Section E: Student Activity 1**, what is the alternative hypothesis?	➢ 396 is the mean mark for the sample and not the population. ➢ This is not the literal contradiction of the null hypothesis. ➢ There are many other value of μ which would make the null hypothesis false. We can't just look at one of them. ➢ This is only one possible statement of $\mu \neq 385$. We need to consider all the values which makes this true. ➢ The mean annual wage for people in full time employment is €39,400. ➢ The mean annual wage in 2016 is the same as it was in 2012. ➢ The mean annual wage has not changed since 2012. ➢ $\mu = €\ 39{,}400$ ➢ The mean annual wage is more than €39,400. (incorrect) ➢ The mean annual wage is less than €39,400. (incorrect) ➢ The mean annual wage is €38,280. (incorrect)		• Can students explain why $\mu = 396$ is not a correct alternative hypothesis? • Can students state the alternative hypothesis correctly?

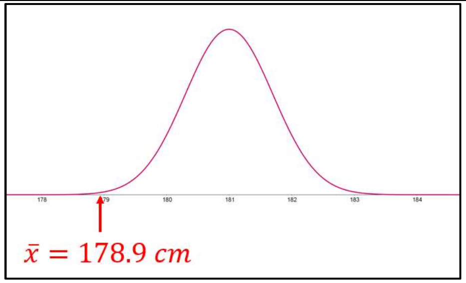
• Can you identify which statements are acceptable alternative hypotheses and which are unacceptable? Give reasons for your choice. • Working in pairs I want you to now check that the remaining alternative-hypothesis statements in **Section E: Student Activity 1** are suitable. • It can be easy to write down an alternative hypothesis which is incorrect. • By writing the null hypothesis as an equation, it is simple to then write down the alternative hypothesis. Once this is done it should make it easier for you to write down both	➢ The last three alternative hypotheses are incorrect. ➢ The statements which say the mean wage is less than or more than are incorrect as they are not counter statements of the null hypothesis. ➢ The final statement is incorrect as it is not a contrary statement of the null hypothesis.	➢ Go through some examples of suitable alternative hypotheses for statements C – J of **Section E: Student Activity 1**. ➢ Encourage students to ask for help if they are unsure if their statements are acceptable or not. ➢ Remind students that if $\mu = a$ is the null hypothesis then $\mu \neq a$ is the alternative hypothesis.	• Can students identify correct and incorrect statements of the alternative hypotheses? • Can students explain what makes a statement a suitable alternative hypothesis? • Can students change their incorrect alternative hypotheses so that they are suitable? • Do students understand that writing the null hypothesis as an equation and then writing the alternative hypothesis as an equation can help to state the alternative hypothesis as a sentence?

hypotheses using everyday language.			
• Now that we have our claim and counter claim, how can we test our claim? • If we return to the claim in part (i) of **Section E: Student Activity 1**, what other information are we given? • Our null hypothesis states that $\mu = 181\ cm$. Our sample mean is $\bar{x} = 178.9\ cm$. Does this data show that the null hypothesis is false? Explain.	➢ We need some data. ➢ We use the sample information. ➢ We use the other information provided in the question. ➢ We are given information about a sample. ➢ We are given the standard deviation of the population, the size of the sample and the mean of that sample. ➢ Yes (incorrect). ➢ Yes because we get a different value to what is claimed (incorrect). ➢ Yes because the data collected gives us a different answer to the null hypothesis (incorrect) ➢ No because the sample mean is very close to the population mean. ➢ No – this is just one of the possible sample means that you could get. A different sample could produce a value equal to the null hypothesis value.	• Write the population and sample information for part (i) of **Section E**: Student Activity 2 under **Section E** of the board plan. • Encourage students to explain their reasoning. • Write the following under **Section E** of the board plan: "Because of sampling variability, we expect $\bar{x} \neq \mu$. This does not necessarily imply that the claim is false."	• Do students understand that to test the claim we need some data? • Can students identify that they are given the hypothesised population mean, the standard deviation of the population, a sample mean and a sample size? • Do students understand that due to sampling variability, the chances of getting a sample from the population with a mean equal to the population mean is miniscule? • Do students understand that a sample mean which is different to the hypothesised mean does not necessarily imply that the hypothesised value of the population mean is incorrect?

• So we are faced with a similar problem to that which we faced when we tried to use a sample mean to make a statement about the population mean. This problem is sampling variability and means that when we sample we are unlikely to get a mean equal to the hypothesised mean but that this is not necessarily because the hypothesised mean is incorrect, it may just be because of sampling variability. • The solution to the problem lies in our understanding of the distribution of sample means.	➢ No it doesn't. Because of sampling variability this is just one of a huge number of possible sample means that are possible. Just because one of these is different to the hypothesised mean doesn't make the claim incorrect.	• Distribute copies of **Section E: Student Activity 2** to all students.	• Do students understand the challenge that sampling variability presents when trying to test a claim using a single sample? • Do students appreciate that by understanding sampling variability we can overcome this problem?

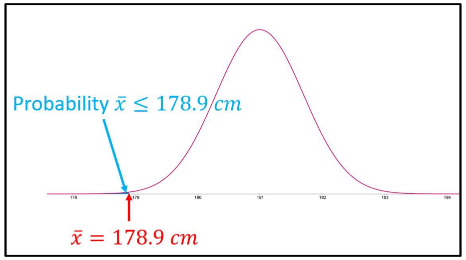
• Let’s start to understand how we can solve this problem by first assuming that the population mean really is $\mu = 181\ cm$. I want you to answer Q1. of **Section E: Student Activity 2**.	➢ Students sketch the distribution of sample means.	• Encourage students to draw an accurate sketch of the distribution of sample means. • Refer to **Section B** of the board plan to remind students of the properties of the distribution of sample means which they have already discovered. • Place a print-out of the distribution of sample means from **Appendix B** on the boar, under **Section E**.	• Can students create an accurate sketch of the hypothesised distribution of sampling means?
• Describe the shape of your sketch.	➢ It’s normal. ➢ It’s symmetric. ➢ It’s a normal curve.	• Mark in the mean of the distribution as the hypothesised mean ($\mu = 181\ cm$).	• Do students recall that the distribution of sample means is normal?
• Where is the distribution of sample means centred?	➢ It’s centred on the population mean. ➢ It’s centred on $181\ cm$. ➢ It’s centred on the hypothesised mean.		• Do students recall that the mean of the sample means is equal to the mean of the population?
• How wide is the distribution of sample means?	➢ It has a standard deviation of 7 cm (incorrect). ➢ It has a standard deviation of 0.7 cm. ➢ It has a standard deviation of $\frac{7}{\sqrt{100}}$.	• Mark in the standard deviation of the distribution as $\frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{100}} = 0.7$. • Sketch what a normal distribution with a standard deviation of 7 cm would look like on the board.	• Can students calculate the standard deviation of the distribution of sample means?
• Why is a standard deviation of 7 cm incorrect?	➢ Because this is the standard deviation of the population. We want the standard deviation of the sample means.		• Can students explain why a standard deviation of the distribution of sample means is not 7 cm?

• So if we're given a hypothesised population mean and standard deviation we can easily predict what the sample means from this population will look like i.e. we can predict the associated distribution of sample means. • I'd like you to mark on the distribution, the single sample mean	➢ The standard deviation of the distribution of sample means is always smaller than the standard deviation of the population. ➢ The standard deviation of the sample means is $\sqrt{n}$ times smaller than the standard deviation of the population. ➢ Well if the standard deviation was 7 we wouldn't be able to fit the distribution of sample means on the axis we were given.	• Mark in the single sample mean quoted in **Section E**: Student Activity 2 ($\bar{x} = 178.9\ cm$) on the diagram on the board.	• Do students understand that we can use the information given in a claim about a population to predict the range of sample means we are likely to get when we sample from this population?

quoted by the Irish newspaper.	➢ Students mark in the position of the sample mean on their distribution of sample means.		• Can students locate the mean of their single sample on the sketch of the distribution of sample means?
• Do you think it's possible to get a mean of 178.9 cm from a single sample of size 100, given a distribution of sample means which is centred on 181 cm? Explain your reasoning.	➢ Yes it's possible. ➢ Yes because it is part of the distribution. ➢ It's possible but not very likely. ➢ It's not likely, as the value is far away from the centre of the distribution. ➢ It's unlikely as the value is close to one the tails of the distribution. ➢ Yes but there's only a small chance since the area under the curve there is tiny.	• Write the following under **Section E** of the board plan: "Assuming H_0 is true, is sampling variability likely to give a sample mean of $\bar{\mathrm{x}} = 178.9$?" "Assuming H_0 is true and that this is the distribution of sample means, is a sample mean of $\bar{x} = 178.9$ likely?" • Use the poster of the distribution of sample means to highlight the sample-mean values which are more likely to result from this distribution and those which are less likely to result from it.	• Can students interpret the distribution of sample means to get a sense of how likely a value of 178.9 cm is if a sample of size 100 is selected from the population?
• We already know that if we sample from a population with a mean of 181 cm we are likely to get a sample mean different to 181 cm. At the same time our sketch shows that it is more			• Do students understand what the distribution of sample means tells us about sampling?

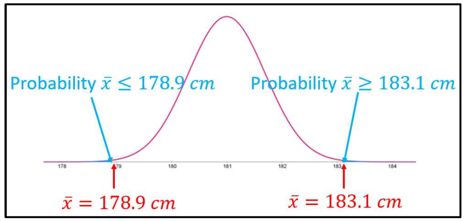
likely to get some sample means compared to others and that a sample mean of 178.9 cm is unlikely. • In other words given the hypothesised population mean, sampling variability is unlikely to produce a sample mean of 178.9 cm. • How then could we have gotten a sample mean of 178.9 cm?	➢ While it's unlikely to be down to sampling variability, it may still be. ➢ If the population mean is not 181 cm. ➢ If the distribution of sample means is shifted to the left. ➢ If the distribution of sample means was centred on a value less than 181 cm, sampling variability could more-easily produce a sample mean of 178.9 cm. ➢ If the mean of the sample means is less than 181 cm. ➢ If the null hypothesis is not true.	• Under **Section E** of the board plan write in the answer to the previous two questions as "No". • On the diagram of the distribution of sample means, sketch in a distribution of sample means with the same standard deviation, but shifted to the left.	• Do students understand that a sample mean of 178.9 cm is unlikely due to sampling variability alone? • Do students understand that a result of 178.9 cm suggests that the underlying distribution of sample means is incorrect? • Do students understand that this result suggests that the distribution of sample means is more likely to be centred on a value less than 181 cm?

• If the sample mean is far enough away from the given hypothesised mean this suggests that this is not the correct distribution of sample means and that the hypothesised mean is incorrect.	➢ If the hypothesised mean is incorrect. ➢ If the distribution of sample means is centred on a different value. This would mean that the hypothesised mean is not correct.	• Under **Section E** of the board plan write the following: "A large difference between $\bar{x}$ & μ suggests that the distribution of sample means is centred elsewhere and that H_0 is false."	• Do students understand that a sample mean far from the hypothesised mean suggests the hypothesised mean is incorrect?
• Up to now we have agreed that getting a value of 178.9 cm is unlikely due to sampling variability. We would like to quantify this uncertainty by working out the probability of getting a value of 178.9 cm, assuming the hypothesised distribution of sample means. Is this possible?	➢ Yes. ➢ Yes – we're dealing with a normal distribution so we can use z-tables to calculate probability. ➢ We could use z-tables. ➢ No – we cannot calculate the probability of a single value using z-tables. All we can do is calculate the probability of getting a range of values.		• Do students understand that z-tables have a role to play in calculating the probability of getting a value of 178.9 cm?

• While we cannot use z-tables to calculate the probability of getting a single value of 178.9 cm (this probability is zero), we can calculate the probability of getting a value of 178.9 cm or a value even more extreme than this.		• Under **Section E** of the board plan, on the diagram of the normal distribution shade in the area to the left of 178.9 cm. • Label this area as “$P(\bar{x} \leq 178.9)$”	• Do students understand that it’s not possible to use z-tables to calculate the probability of a single value, rather all you can do is work out the probability of a range of values? • Do students understand that we will now calculate the probability of getting a value at least as extreme as 178.9 cm?
• Given the hypothesised distribution of sample means, I now want you to calculate the probability of getting a sample mean at least as extreme as 178.9 cm or at least as far away from 181 cm as 178.9 cm is. You can show your calculation in Q3. of **Section E: Student Activity 2**.	➢ Students work on calculating the probability of the given event.	 • If students are having difficulties remind them of their understanding of the normal distribution and z-scores.	• Can students work out a z-score and calculated the corresponding probability value using their z-tables?
• How did you calculate the probability of	➢ I calculated the area under the distribution to the left of 178.9 cm.		• Do students use the correct value for standard deviation?

getting a sample mean which is at least as extreme as 178.9 cm given the hypothesised distribution of sample means?	➢ I converted 178.9 cm to a z-score and then looked up this value in my z-tables to find the probability of getting a value less than or equal to z. ➢ I calculated a z-score and found the probability of this using the standard normal distribution in my Formulae & Tables booklet. ➢ $z = \frac{178.9-18}{0.7} = -3.$ ➢ $P(z \leq -3) = 0.0013.$ ➢ $P(z \leq -3) = P(z \geq 3) = 1 - P(z \leq 3) = 0.0013.$ ➢ $1 - 0.9987.$	• Under **Section E** of the board plan, write up the calculation of the z-score. $p = P(\bar{x} \leq 178.9)$ $p = P(z \leq \frac{178.9 - 181}{0.7})$ $p = P(z \leq -3)$ • Remind students how to use their z-tables to calculate p: $p = 0.0013$	• Can students manipulate their z-table values to find the required probability?
• What is the probability of getting a sample mean with a value at least as extreme as 178.9 cm given the hypothesised distribution of sample means?	➢ $0.0013.$		
• Is this value consistent with our earlier assumption about the likelihood of getting a	➢ Yes – we said it's unlikely and the probability value backs this up.	• Change the probability value written on the board from 0.0013 to 0.0026.	• Do students recognise 0.0013 as a low probability? • Do students understand that this result backs up our initial thoughts that the probability of getting a value of 178.9 cm, given the hypothesised mean, is small?

value of 178.9 cm? Explain. • Actually, the calculation is incorrect. The probability of getting a value at least as extreme as 178.9 cm is, is not 0.0013. Rather the probability is 0.0026. • Where did I get this probability value from? • Why is the probability double our original calculation? • This may be understood in a couple of ways. • We agreed that we wanted to calculate the probability of getting a sample mean at least as extreme as 178.9 cm.	➢ It's twice the probability that we calculated. ➢ By multiplying our result by two. ➢ I don't know. ➢ Values which are less than 178.9 cm.		• Do students understand that this result backs up our assertion that the distribution of sample means is not as hypothesised and thus that the null hypothesis is false? • Can students identify that the correct probability is twice the value that they calculated?

• What do we mean by “at least as extreme as 178.9 cm”?	➢ Values which are further from the population mean than 178.9 cm is. ➢ Values which are more than 2.1 cm away from the population mean. ➢ Values which are more than three standard deviations from the population mean.		
• So we need to identify all the sample means which are at least as far from the population mean as 178.9 cm is (i.e. sample means which are at least 2.1 cm or three standard deviations away from the population mean).		• Under **Section E** of the board plan, use the poster of the distribution of sample means to highlight the area of the distribution of sample means which is to the left of 178.9 cm. • Highlight that values above 183.1 cm are just as extreme as values below 178.9 cm are and shade in the upper tail of the distribution above 183.1 cm to highlight this.  	
• Are the set of values to the left of 178.9 cm the only sample means which fall into this category?	➢ Yes (incorrect). ➢ No – there are a set of sample means on the other side of the population mean which satisfy the criterion. ➢ Yes there are values in the other tail of the distribution which are the same distance away from 181 cm as 178.9 cm is. ➢ All the values to the right of 183.1 cm are also at least as		• Do students understand that values at least as extreme as 178.9 cm include all the values below 178.9 cm and all the values above 183.1 cm?

 • So our calculation missed out on the area of the distribution of sample means which is the same distance from the population mean as 178.9 cm is but on the other side of the population mean i.e. the area of the distribution above 183.1 cm. • We can calculate this area (or probability) separately using our z-tables and when we include it in our calculation we get $p = 0.0026$ as expected. • Could anybody suggest a more efficient way to complete the	far away from 181 cm as 178.9 cm is. ➢ Yes we can just double our initial probability. ➢ We can double the probability as the area to the left of 178.9 cm is the same as the area to the right of 183.1 cm.	• Under **Section E** of the board plan, amend the p-value calculation to include the calculation of the area of the upper tail as follows: $$p = P(\bar{x} \leq 178.9) + P(\bar{x} \geq 183.1)$$ $$p = P\left(z \leq \frac{178.9 - 181}{0.7}\right) + P\left(z \geq \frac{183.1 - 181}{0.7}\right)$$ $$p = P(z \leq -3) + P(z \geq 3)$$ $$p = 0.0013 + 0.0013$$ $$p = 0.0026$$ • Point out that the two shaded areas of the distribution of sample means are identical in size. • Point out the symmetry of the normal distribution.	• Do students understand why we are include a second area in our p-value calculation? • Can students use the symmetry of the distribution of sample means to perform the p-value calculation more efficiently?

calculation? Explain your reasoning. • The inclusion of the upper tail of the distribution may also be understood by examining the original claim and the associated hypotheses. • For the given claim, the null hypothesis states that the mean height of adult Irish males is 181 cm while the alternative hypothesis states that the mean height of adult Irish males is not 181 cm. • If the alternative hypothesis was that the mean height of an adult Irish make is less than 181 cm we would only have been interested in the area	➢ Yes we can double the area to the left of 178.9 cm because of the symmetry of the normal distribution.	• Add the term “Two-Tailed Test” to a suitable location under **Section E** of the board plan.	• Do students understand that the inclusion of the values above 183.1 cm in our probability calculation is because of the way our hypothesis test is set up?

below 178.9 cm as these are the values which pertain to this. • For this reason, because of the nature of the claim and counter claim, we must also include the area above 183.1 cm in our probability calculation. • The type of hypothesis test used to test a claim which is of the form "$H_0: \mu = a$ and $H_A: \mu \neq a$" is usually called a two-tailed test. Why do you think this is?	➢ Because the test is based around the probability of getting values in the two tails of the distribution. ➢ Because sample means which are far from the hypothesised mean on each side are consistent with the null hypothesis being false. ➢ Because it doesn't matter if the sample means are less than or greater than the hypothesised mean. All that matters is that they are sufficiently different to be unlikely because of sampling variability. ➢ Because the alternative hypothesis is that the hypothesised mean is not true. Values which are much less than the hypothesised mean and values which are much more than the hypothesised mean are both	• Use the poster of the distribution of sample means to remind students that extreme values on either side of the hypothesised mean are important in hypothesis tests of the form "$H_0: \mu = a$ and $H_A: \mu \neq a$". • Write the following under **Section E** of the board plan: "The p-value is the probability of getting a sample	• Do students understand why this type of hypothesis test is known as a two-tailed test? • Do students understand that hypothesis tests of the form "$H_0: \mu = a$ and $H_A: \mu \neq a$" are always two-tailed tests?

• There are other types of hypothesis test which are based on one tail of a distribution. We are not going to concern ourselves with these. • The probability we calculated (0.0026) using the hypothesised distribution of sample means and the mean of our single sample is known as a p-value. Why do you think this is? • The p-value is the probability of getting a sample mean at least as extreme as the given value, assuming the hypothesised population mean is correct.	consistent with this alternative hypothesis. ➢ "p" stands for probability. ➢ Because it tells us the probability of getting a value at least as extreme as our sample mean assuming the null hypothesis is true.	mean at least as extreme as the given value, assuming the hypothesised population mean is correct".	• Do students understand that there are other types of hypothesis test? • Do students understand that the probability of getting a sample mean at least as extreme as the given value, assuming the hypothesised population mean is correct, is known as the p-value of the test? • Do students understand that a small p-value suggests that the difference between the sample mean and the hypothesised mean is unlikely to be due to sampling variability alone?

• If the p-value is sufficiently small, it is highly unlikely that such a sample mean could come from the hypothesised distribution of sample means i.e. it is highly unlikely that given the hypothesised population mean, such a sample mean could be caused by sampling variability. • This leads us to the conclusion that for a small p-value, the hypothesised distribution of sample means is not correct so the hypothesised mean and null hypothesis are incorrect. • The next question is how small does a p-value have to be to reject the hypothesised mean? • Different tests use a different cut-off point. For example, when searching for the Higgs	➢ 10% or 0.1 ➢ 5% or 0.05. ➢ 1 % or 0.01.	• Write the term “Significance Level” under **Section E** of the board plan and add the following: “If $p < 0.05$ we reject H_0 in favour of H_A” and “ If $p \geq 0.05$ we fail to reject (or we accept) H_0”. • Write the following under **Section E** of the board plan: “If p is low ($< 5\%$) it’s unlikely that the difference between our sample mean and hypothesised mean is due to sampling variability alone. The hypothesised mean is unlikely to be correct and we reject the null hypothesis in favour of the alternative”	• Do students understand that a small p-value suggests a false null hypothesis? • Do students understand that we have yet to decide what p-value is small enough to imply a false null hypothesis? • Do students understand that a p-value can take on any value between 0 & 1? • Do students understand that different tests may use different p-values as the boundary for deciding to reject or fail to reject the null hypothesis? • Do students understand that we will use a p-value of 5% or 0.05? • Do students understand that the cut-off point in the p-value is

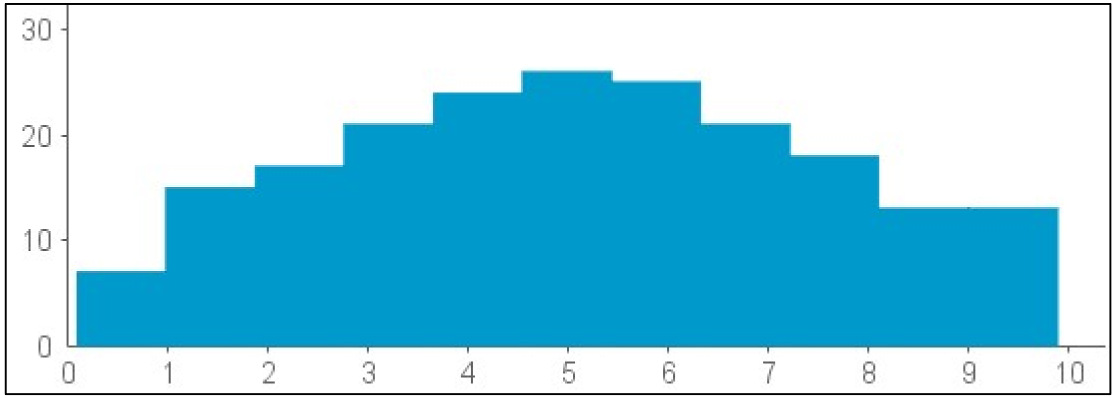
Boson, CERN used a probability of 0.000000286 as their cut-off point. • We are going to use a much larger cut-off of 5% or 0.05. • The cut-off point is often called the level of significance of the test. Our level of significance is 0.05. • If we calculate a p-value less than 0.05, it is unlikely that the difference between our sample mean and population mean is simply caused by sampling variability and we reject the null hypothesis in favour of the alternative. • If we calculate a p-value of 0.05 or greater then it is reasonable to assume that any difference between the mean of our sample and the hypothesised mean is purely down to sampling variability.	➢ We are rejecting the null hypothesis.	• Write the following under **Section E** of the board plan: If p is not low ($\geq 5\%$) it is reasonable to assume that the difference between our sample mean and hypothesised mean is simply due to sampling variability. Therefore there is no evidence to suggest that the hypothesised mean is incorrect and we fail to reject (accept) the null hypothesis. • Write the conclusion for the hypothesis test on the board: "Adult Irish males are not the same height as adult European males".	known as the significance level of the test? • Do students understand that a p-value less than 0.05 means that we reject the null hypothesis in favour of the alternative? • Do students understand that a p-value of 0.05 or greater means that we fail to reject the null hypothesis? • Can students determine if we are rejecting or failing to reject the

• In such case we fail to reject the null hypothesis (or we accept the null hypothesis). • In our case, are we rejecting or failing to reject the null hypothesis? Explain. • In the context of the original claim, what does our rejection of the null hypothesis mean? • It is important to complete the data-handling cycle and make some conclusion. • We've discovered how a deep understanding of sampling variability underpins hypothesis testing. Let's apply our understanding by carrying out a hypothesis test on each claim in **Section E: Student Activity 1**. As already discussed,	➢ We are rejecting the null hypothesis because our p-value is less than 5%. ➢ Students work on completing the hypothesis tests in **Section E: Student Activity 1**.	• Move around the room and make sure students understand what they are expected to do. • If students are having difficulty use appropriate questions to help them understand the concepts needed to complete the hypothesis tests. • See **Appendix C** for the solution to **Section E: Student Activity 1**.	null hypothesis in **Section E: Student Activity 2**? • Can students explain what rejecting the null hypothesis means in the context of the claim? • Do students understand the importance of completing the data-handling cycle and making a conclusion about the claim? • Can students correctly sketch the distribution of sample means? • From their sketch, can students get a sense of whether the null hypothesis is likely to be false or not? • Can students calculate a z-score? • Can students calculate a p-value? • Do students remember to include both tails of the distribution in their calculation of the p-value? • Can students use the p-value to correctly reject or fail to reject the null hypothesis? • Can students use the result of the hypothesis test to make a conclusion about the population?

we will use a significance level of 5% in each test.			

Section B: Student Activity 1

The diagram shows the distribution of schoolbag weights for a population of 200 students.



Q1. Looking at the distribution, describe its:

(a) Shape

(b) Centre

(c) Spread

Q2. If I take a sample of 36 from my population and calculate its mean, what is the likelihood that I will get an answer less than 3?

Very likely	☐
Likely	☐
Somewhat likely	☐
Somewhat unlikely	☐
Unlikely	☐
Very unlikely	☐
Extremely unlikely	☐

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Explain your reasoning.

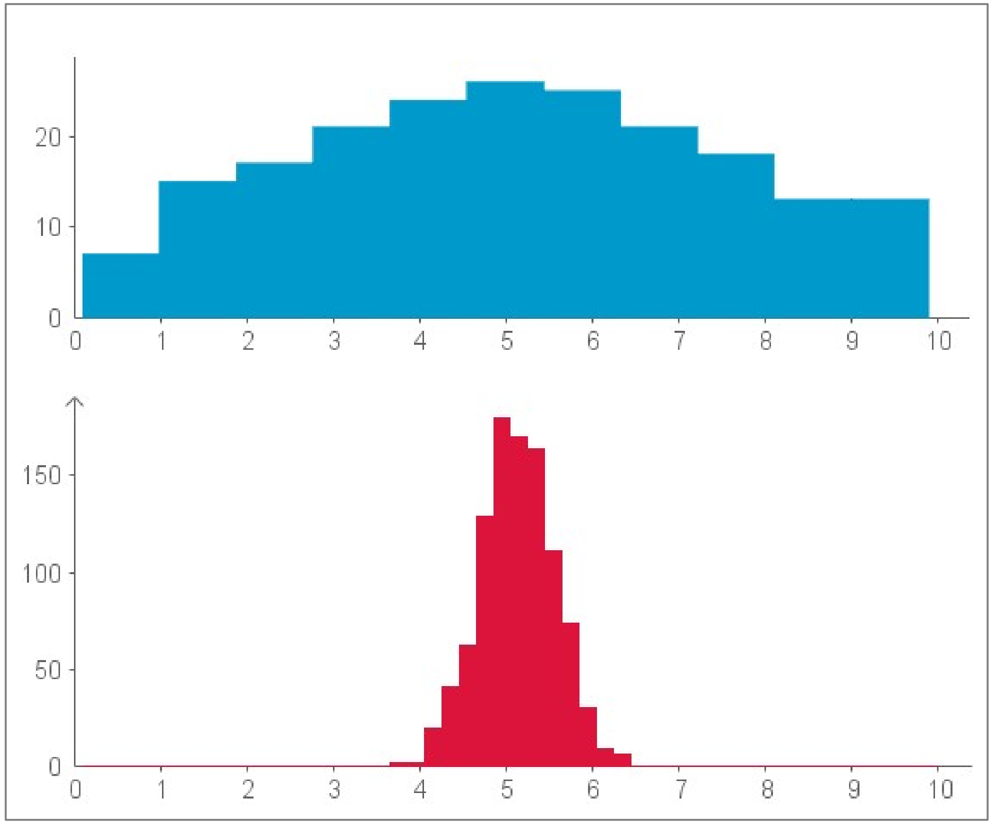
Q3. If a take a sample of 36 from my population and calculate its mean, what is the likelihood that I will get an answer between 4 and 6?

Very likely	☐
Likely	☐
Somewhat likely	☐
Somewhat unlikely	☐
Unlikely	☐
Very unlikely	☐
Extremely unlikely	☐

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Explain your reasoning.

Q4.



Looking at the distribution of sample means, describe its

(a)Shape

(b)Centre

(c)Spread

Q5. Make a statement comparing the centre of the distribution of the sample means to the centre of the population distribution.

Q6. Make a statement comparing the spread of the distribution of the sample means to the spread of the population distribution.

Q7. By examining the distribution of the sample means, complete the following statement:

“For a sample size of 36, it is most likely that I will get a sample mean between __ and __.”

Section B: Student Activity 2

Q1. By referring to the GeoGebra file "**Distribution of Sample Means.ggb**" complete the following table:

Sample size	σ	$\sigma_{\bar{x}}$	Ratio of $\sigma: \sigma_{\bar{x}}$ (to the nearest integer)
16			
25			
36	**2.57**	**0.43**	**6**
49			
64			
81			
100			
n			

Q2. Describe what is happening to the standard deviation of the population (σ) as the size of our 1000 samples increases.

Q3. Describe what is happening to the standard deviation of the sample means ($\sigma_{\bar{x}}$) as the size of our 1000 samples increases.

Q4. Describe the relationship between $\sigma_{\bar{x}}$ and sample size.

Q5. Do the graphs of the distributions back up this relationship? Explain.

Q6. Why does this relationship between $\sigma_{\bar{x}}$ and sample size exist?

Q7. For the given population ($\mu = 5.12, \sigma = 2.57$), a statistician takes 1000 samples of size 100. Write the corresponding values of σ, $\sigma_{\bar{x}}$ and $\sigma : \sigma_{\bar{x}}$ into the Table.

Q8. Describe the relationship between $\sigma_{\bar{x}}$, σ and n(note it may be helpful to refer to the fourth column of the table).

Q9. If, for the given population ($\mu = 5.12, \sigma = 2.57$), a statistician took 1000 samples of size 50, what would the value of $\sigma_{\bar{x}}$ be?

Section C: Student Activity 1

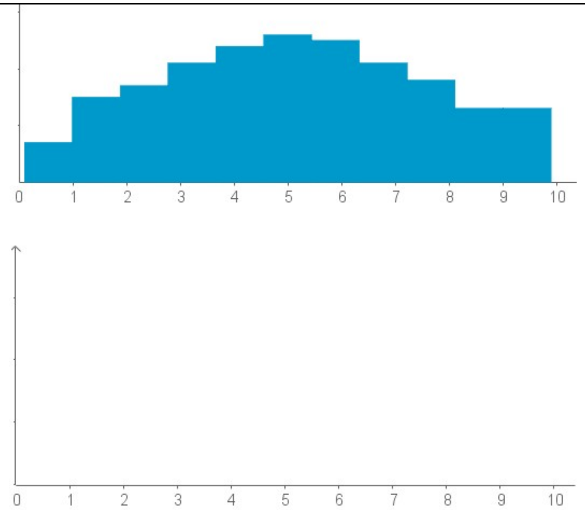
Q1. 10,000 samples of size 100 are to be chosen from our population of schoolbag weights ($\mu = 5.12$, $\sigma = 2.57$).

(a) What shape will the distribution of sample means be?

(b) Write down the value of the mean of the sample means ($\mu_{\bar{x}}$).

(c) Calculate the value of the standard deviation of the sample means ($\sigma_{\bar{x}}$).

(d) In the space below the population distribution, sketch the distribution of sample means. In doing so mark in the values for $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$. Use your information to complete the statements in parts (i) – (iii).



(i) 68% of the sample means will have a value between ___ & ___.

(ii) 95% of the sample means will have a value between ___ & ___.

(iii) 99.7% of sample means will have a value between ___ & ___.

Calculations & Rough Work

(e) What prior knowledge of statistics did you use to complete part (d)?

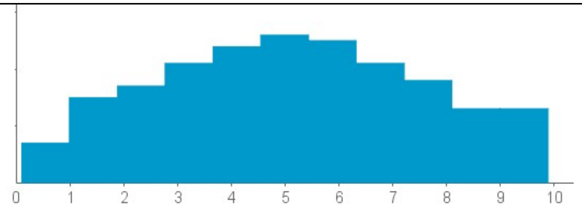
Q2. 10,000 samples of size 50 are to be chosen from our population of schoolbag weights ($\mu = 5.12$, $\sigma = 2.57$).

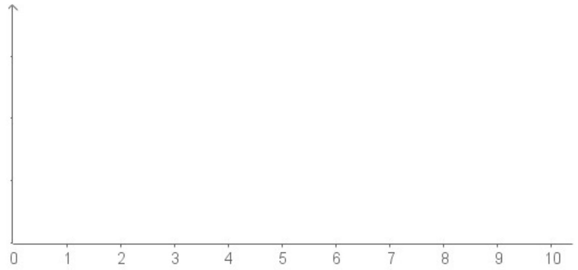
(a) What shape will the distribution of sample means be?

(b) Write down the value of the mean of the sample means ($\mu_{\bar{x}}$).

(c) Calculate the value of the standard deviation of the sample means($\sigma_{\bar{x}}$).

(d) In the space below the population distribution, sketch the distribution of sample means. In doing so mark in the values for $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$. Use your information to complete the statements in parts (i) – (iii).





(i) 95% of the sample means will have a value between ___ & ___.

(ii) Before I choose a random sample of size 50 from my population I can be 95% confident that its mean will be between ___ & ___.

(iii) The 95% confident interval for my sample mean is ___ $\leq \bar{x} \leq$ ___.

Calculations & Rough Work

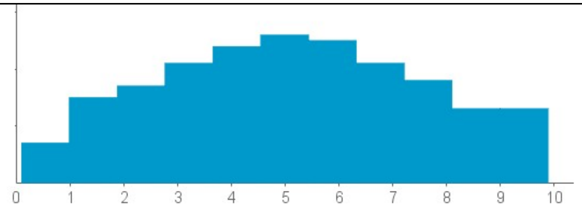
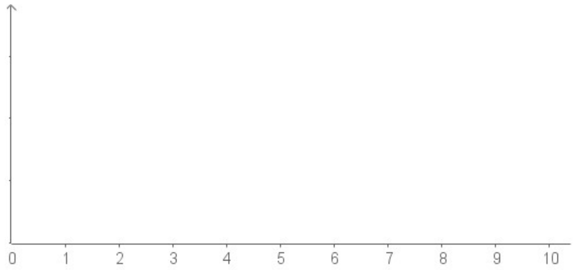
Q3. 10,000 samples of size 30 are to be chosen from our population of schoolbag weights ($\mu = 5.12$, $\sigma = 2.57$).

(a) What shape will the distribution of sample means be?

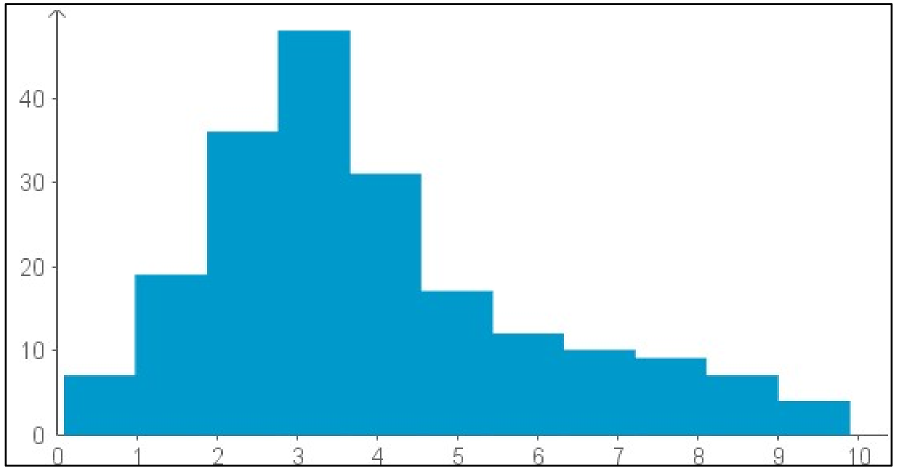
(b) Write down the value of the mean of the sample means ($\mu_{\bar{x}}$).

(c) Calculate the value of the standard deviation of the sample means($\sigma_{\bar{x}}$).

(d) In the space below the population distribution, sketch the distribution of sample means. In doing so mark in the values for $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$. Use your information to complete the statements in parts (i) – (iii).

 	(i) 95% of the sample means will have a value between ___ & ___. (ii) Before I choose a random sample of size 30 from my population I can be 95% confident that its mean will be between ___ & ___. (iii) The 95% confident interval for my sample mean is ___$\leq \bar{x} \leq$___.	Calculations & Rough Work

Q4. A population distribution is shown. The population has a mean of 3.97 ($\mu = 3.97$) and a standard deviation of 2.1 ($\sigma = 2.1$).



(a) What shape is the population distribution?

(b) You decide to take 1000 samples of size 50 from the population and histogram the sample means. What shape will the distribution of sample means be? Explain your reasoning.

(c) John also decides to take 1000 samples, this time of size 20, and histogram their sample means. Will John get a distribution of sample means which is normal? Explain your reasoning.

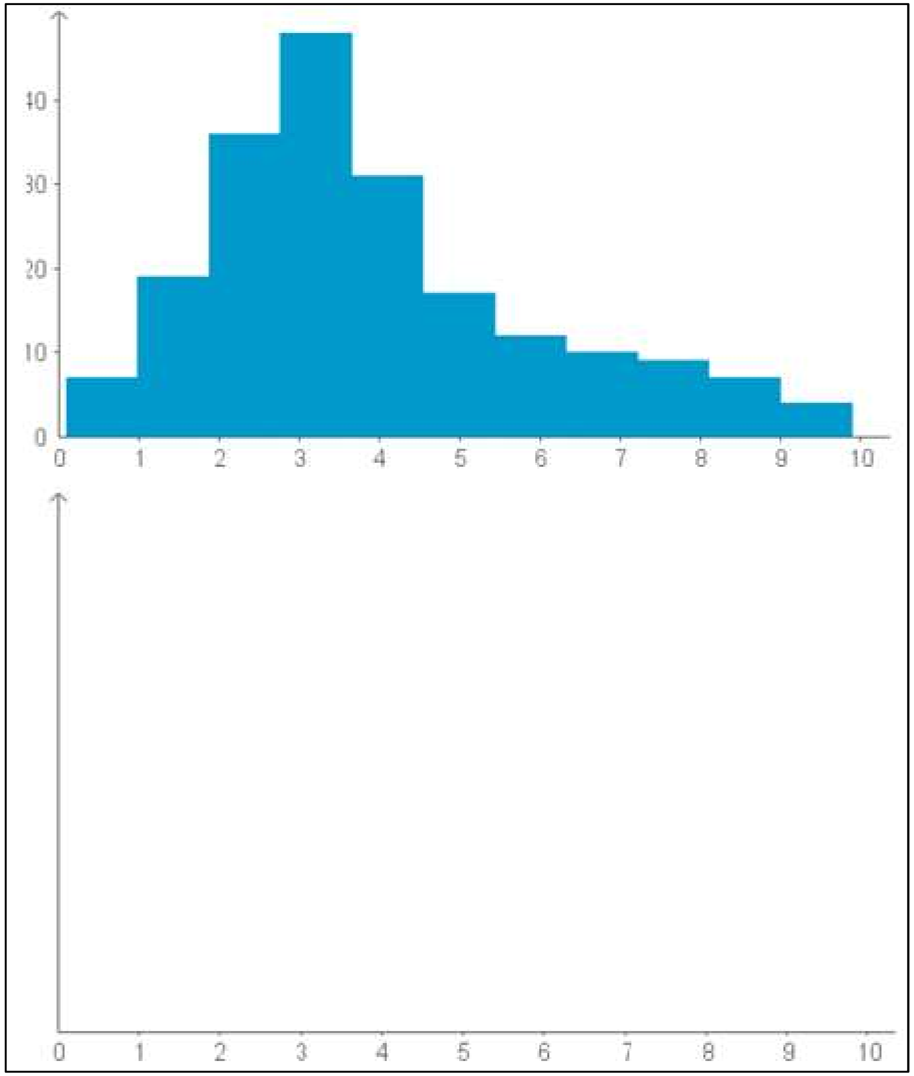
(d) Calculate the mean of your distribution of sample means.

$\mu_{\bar{x}} =$

(e) Calculate the standard deviation of your distribution of sample means.

$\sigma_{\bar{x}} =$

(f) In the space below the population distribution, sketch the distribution of sample means for a sample size of 50.

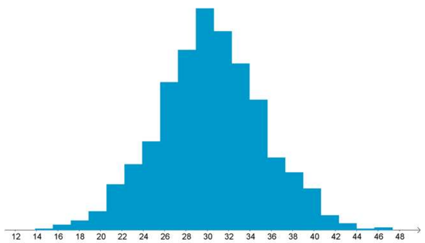
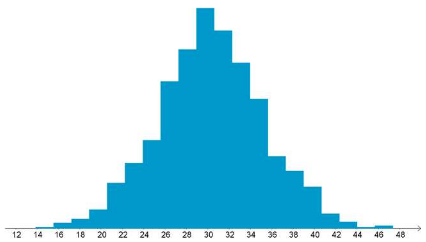
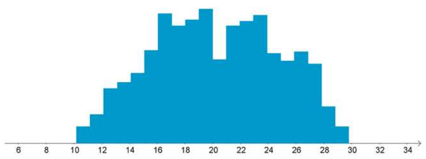
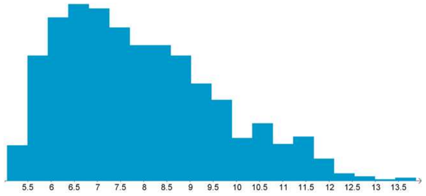
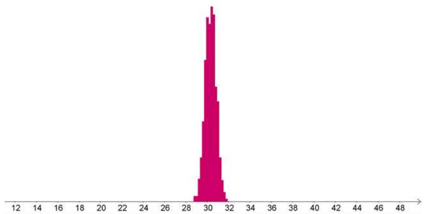
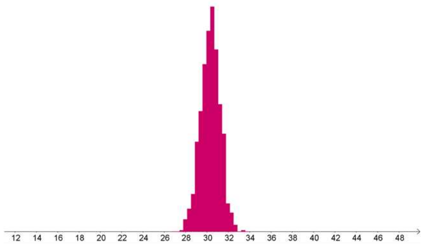
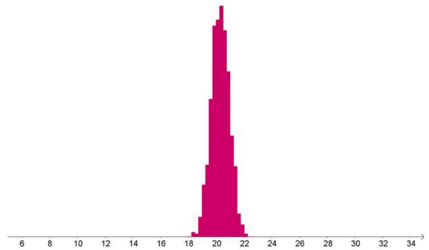
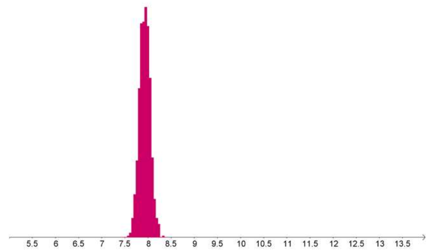


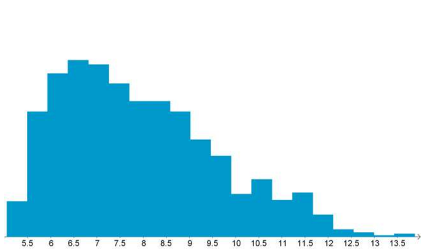
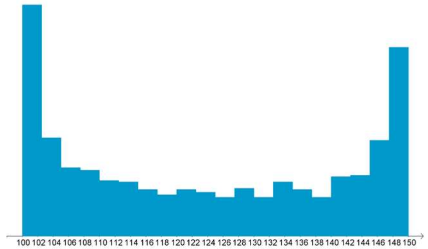
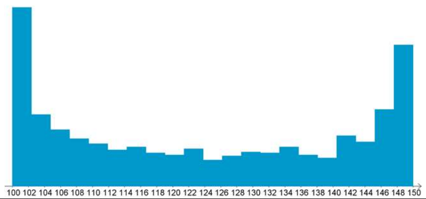
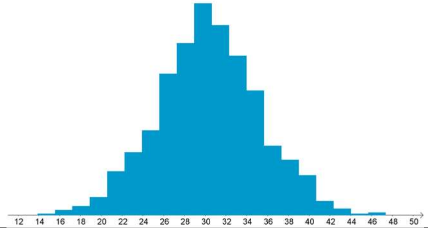
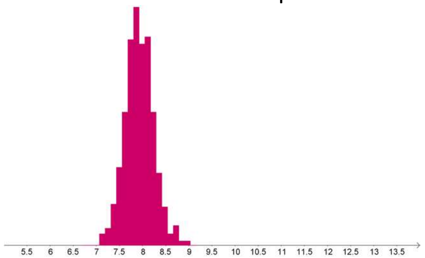
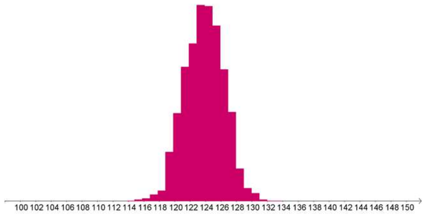
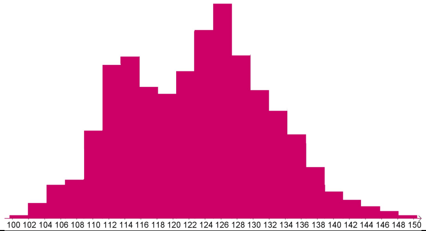
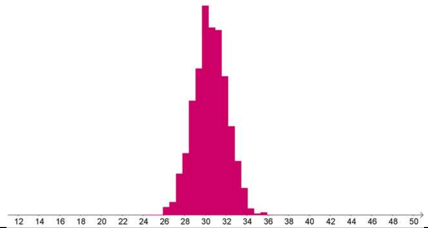
(g) I decide to choose a sample of size 50 from the population. Construct the 95% confidence interval for the mean of this sample.

The 95% C.I. for the mean of a sample of size 50 is _______ $\leq \bar{x} \leq$ _______	Calculations & Rough work

(h) Explain the meaning of the 95% confidence interval for your sample mean.

Section C: Student Activity 2

A1 Population Distribution 	**A2** Population Distribution 	**A3** Population Distribution 	**A4** Population Distribution 
B1 Distribution of 1000 sample means 	**B2** Distribution of 1000 sample means 	**B3** Distribution of 1000 sample means 	**B4** Distribution of 1000 sample means 
C1 $\mu = 30.3 \quad \sigma = 4.94$	**C2** $\mu = 30.3 \quad \sigma = 4.94$	**C3** $\mu = 20.25 \quad \sigma = 4.48$	**C4** $\mu = 7.93 \quad \sigma = 1.68$
D1 $\mu_{\bar{x}} = 30.3 \quad \sigma_{\bar{x}} = \frac{4.94}{\sqrt{100}}$	**D2** $\mu_{\bar{x}} = 30.3 \quad \sigma_{\bar{x}} = \frac{4.94}{\sqrt{30}}$	**D3** $\mu_{\bar{x}} = 20.25 \quad \sigma_{\bar{x}} = \frac{4.48}{\sqrt{50}}$	**D4** $\mu_{\bar{x}} = 7.93 \quad \sigma_{\bar{x}} = \frac{1.68}{\sqrt{200}}$
E1 $\mu_{\bar{x}} = 30.3 \quad \sigma_{\bar{x}} = 0.49$	**E2** $\mu_{\bar{x}} = 30.3 \quad \sigma_{\bar{x}} = 0.9$	**E3** $\mu_{\bar{x}} = 20.25 \quad \sigma_{\bar{x}} = 0.63$	**E4** $\mu_{\bar{x}} = 7.93 \quad \sigma_{\bar{x}} = 0.12$
F1 95% of $\bar{x}$ values lie between 29.35 & 31.29	**F2** 95% of $\bar{x}$ values lie between 28.55 & 32.09	**F3** 95% of $\bar{x}$ values lie between 19.01 & 21.49	**F4** 95% of $\bar{x}$ values lie between 7.69 & 8.16
G1 n=100	**G2** n=30	**G3** n=50	**G4** n=200
H1 95% of $\bar{x}$ values lie between $30.3 \pm 1.96\frac{4.94}{\sqrt{100}}$	**H2** 95% of $\bar{x}$ values lie between $30.3 \pm 1.96\frac{4.94}{\sqrt{30}}$	**H3** 95% of $\bar{x}$ values lie between $20.25 \pm 1.96\frac{4.48}{\sqrt{50}}$	**H4** 95% of $\bar{x}$ values lie between $7.93 \pm 1.96\frac{1.68}{\sqrt{200}}$

A5 Population Distribution 	**A6** Population Distribution 	**A7** Population Distribution 	**A8** Population Distribution 
B5 Distribution of 1000 sample means 	**B6** Distribution of 1000 sample means 	**B7** Distribution of 1000 sample means 	**B8** Distribution of 1000 sample means 
C5 $\mu = 7.93 \quad \sigma = 1.68$	**C6** $\mu = 123.9 \quad \sigma = 17.9$	**C7** $\mu = 123.9 \quad \sigma = 17.9$	**C8** $\mu = 30.3 \quad \sigma = 4.94$
D5 $\mu_{\bar{x}} = 7.93 \quad \sigma_{\bar{x}} = \frac{1.68}{\sqrt{30}}$	**D6** $\mu_{\bar{x}} = 123.9 \quad \sigma_{\bar{x}} = \frac{17.9}{\sqrt{50}}$	**D7** $\mu_{\bar{x}} = 123.9 \quad \sigma_{\bar{x}} = \frac{17.9}{\sqrt{10}}$	**D8** $\mu_{\bar{x}} = 30.3 \quad \sigma_{\bar{x}} = \frac{4.94}{\sqrt{10}}$
E5 $\mu_{\bar{x}} = 7.93 \quad \sigma_{\bar{x}} = 0.31$	**E6** $\mu_{\bar{x}} = 123.9 \quad \sigma_{\bar{x}} = 2.53$	**E7** $\mu_{\bar{x}} = 123.9 \quad \sigma_{\bar{x}} = 5.67$	**E8** $\mu_{\bar{x}} = 30.3 \quad \sigma_{\bar{x}} = 1.56$
F5 95% of $\bar{x}$ values lie between 7.33 & 8.53	**F6** 95% of $\bar{x}$ values lie between 118.9 & 128.8	**F7** Cannot construct a 95% C.I. using z-scores as distribution of sample means is not normal.	**F8** Even though sample size is small, because population is normal can still construct a 95% C.I. using z-scores.
G5 n=30	**G6** n=50	**G7** n=10	**G8** n=10
H5 95% of $\bar{x}$ values lie between $7.93 \pm 1.96\frac{1.68}{\sqrt{30}}$	**H6** 95% of $\bar{x}$ values lie between $123.9 \pm 1.96\frac{17.9}{\sqrt{50}}$	**H7** Cannot construct a 95% C.I. using z-scores as distribution of sample means is not normal.	**H8** 95% of $\bar{x}$ values lie between $30.3 \pm 1.96\frac{4.94}{\sqrt{10}}$

Section D: Student Activity 1

The Gaelic Players Association (GPA) is interested in finding out the typical length of time a club player spends training every week during the football/hurling season. In their research they ask a sample of 100 club players the total time they spent training the previous week. The results from the sample were as follows:

Mean time spent training: 7.2 hours
Standard deviation (s): 1.6 hours
Sample size: 100

The GPA make a statement about the mean amount of time spent training each week **by all Gaelic club players in Ireland**. What should this statement say?

Section E: Student Activity 1

Part (i)

An EU survey reports that the mean height of adult male Europeans is 181 cm with a standard deviation of 7 cm. An Irish newspaper carries out some research on this. They sample 100 adult Irish males and find that the mean of the sample is 179 cm. Based on this sample, are Irish males different in height to other Europeans?

Part (ii)

For each of the claims below state the null hypothesis and the alternative hypothesis.

Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
A. The mean mark in maths for all Leaving Certificate candidates in 2014 was 385 with a standard deviation of 45. A survey of 50 leaving Cert. students in 2016 finds a mean mark of 396 in maths with a standard deviation of 45. John suggests that maths results have not changed since 2014. Is he correct?				

Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
B. The Irish Government published information on the earnings of the population in 2012. This information states that, for people in full-time employment, the mean annual earnings are €39,400 with a standard deviation of €12,920. In 2016, to see if the situation has changed, a research institute surveys 1000 workers and finds that their mean annual income is €38,280. Has the mean annual income changed since 2012?				
C. A car-rental company uses *Evertread tyres* on their cars. Over a number of years the *tyres* have been shown to have lifespan which is normally distributed with a mean of 45 000 km and a standard deviation of 8000 km. The rental company want to see if a new, cheaper brand of tyre – *Saferun tyres* – have as good a lifespan. They fit 25 of their cars with *Saferun tyres* and record their lifespan. For these cars they find a mean lifespan of 43,850 km. Do *Saferun* tyres perform at the same level as *Evertread* tyres?				

Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
D. A national newspaper reported in January 2010 that the mean rent for a 3-bedroom house in Ireland was €824 per month. A Dublin estate agent surveys 40 such properties in the greater Dublin area to see if the same was true there. The estate agent's sample had a mean rent of €1090 with a standard deviation of €480. The estate agent says that his survey shows that the mean rent in Dublin is not the same as the mean rent in the country as a whole. Is he correct?				
E. *Milko* Milk Powder is sold in packets with an advertised mean weight of 1.5 kg. The standard deviation is known to be 0.184 kg. A consumer group wishes to check the accuracy of the advertised mean and takes a sample of 52 packets finding a mean weight of 1.49kgs. Are *Milko* Milk Powder correct with their advertised weight?				

Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
F. In 2007, the EPA reports that the mean pH level in the river Barrow is 6.2 with a standard deviation of 0.9. A new chemical manufacturing company opens along the banks of the Barrow in 2010. In 2016 the company releases a statement to highlight its “green credentials. The report states that its presence on the banks of the river Barrow has had no effect on the river’s pH level. To support their claim they sampled the river water at 30 different locations and found a mean pH level of 5.91. Are the company correct in their claim?				
G. Studies conducted in the 1990s stated that the mean number of cigarettes used daily by smokers was 6.3 with a standard deviation of 1.7. The Irish Government claims that, following the introduction of the smoking ban, this has changed. To check this claim, *GoodHealth* magazine samples 100 smokers and finds the mean number of cigarettes smoked daily by them is 5.8 with a standard deviation of 1.9. Is there evidence to support the government’s claim?				

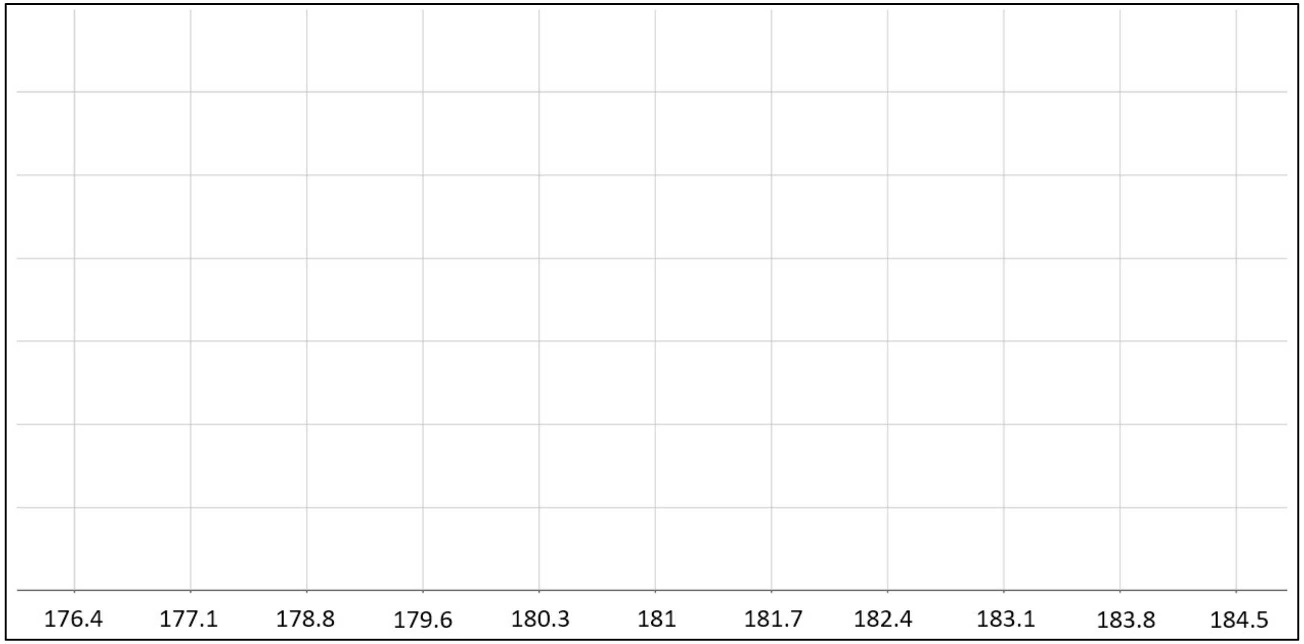
Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
H. A World Health Organisation report says that Irish people have a mean life expectancy of 78.3 years with a standard deviation of 5.7 years. The Donegal Echo newspaper commissions some research to see if people in Donegal have a different life expectancy. The ages of 1000 people from Donegal who died in 2015 are recorded and a mean age of 78.6 years is calculated. Is there evidence to support the Donegal Echo's claim that Donegal people do not have the same life expectancy as the rest of the country?				
I. The 1911 Census reported that the mean number of people living in a house in Ireland was 8.4 with a standard deviation of 2.6. As part of the 1916 celebrations, the property website Duft.ie survey 50 households to see if this situation has changed. Their sample returns a mean of 6.9 people. Is there evidence to support the claim that the number of people living in a house in Ireland has changed?				

Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
J. It is known that when taking a commonly used pain medication, patients usually report that their pain is reduced by a mean of 3.5 points on the pain scale. We are testing a new pain medication in a sample of 22 patients and don't know whether this new medication works better or worse, but in this sample, we find a mean pain reduction of 4.2 points, with a standard deviation of 1.3 points. Does the new drug perform differently to commonly-used pain medication?				

Section E: Student Activity 2

An EU survey reports that the mean height of adult male Europeans is 181 cm with a standard deviation of 7 cm. An Irish newspaper carries out some research on this. They sample 100 adult Irish males and find that the mean of the sample is 178.9 cm. Based on this sample, are Irish males different in height to other Europeans (you may use a significance level of 5%)?

1. On the grid below, sketch the distribution of sample means for samples of size 100 drawn from a population with a mean of 181 cm and a standard deviation of 7 cm.



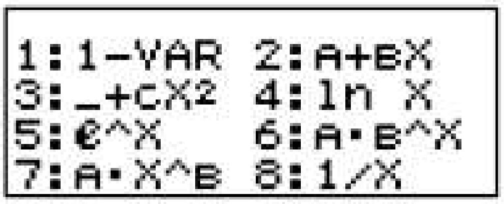
2. If the population distribution is as claimed by the EU survey, how likely is it to get a mean of 178.9 cm from a sample of size 100? Explain your reasoning.

3. Use z-tables to calculate the probability of getting a value of 178.9 cm or less, given the hypothesised distribution.

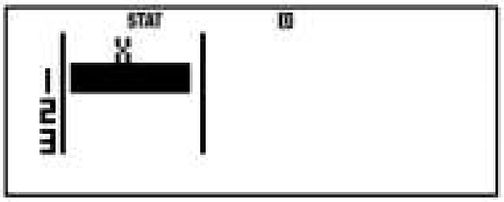
Sketch	Calculation

Mean & Standard Deviation

1. [MODE] [2] [1]



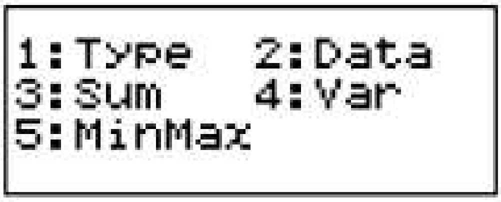
2. After each number is entered press [=]

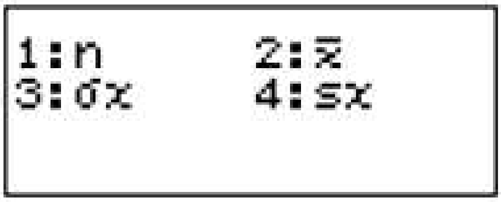


Now that the data is entered press [AC] to begin analysis.
Note: This does not clear the data.

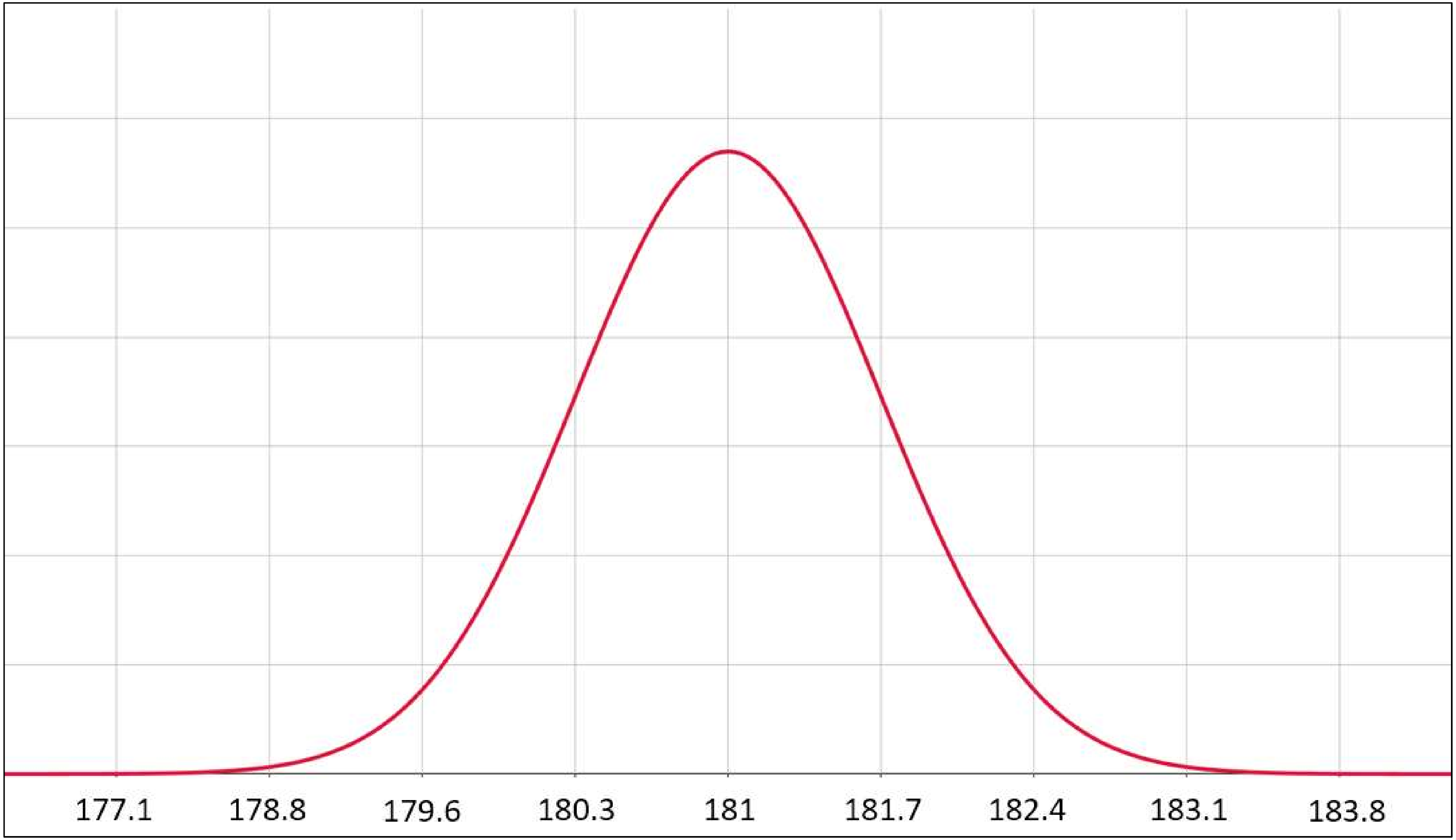
3. **Mean:**
[SHIFT] [1] [4] [2] [=]

4. **Standard Deviation**
press [AC]
[SHIFT] [1] [4] [4] [=]

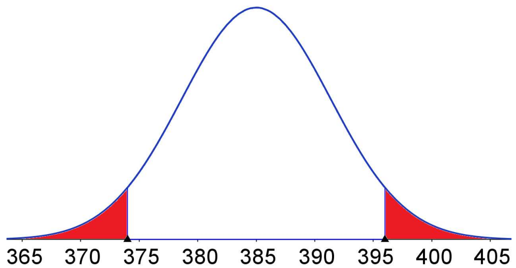
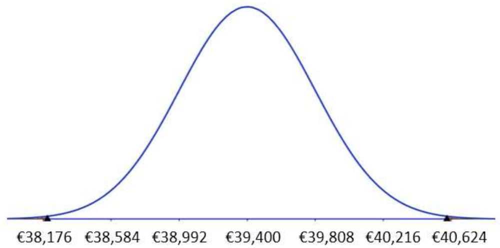


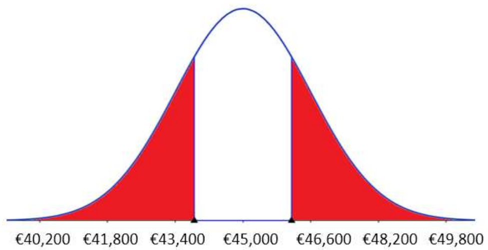
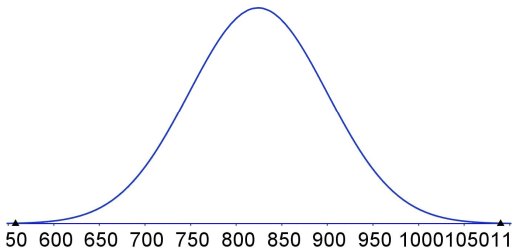


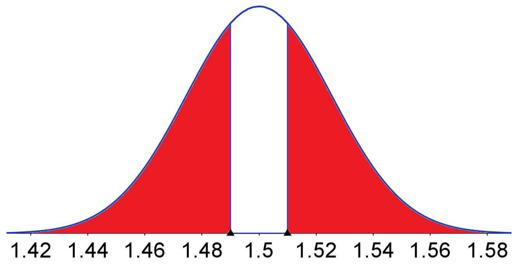
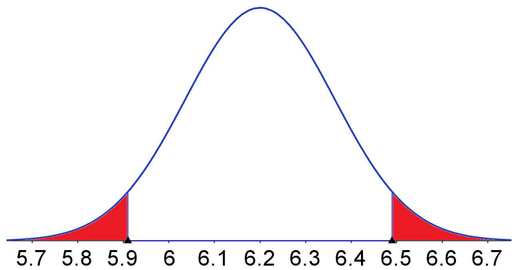
Appendix B – Diagram of the Distribution of Sample Means for **Section E**: Student Activity 2

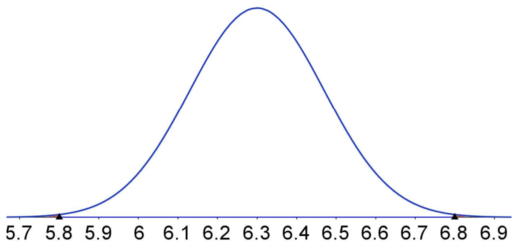
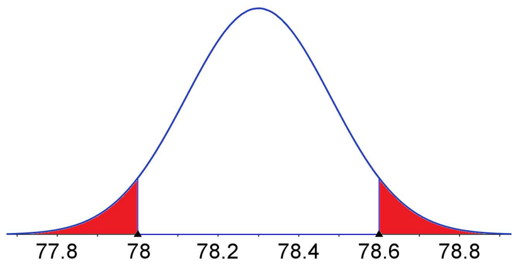


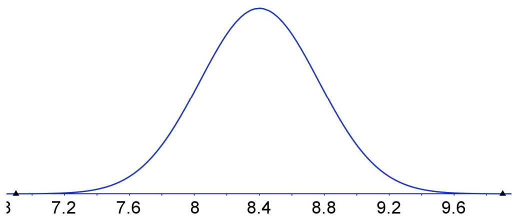
Appendix C: Solutions to **Section E**: Student Activity 1 Part (ii)

Claim	Null Hypothesis	Alternative Hypothesis	Sketch & Calculation of p-value	Conclusion
A. The mean mark in maths for all Leaving Certificate candidates in 2014 was 385 with a standard deviation of 45. A survey of 50 leaving Cert. students in 2016 finds a mean mark of 396 in maths with a standard deviation of 45. John suggests that maths results have not changed since 2014. Is he correct?	$H_{0:}$The mean mark in maths for all LC candidates in 2016 is 385. $H_0: \mu = 385$	$H_{A:}$ The mean mark in maths for all LC candidates in 2016 is not 385. $H_A: \mu \neq 385$	 $z = 1.7285$ $p = 0.0839$	I fail to reject the null hypothesis. It is reasonable to claim that the mean mark in maths for all LC candidates in 2016 is 385.
B. The Irish Government published information on the earnings of the population in 2012. This information states that, for people in full-time employment, the mean annual earnings are €39,400 with a standard deviation of €12,920. In 2016, to see if the situation has changed, a research institute surveys 1000 workers and finds that their mean annual income is €38,280. Has the mean annual income changed since 2012?	$H_{0:}$ The mean earnings for people in full-time employment in 2016 is €39,400. $H_0: \mu = €39{,}400$	$H_{A:}$ The mean earnings for people in full-time employment in 2016 is not €39,400. $H_A: \mu \neq €39{,}400$	 $z = -2.937$ $p = 0.003$	I reject the null hypothesis in favour of the alternative hypothesis. It is not correct to claim that the mean earnings for the full-time employed in 2016 is €39,400.

C. A car-rental company uses *Evertread tyres* on their cars. Over a number of years the *tyres* have been shown to have lifespan which is normally distributed with a mean of 45 000 km and a standard deviation of 8000 km. The rental company want to see if a new, cheaper brand of tyre – *Saferun tyres* – have as good a lifespan. They fit 25 of their cars with *Saferun tyres* and record their lifespan. For these cars they find a mean lifespan of 43,850 km. Do *Saferun* tyres perform at the same level as *Evertread* tyres?	$H_{0:}$ The mean lifespan of Saferun tyres is 45,000 km. $H_0: \mu = 45{,}000\ km$	$H_{A:}$ The mean lifespan of Saferun tyres is not 45,000 km. $H_A: \mu \neq 45{,}000\ km$	 $z = -0.71875$ $p = 0.4723$ Note: Sample size is smaller than 30 but we can still assume that the distribution of sample means is normal because we are told that the population is normal.	I fail to reject the null hypothesis. It is fair to claim that the mean lifetime of Saferun tyres is 45,000 km.
D. A national newspaper reported in January 2010 that the mean rent for a 3-bedroom house in Ireland was €824 per month. A Dublin estate agent surveys 40 such properties in the greater Dublin area to see if the same was true there. The estate agent's sample had a mean rent of €1090 with a standard deviation of €480. The estate agent says that his survey shows that the mean rent in Dublin is not the same as the mean rent in the country as a whole. Is he correct?	$H_{0:}$ The mean rent for a 3-bedroom house in the greater Dublin area is €824. $H_0: \mu = €824$	$H_{A:}$ The mean rent for a 3-bedroom house in the greater Dublin area is not €824. $H_A: \mu \neq €824$	 $z = 3.505$ $p = 0.0005$ (=0)	I reject the null hypothesis in favour of the alternative. It is not fair to claim that the mean rent for a 3-bedroom house in the greater Dublin area is €824.

E. *Milko* Milk Powder is sold in packets with an advertised mean weight of 1.5 kg. The standard deviation is known to be 0.184 kg. A consumer group wishes to check the accuracy of the advertised mean and takes a sample of 52 packets finding a mean weight of 1.49kgs. Are *Milko* Milk Powder correct with their advertised weight?	$H_{0:}$ The weight of Milko Milk Powder packets is 1.5 kg. $H_0: \mu = 1.5\ kg$	$H_{A:}$ The weight of Milko Milk Powder packets is not 1.5 kg. $H_A: \mu \neq 1.5\ kg$	 $z = -0.392$ $p = 0.695$	I fail to reject the null hypothesis. It is fair to claim that the mean weight of Milko Milk Powder packets is 1.5 kg.
F. In 2007, the EPA reports that the mean pH level in the river Barrow is 6.2 with a standard deviation of 0.9. A new chemical manufacturing company opens along the banks of the Barrow in 2010. In 2016 the company releases a statement to highlight its “green credentials”. Part of the report states that its presence on the banks of the river Barrow has had no effect on the river’s pH level. To support their claim they sampled the river water at 30 different locations and found a mean pH level of 5.91. Are the company correct in their claim?	$H_{0:}$ The mean pH level in the river Barrow in 2016 is 6.2. $H_0: \mu = 6.2$	$H_{A:}$ The mean pH level in the river Barrow in 2016 is not 6.2. $H_A: \mu \neq 6.2$	 $z = -1.765$ $p = 0.078$	I fail to reject the null hypothesis. It is fair to claim that the mean pH level in the river Barrow in 2016 is 6.2.

G. Studies conducted in the 1990s stated that the mean number of cigarettes used daily by smokers was 6.3 with a standard deviation of 1.7. The Irish Government claims that, following the introduction of the smoking ban, this has changed. To check this claim, *GoodHealth* magazine samples 100 smokers and finds that the mean number of cigarettes smoked daily by them is 5.8 with a standard deviation of 1.9. Is there evidence to support the government's claim?	$H_{0:}$ The mean number of cigarettes smoked daily in Ireland following the smoking ban is 6.3. $H_0: \mu = 6.3$	$H_{A:}$ The mean number of cigarettes smoked daily in Ireland following the smoking ban is not 6.3. $H_A: \mu \neq 6.3$	 $z = -2.941$ $p = 0.003$	I reject the null hypothesis in favour of the alternative. It is unfair to claim that the mean number of cigarettes smoked daily in Ireland since the smoking ban is 6.3.
H. A World Health Organisation report says that Irish people have a mean life expectancy of 78.3 years with a standard deviation of 5.7 years. The Donegal Echo newspaper commissions some research to see if people in Donegal have a different life expectancy. The ages of 1000 people from Donegal who died in 2015 are recorded and a mean age of 78.6 years is calculated. Is there evidence to support the Donegal Echo's claim that Donegal people do not have the same life expectancy as the rest of the country?	$H_{0:}$ The mean life expectancy of Donegal people is 78.3 years. $H_0: \mu = 78.3$	$H_{A:}$ The mean life expectancy of Donegal people is not 78.3 years. $H_A: \mu \neq 78.3$	 $z = 1.664$ $p = 0.096$	I fail to reject the null hypothesis. It is fair to claim that the mean life expectancy of Donegal people is 78.3 years.

I. The 1911 Census reported that the mean number of people living in a house in Ireland was 8.4 with a standard deviation of 2.6. As part of the 1916 celebrations, the property website Duft.ie survey 50 households to see if this situation has changed. Their sample returns a mean of 6.9 people. Is there evidence to support the claim that the number of people living in a house in Ireland has changed?	$H_{0:}$ The mean number of people living in a house in Ireland in 2016 is 8.4. $H_0: \mu = 8.4$	$H_{A:}$ The mean number of people living in a house in Ireland in 2016 is not 8.4. $H_A: \mu \neq 8.4$	 $z = -4.079$ p =0	I reject the null hypothesis in favour of the alternative. It is not fair to claim that the mean number of people living in a house in Ireland in 2016 is 8.4.
J. It is known that when taking a commonly used pain medication, patients usually report that their pain is reduced by a mean of 3.5 points on the pain scale. We are testing a new pain medication in a sample of 22 patients and don't know whether this new medication works better or worse, but in this sample, we find a mean pain reduction of 4.2 points, with a standard deviation of 1.3 points. Does the new drug perform differently to commonly-used pain medication?	$H_{0:}$ The mean reduction in pain level by patients using the new medication is 3.5 points. $H_0: \mu = 3.5$	$H_{A:}$ The mean reduction in pain level by patients using the new medication is not 3.5 points. $H_A: \mu \neq 3.5$	Cannot carry out a z-test as we cannot guarantee that the distribution of sample means is normal – due to small sample size (n<30) and not knowing if population is normally distributed.	Cannot make a conclusion with given data.

Appendix D: Schoolbag Weights for Printing

The values below represent a population of 200 schoolbag weights and are to be used to allow students experience sampling during **Section A** of this Teaching and Learning plan.

To prepare for the sampling activity you need to:

1. Print out the weights below on A4 sticker sheets and stick each one onto a piece of card.
2. Shuffle the cards and place them into some container (a bag, a box) ready for sampling.

To save time during the lesson, you might want to prepare a few sets of weights so that several groups of students can carry out sampling simultaneously.

0.1	1	1	11	1.6	21	2	31	3	41
0.1	2	1	12	1.7	22	2	32	3	42
0.5	3	1	13	1.9	23	2	33	3	43
0.5	4	1.1	14	2	24	2	34	3	44
0.5	5	1.1	15	2	25	2.5	35	3	45
0.6	6	1.2	16	2	26	2.5	36	3	46
0.8	7	1.3	17	2	27	2.5	37	3	47
1	8	1.5	18	2	28	2.5	38	3	48
1	9	1.5	19	2	29	2.6	39	3	49
1	10	1.5	20	2	30	3	40	3	50

3	51	3.9	61	4	71	4	81	5	91
3	52	4	62	4	72	4.1	82	5	92
3	53	4	63	4	73	4.1	83	5	93
3	54	4	64	4	74	4.5	84	5	94
3	55	4	65	4	75	4.6	85	5	95
3.5	56	4	66	4	76	4.7	86	5	96
3.5	57	4	67	4	77	4.8	87	5	97
3.5	58	4	68	4	78	4.9	88	5	98
3.5	59	4	69	4	79	5	89	5	99
3.5	60	4	70	4	80	5	90	5	100

5	101	5.5	111	6	121	6	131	6.6	141
5	102	5.5	112	6	122	6	132	6.7	142
5	103	5.5	113	6	123	6.1	133	6.7	143
5	104	5.6	114	6	124	6.2	134	6.8	144
5	105	5.6	115	6	125	6.2	135	6.8	145
5.1	106	5.7	116	6	126	6.5	136	6.8	146
5.1	107	5.8	117	6	127	6.5	137	6.9	147
5.2	108	5.9	118	6	128	6.5	138	7	148
5.3	109	5.9	119	6	129	6.5	139	7	149
5.4	110	6	120	6	130	6.6	140	7	150

7	151	**7.9**	161	**8**	171	**9**	181	**9.5**	191
7	152	**7.9**	162	**8**	172	**9**	182	**9.5**	192
7	153	**8**	163	**8**	173	**9**	183	**9.5**	193
7	154	**8**	164	**8.1**	174	**9**	184	**9.7**	194
7	155	**8**	165	**8.4**	175	**9**	185	**9.7**	195
7.1	156	**8**	166	**8.5**	176	**9**	186	**9.9**	196
7.4	157	**8**	167	**8.8**	177	**9**	187	**9.9**	197
7.5	158	**8**	168	**8.9**	178	**9.1**	188	**9.9**	198
7.8	159	**8**	169	**9**	179	**9.3**	189	**9.9**	199
7.8	160	**8**	170	**9**	180	**9.4**	190	**9.9**	200