Teaching & Learning Plans

Inequalities

Junior Certificate Syllabus

Project Maths Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plans: Inequalities

Project Maths Tionscadal Mata
Development Team

Aims

The aim of this series of lessons is to enable students to:

- enable students to understand the relationship between numbers
- enable students to represent inequalities on the number line
- enable students to solve linear inequalities and relate these to everyday life

Prior Knowledge

Students have prior knowledge of:

- Sets
- Number systems
- How to represent all number systems on the number line
- Order of numbers on the number line
- Patterns including: completing tables and drawing graphs of patterns
- Linear equations in one unknown

Learning Outcomes

As a result of studying this topic, students will be able to:

- determine if a number is less than, less than or equal to, greater than or greater than or equal to another number
- represent solutions to inequalities on the number line
- simplify and solve linear inequalities by table, graph and/or formula

Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students, may engage with the activities in a relatively straightforward way while the more able students should engage in more open-ended and challenging activities.

In interacting with the whole class, teachers can make adjustments to meet the needs of all of the students. **For example, some students may engage with some of the more challenging questions for example question 6 in Section H: Student Activity 1.**

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to articulate their mathematics openly and to share their learning.

Relationship to Junior Certificate Syllabus

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
4.7 Equations and Inequalities	Using a variety of problem solving strategies to solve equations and inequalities. They identify the necessary information, represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.	Solve linear inequalities in one variable of the form $g(x) \leq k$ where $g(x) = ax + b$, $a \in N$ and $b, k, \in Z$; **$k \leq g(x) \leq h$ where** **$g(x) = ax + b$ and** **$k, a, b, h, \in Z$ and** **$x \in R$**

Resources Required

Graph matching exercises and Tarsias contained in this Teaching and Learning Plan need to be laminated and cut up.

For Graph matching exercises see **Section C: Student Activity 1** page 37 and **Section I: Student Activity 3** page 58.

For Tarsias see **Section D: Student Activity 1** page 41 and **Section I: Student Activity 2** page 56.

<table>
<tr><th colspan="4">Lesson Interaction</th></tr>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Checking Understanding</th></tr>
<tr><td colspan="4">Section A: Revision of <, >, ≤ and ≥ symbols.</td></tr>
<tr><td>» Which would you prefer to have, 4 euro or 6 euro?

» Is 4 less than or greater than 6?

» Is 3 less than or greater than 1?

» Can you think of words or phrases that mean less than?

» Can you think of words or phrases that mean greater than?</td><td>• 6

• Less than

• Greater than

• Smaller

• Bigger</td><td>» Write "4 is less than 6" on the board.

» Write "3 is greater than 1" on the board.</td><td>» Do students understand the difference between less than and greater than?</td></tr>
<tr><td>» In mathematics we use the "<" symbol to represent less than.

» Look at the shape of the symbol. The shape of this symbol goes from small to big.

» We use the ">" symbol to represent greater than. The shape of this symbol goes from big to small.</td><td></td><td>» Write the following table on the board.
<table><tr><td>Less than</td><td><</td></tr><tr><td>Greater than</td><td>></td></tr></table></td><td></td></tr>
<tr><td>» Write 8 is greater than 5 in symbol format.

» Write 4 is less than 5 in symbol format.</td><td>• 8 > 5

• 4 < 5</td><td></td><td>» Do students know that < means less than and > means greater than?</td></tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Is -3 less than or greater than 4? » Which would you prefer to owe your friend, €3 or €2? » Is -3 less than or greater than -2? Is -3 less than or greater than -4? » Is -9 less than or greater than -7? » Is 5 < or > 5?	• -3 < 4 • €2 • -3 < -2 • -3 > -4 • -9 < -7 • Neither, 5 is equal to 5	» Write each expression on the board in words and symbols.	» Do students understand that -3 < -2 etc.?
» Do questions 1 and 2 in **Section A: Student Activity 1**.	• Students complete questions 1 and 2 in **Section A: Student Activity 1**.	» Distribute **Section A: Student Activity 1**.	
» Mathematicians use ≤ to represent less than or equal to and ≥ to represent greater than or equal to. These are also known as the inclusive inequalities. » Can you think of other words or phrases that might be used to describe less than or equal to? » Can you think of other words or phrases that might be used to describe greater than or equal to? » Complete the table on the board.	• At most No more than • At least Not less than • Students complete the table on the board.	» Draw the table containing the following words on the board and add the symbols when students have had an opportunity to complete the table in their exercise books. Less than < Greater than > Less than or equal to ≤ Greater than or equal to ≥	» Do students understand the difference between less than and less than and equal to?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» When the statement uses the symbols <, >, ≤ or ≥ we call it an inequality and when = is used we call it an equation. » Write, using the symbols above, examples of inequalities and equations.	• Students write out their own examples.	» Ask students to come to the board to present their own examples.	» Do students understand what is meant by an inequality and how it differs from an equation? » Can students use the ≤ and ≥ symbols correctly?
» How many solutions does the equation $x = 7$ have? » How many solutions does the inequality $x > 7$, $x \in N$ have? » Emily has at least €200 in her savings account. How would you write this using mathematical symbols? » In pairs, complete questions 3 and 4 in **Section A: Student Activity 1**.	• One, since the equation states that x is equal to 7. • Infinite, x could be 8, 9, 10, … • $s \geq 200$ • In pairs students discuss and compare their answers.	» Circulate the room and offer assistance to students when required. » Show students where to find questions 3 and 4 in **Section A: Student Activity 1**, which has been already distributed.	» Can students convert context based questions to mathematical language?

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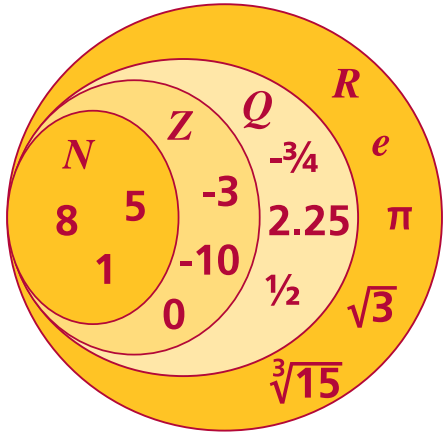
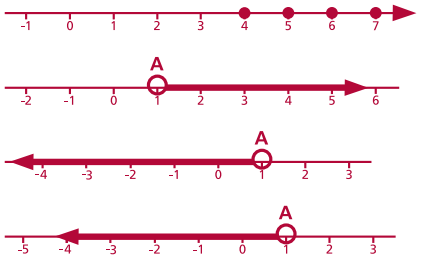
Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section B: Revision of Number Systems.			
» Give me some examples of Natural Numbers?	• {1, 2, 3, 4, 5…}	» Draw the following diagram on the board and add numbers students suggest where appropriate. 	» Can students use sets and set notation to show what they understand by Natural Numbers?
» What letter is normally used to denote the set of Natural Numbers?	• N		
» Can Natural Numbers be represented on the number line?	• Yes		
» Give me some examples of Integers?	• {…-3.-2,-1,0,1,2,3…}		
» What letter is normally used to denote the set of Integers?	• Z		
» Are all Natural Numbers Integers? Give examples.	• Yes. {1, 2, 3…}		
» Can Integers be represented on the number line?	• Yes		
» Draw a number line and represent the Natural Numbers and Integers represented on the diagram on the board on this number line.	• Students draw a number line and represent the Natural Numbers and Integers on it.		» Can students represent Natural Numbers and Integers on the number line?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» What are Rational Numbers?	• Numbers that can be written as fractions.		
» Can you give me an example of a Rational Number?	• For example -¾, 2.25, ½.		
» What letter is normally used to represent the set of Rational Numbers?	• $\mathbb{Q}$		
» Is 3 a Rational Number?	• Yes. It can be written as a fraction i.e. $^3/_1$ or $^6/_2$ etc.	» Write $^3/_1$ or $^6/_2$ on the board.	» Can students recall how to write equivalent fractions?
» Are all Integers Rational Numbers? Why?	• Yes. All Integers can be written as fractions e.g. -8 = $^{-24}/_3$ or $^{-16}/_2$. Also 5 = $^{10}/_2$ or $^{20}/_4$ etc.	» Write the following on the board: -8 = $^{-24}/_3$ or $^{-16}/_2$ 5 = $^{10}/_2$ or $^{20}/_4$	
» Can Rational Numbers be represented on the number line?	• Yes		
» Is 2.5 a Rational Number and why?	• Yes. It can be written as a fraction.	» Write $^5/_2$ = $^{10}/_4$ on the board.	
» What are the numbers that cannot be written as a fraction called?	• Irrational		
» Give examples of Irrational Numbers?	• $\sqrt{2}$,$\sqrt{3}$ or π are examples of irrational numbers. The square root of every prime number is irrational.		» Do students remember what a prime number is?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» What are the combined Rational and Irrational Numbers called?	• Real Numbers.	**Explain** clearly that Z is a subset of R, while the set of Rational and Irrationals are disjoint.	» Are students aware of and do they understand the different types of numbers?
» What letter represents set of Real Numbers?	• R		
» Are Rational Numbers also Real Numbers?	• Yes		» Do students see that asking for an example of a Real Number that is not a Rational Number is another way of asking for an irrational number?
» Are Irrational Numbers also Real Numbers?	• Yes		
» Give examples of Real Numbers, Rational Numbers and Irrational Numbers.	• Students give examples of Real Numbers, Rational Numbers and Irrational Numbers.		» Do students understand the diagram that is on the board to represent the number systems?
» Can you give an example of a Real Number that is not a Rational Number?	• For example $\sqrt{5}$,$\sqrt{7}$ or π.		
» Is it possible to represent the Irrational Numbers accurately on the number line? Why?	• No, not accurately. We can make estimates, but as they are non-repeating decimals it depends on the degree of accuracy required.	**Note:** It is possible to use constructions to locate irrationals along the number line. The inherent inaccuracy then arising from the width of the pencil and other limitations associated with constructions can be explored.	
» Answer the questions in **Section B: Student Activity 1**.	• Students work on **Section B: Student Activity 1**.	» Distribute **Section B: Student Activity 1**. » Circulate the room and offer assistance where required.	» Can students distinguish between the different number systems?

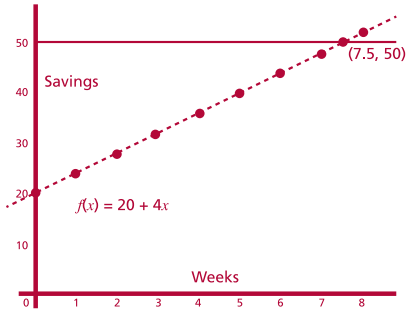
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section C: Number Systems and the Number Line.			
» Where along the number line on the board does the set of Natural Number begin? Does it include 1? Which direction is the arrow pointing? As the arrow is on a continuous line, what set of numbers do you think is being represented on the following number line.	• 1 Yes. To the right. $x \geq 1, x \in R$	» Draw the number line on the board. 	
» If instead of a closed circle at 1 we had an open circle, how do you think the inequality would differ? **Note:** When solving inequalities it is very important that the number system to which the number belongs is stressed. » What does each of the diagrams on the board represent?	• It would be greater than rather than greater than or equal to. • $x \geq 4, x \in N$ • $x > 1, x \in R$ • $x \leq 1, x \in R$ • $x < 1, x \in R$	» Draw the following diagrams on the board and discuss their differences. 	» Can students distinguish between the different inequalities represented on these number lines?
» These packages contain a number of statements, mathematical inequalities and number lines. For every statement I want you to match the statement, with the appropriate mathematical inequality and number line.	• Students work in pairs discussing and comparing their work.	» Distribute packages containing the activity from **Section C: Student Activity 1**. » Circulate the room and offer assistance when required.	» Can students represent the inequalities correctly on the number line?

Section D: Solving inequalities of the form $ax + b < k$.

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» See question 1 **Section D: Student Activity 1**. » Annika has seen three pairs of trainers she likes. They cost €50, €55 and €68. She already has saved €20 and gets €4 pocket money per week at the end of each week. Annika is wondering how soon she can buy one of these pairs of trainers. What different methods can you use to help her solve this problem? » Which trainers will she be able to buy first? » Now I want you to arrive at a strategy to solve this problem. I will be asking some of you to present your strategy to the class and I will want you to discover the earliest, when Annika can afford to buy one of these pairs of trainers.	• €50 **Trial and error:** She has €20 and will need another €30 so 30 divided by 4 equals 7.5. So after 8 weeks she will have more than €50 and she will be able to buy the trainers. • A table She can afford the €50 trainers at the end of week 8.	» Distribute **Section D: Student Activity 1**. » Give students time to come up with their strategies and then allow as many students as possible to present their strategy, writing their tables, graphs, calculation etc. on the board. Allow all strategies to remain on the board if possible until a solution involving an inequality is produced. » Students may begin with a table and then move to algebra. This is perfectly acceptable. » Students may produce other strategies, for example diagrams, and if so, acknowledge these strategies.	» Can students solve the problem by trial and error? » Can students solve the problem by the table method?

Week	Amount saved
0	€20
1	€24
2	€28
3	€32
4	€36
5	€40
6	€44
7	€48
8	€52

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Do you see any pattern forming?	• $20 + 4x$, where x is the number of weeks. • A graph 	**Note:** It is also worth discussing that she will be able to afford the cheapest pair at any time after the earliest date and this underlines the difference between an equation and an inequality.	» Can students solve the problem by means of a graph? » Do students understand that as the pocket money is paid at the end of the week it will be the end of week 8 before Annika can afford the trainers and not half way through week 7 as the graph appears to indicate?
» What formula is represented in your graph?	• $f(x) = 20 + 4x$		
» Why did you use a dashed line?	• Because the data is discrete	» A student, who solved the problem using an equation, may be asked to present their solution at this stage.	
» Now we will try and put this problem in mathematical language.			
» In x weeks how much has she saved in total?	• $4x$	» Write the inequality on the board as it evolves and find its solution.	
» How much will she have saved in total after x weeks?	• $20 + 4x$		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» In order to be able to buy the trainers, what is the minimum amount she will have to save?	• 50	» A student, who solved the problem using algebra solely, may be asked to present their solution at this stage.	
» Write this using mathematical symbols.	• $20 + 4x \geq 50$		
» Now remembering how we solved an equation, can you solve this inequality?	• Stabilisers and as follows: $\begin{array}{l\|c\|l} -20 & 20 + 4x \geq 50 & -20 \\ \div 4 & 4x \geq 30 & \div 4 \\ & x \geq 7.5 & \end{array}$	» Write the inequality on the board as it evolves and find its solution.	» Can students translate the problem into an inequality? » Can students solve the inequality?
» When does she get her pocket money?	• At the end of each week.		» Do students understand that solving a linear inequality is similar to solving a linear equation?
» So when will she be able to afford the €50 trainers?	• It will be at least 8 weeks before she can afford these trainers.		» Can students explain their answer in the context of the question?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» What does it mean to solve an equation?	• Find a value(s) for x that makes the equation true.		
» What does it mean to solve an inequality?	• Find the set of values for x that make the inequality true.		
» How does an equation and an inequality differ?	• An equation uses the = symbol and an inequality uses one of the <, >, ≤ or ≥ symbols. An inequality can have more than one solution.	» Write the differences on the board. » Tell the students how it is possible for equations to have more than one solution and distinguish this from the solution to an inequality whose solution set contains a range of values of x.	
» Solve the following inequality $2x - 3 < 7$, $x \in N$. Show your calculations.	$\begin{array}{l\|l\|l} +3 & 2x - 3 < 7 & +3 \\ \div 2 & 2x < 10 & \div 2 \\ & x < 5 & \end{array}$	» Write the requisite inequality and calculations on the board.	
» List the possible outcomes of the inequality and show on the number line.	• {1, 2, 3, 4}  	» Write the possible outcomes and their representation on the number line on the board.	» Can students see the significance of the different domains ($x \in N$ and $x \in R$) for example, when representing the solutions to inequalities?
» If x had been an element of R ($x \in R$) how would the solution appear on number line?	 		
» Complete questions 2-4 in **Section D: Student Activity 1**.	• Students work on questions 2-4 in **Section D: Student Activity 1**.	» Circulate the room and offer assistance when required.	» Do students distinguish between being asked for a possible solution to an inequality and the solution?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Solving inequalities of the form $ax + b < cx + d$.			
» Read question 1 **Section D: Student Activity 2** in your worksheet. John has 18 ten-cent coins in his wallet and Owen has 22 five-cent coins in his wallet. Each day, they decide to take one coin from their wallets and put it into a money box, until one of them has no more coins left in their wallet. When did Owen have more money than John in his wallet? » Now I want you to arrive at a strategy to solve this problem. I will be asking some of you to present your strategy to the class and explain how you arrived at the answer.		» Distribute **Section D: Student Activity 2**. » In this case get students to work individually. » Give the students time to individually develop a strategy. » Observe the students' reactions and strategies. » Encourage students to develop more than one strategy. » Circulate the room and pick strategies used by students that you would like presented on the board. Note students may present different strategies or combinations of strategies to those listed in this T&L and it is fine to use those.	

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Checking Understanding</th></tr>
<tr>
<td></td>
<td>
• Student 1: I designed a picture to represent the ten-cent coins and another set in a different colour to represent the five-cent coins and acted out the problem and got day 15 as my solution.

• Student 2: I made out the following tables and noticed after day 14 Owen had more money than John.
<table>
<tr><td colspan="4">John</td></tr>
<tr><td>~~10~~</td><td>~~10~~</td><td>~~10~~</td><td>~~10~~</td></tr>
<tr><td>~~10~~</td><td>~~10~~</td><td>~~10~~</td><td>~~10~~</td></tr>
<tr><td>~~10~~</td><td>~~10~~</td><td>~~10~~</td><td>~~10~~</td></tr>
<tr><td>~~10~~</td><td>~~10~~</td><td>10</td><td>10</td></tr>
<tr><td>10</td><td>10</td><td></td><td></td></tr>
</table>
<table>
<tr><td colspan="4">Owen</td></tr>
<tr><td>~~5~~</td><td>~~5~~</td><td>~~5~~</td><td>~~5~~</td></tr>
<tr><td>~~5~~</td><td>~~5~~</td><td>~~5~~</td><td>~~5~~</td></tr>
<tr><td>~~5~~</td><td>~~5~~</td><td>~~5~~</td><td>~~5~~</td></tr>
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<tr><td>5</td><td>5</td><td>5</td><td>5</td></tr>
<tr><td>5</td><td>5</td><td></td><td></td></tr>
</table>
• At day 14 they had equal amounts and after that Owen had more as seen from my table.
</td>
<td>
» Now get some of the students to present to the class and request the remainder of the class to record the strategies and solutions they observe.

» There are probably many students who think the answer is only on the 15th day. At this stage do not go into this too deeply, but have them think about it when examining inequalities.

» Get the students who solved the problem using Algebra to use the data in the problem to set up an inequality.

» Ask each student who presents their solution to explain their solution.

» Keep each strategy on the board until you are finished discussing the problem.

» Allow other students to pose questions of the student who is presenting.
</td>
<td>
» Can students explain their strategy and justify it?

» Did students read the problem correctly?

» Do students understand the various solutions?
</td>
</tr>
</table>

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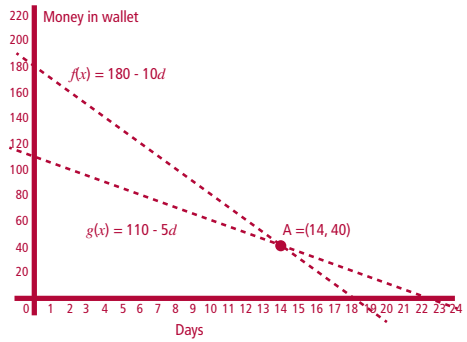
<tr>
<td></td>
<td>
• Student 3: I made out the following table
<table>
<tr><th>Day</th><th>John</th><th>Owen</th></tr>
<tr><td>0</td><td>180</td><td>110</td></tr>
<tr><td>1</td><td>170</td><td>105</td></tr>
<tr><td>2</td><td>160</td><td>100</td></tr>
<tr><td>3</td><td>150</td><td>95</td></tr>
<tr><td>4</td><td>140</td><td>90</td></tr>
<tr><td>5</td><td>130</td><td>85</td></tr>
<tr><td>6</td><td>120</td><td>80</td></tr>
<tr><td>7</td><td>110</td><td>75</td></tr>
<tr><td>8</td><td>100</td><td>70</td></tr>
<tr><td>9</td><td>90</td><td>65</td></tr>
<tr><td>10</td><td>80</td><td>60</td></tr>
<tr><td>11</td><td>70</td><td>55</td></tr>
<tr><td>12</td><td>60</td><td>50</td></tr>
<tr><td>13</td><td>50</td><td>45</td></tr>
<tr><td>14</td><td>40</td><td>40</td></tr>
<tr><td>15</td><td>30</td><td>35</td></tr>
<tr><td>16</td><td>20</td><td>30</td></tr>
<tr><td>17</td><td>10</td><td>25</td></tr>
<tr><td>18</td><td>0</td><td>20</td></tr>
</table>
• From the table I saw that after day 14 Owen had more money than John and I stopped my table at day 18 as John's money ran out at that stage. Hence Owen has more money than John on days 15, 16, 17 and 18.
</td>
<td>» Encourage this student to arrive at the answer days 15, 16 17 and 18 and explain why this is the solution to our problem.</td>
<td>» Do students understand this solution?</td>
</tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
	• **Student 4:** I decided that the amount of money in John's wallet can be represented by $180 - 10d$ and the amount in Owen's wallet can be represented by $110\text{-}5d$, where d is the number of days that have elapsed. I then drew graphs to represent both patterns.  The lines representing the patterns met at (14, 40). This was the day they were equal but the question asked when they were greater. Hence on days 15, 16, 17 and 18 Owen had more money than John. • **Student 5:** I noticed that the amount of money in John's box could be represented by $180\text{-}10d$ and the amount in Owen's by $110\text{-}5d$ where d is the number of days elapsed. Then I solved my equation. $+ 5d \mid 180 - 10d = 110 - 5d \mid + 5d$ $- 180 \mid 180 - 5d = 110 \mid - 180$ $\div (-5) \mid -5d = 70 \mid \div (-5)$ $d = 14$ This means on day 14 they have equal amounts of money, but the question asked when has Owen more money than John in his wallet. Hence on days 15, 16, 17 and 18 Owen has more money than John in his wallet. I stopped at day 18 as John's money ran out on that day and the question stated they stopped when one of them had no more money in their wallet.	» Discuss why dashed lines were used in drawing the graphs and what the point of intersection means. Also discuss what is the significance of the x and y axis intercepts within the context of the problem. » In the case of the student who used an equation to solve the problem, get him/her to present this to the class with an emphasis on setting it up rather than on solving it.	» Do students understand the significance of the point of intersection of the graphs? » Do they appreciate that it is not only day 14 that Owen's wallet contains more money than John's? » Do students remember their methodologies for solving equations.

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Teacher Reflections

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<tr>
<td>» Let us examine this table a little further.

» Insert <, > or = between John and Owen as appropriate.

» So, on what day were the amounts in Owen and John's wallets equal?

» On what days was the amount in Owen's wallet greater than that in John's?</td>
<td>

• Day 14.

• Days 15, 16, 17 and 18.</td>
<td>» Write this on the board if a suitable table has not already emerged.
<table>
<tr><th>Day</th><th>John</th><th></th><th>Owen</th></tr>
<tr><td>13</td><td>50</td><td>></td><td>45</td></tr>
<tr><td>14</td><td>40</td><td>=</td><td>40</td></tr>
<tr><td>15</td><td>30</td><td><</td><td>35</td></tr>
<tr><td>16</td><td>20</td><td><</td><td>30</td></tr>
<tr><td>17</td><td>10</td><td><</td><td>25</td></tr>
<tr><td>18</td><td>0</td><td><</td><td>20</td></tr>
</table>
</td>
<td>» Can the students solve the problem using this method?</td>
</tr>
<tr>
<td>» How could we represent the amount of money John has in his wallet on any particular day algebraically?

» How could we represent the amount of money Owen has in his wallet on any particular day by algebra?

» Hence how could we represent our problem algebraically?

» Can we solve this inequality using a strategy similar to that used for solving simple equations?

» What difficulty occurred if we just used an equation to solve this problem?</td>
<td>• $180 - 10d$

• $110 - 5d$

• $110 - 5d > 180 - 10d$

• Stabilisers and as follows:
<table>
<tr><td>$+10d$
-110
$+5$</td><td>$110 - 5d > 180 - 10d$
$180 - 5d > 180$
$5d > 70$
$d > 14$</td><td>$+10d$
-110
$+5$</td></tr>
</table>
• It only gave us an answer of day 14 and we had to re-read the question and see that it was days 15, 16, 17 and 18.</td>
<td>» Write the student answers on the board and discuss each step as it appears in the inequality.

» Notice that the solution set of the inequality contains a range of values of x.

Note: Be careful to ensure that the inequality does not involve $-d$ as students will not have the skills to deal with this as yet.</td>
<td>» Can students represent the situation using algebra?

» Do students understand that it is insufficient to simply solve the equation and go no further?</td>
</tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» We have just solved an inequality rather than an equation. » How do the approaches to solving linear inequalities and equations differ? How are they the same?	• We used the same method, but with an inequality we can have a range of solutions.		» Do students recognise the differences between an equation and an inequality?
» Complete the remaining questions in **Section D: Student Activity 2**.	• Students work on **Section D: Student Activity 2** comparing and discussing their answers.	» Circulate the room and offer assistance as required.	
» Working in pairs, complete the Tarsia that has been distributed. It is a domino jigsaw. You will need to find the piece with **Start** written on it. This piece also contains an algebraic expression. You must match each piece with another piece containing a corresponding expression. Continue matching until you arrive at **Finish**.	• Working in pairs, students complete the Tarsia.	» Distribute **Section D: Student Activity 3**.	» Can students complete this Tarsia?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section E: Multiplying and Dividing by a Negative Number.			
» Complete the exercise contained in **Section E: Student Activity 1** Question 1.		» Distribute **Section E: Student Activity 1**.	
» Is -3 < 5 true? » When you multiply -3 "by"-1,"what do you get?" » When you multiply 5 "by"-1,"what do you get?" » Is 3 greater or less than -5? » So what happened when you multiplied the inequality by -1? » In groups of two write down four inequalities of your choice and multiply each by -1 and observe the effect in each case. » Repeat the exercise, but multiplying or dividing through by a few different negative numbers. » What happens in each case?	• Yes • 3 • -5 • 3 > -5 • The direction in the inequality reversed. • < becomes > and > becomes <.	» Write the various steps on the board. » Discuss with the class how ≤ becomes ≥ and ≥ becomes ≤.	» Do students recognise that multiplying (or dividing) an inequality by -1 causes the direction of the inequality to reverse?
» Complete **Section E: Student Activity 1**.	• Students complete **Section E: Student Activity 1**.		

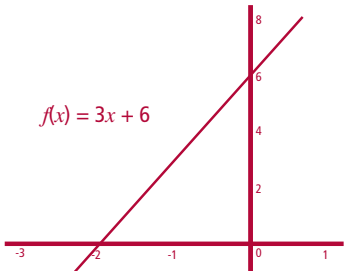
<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Supports and Actions</th><th>Checking Understanding</th></tr>
<tr><td>» When you multiply both sides of an inequality by the same negative number, what happens to the inequality?</td><td>• The direction of the inequality reverses.</td><td></td><td>» Do students understand what happens to the inequality when one multiplies both sides of the inequality by the same negative number?</td></tr>
<tr><td>» Complete the questions in Section E: Student Activities 2 and 3.</td><td>• Students work on Section E: Student Activities 2 and 3 to consolidate their learning.</td><td>» Distribute Section E: Student Activities 2 and 3.</td><td></td></tr>
<tr><td>» Now we are going to complete the following table as a conclusion to the previous exercises.

» What actions applied to both sides of an inequality causes the direction of the inequality to reverse?</td><td><table><tr><th>Action (to both sides of an inequality)</th><th>Does the direction of the inequality change? Yes / No</th></tr><tr><td>Add a positive number</td><td>No</td></tr><tr><td>Subtract a positive number</td><td>No</td></tr><tr><td>Add a negative number</td><td>No</td></tr><tr><td>Subtract a negative number</td><td>No</td></tr><tr><td>Multiply by a positive number</td><td>No</td></tr><tr><td>Multiply by a negative number</td><td>Yes</td></tr><tr><td>Divide by a positive number</td><td>No</td></tr><tr><td>Divide by a negative number</td><td>Yes</td></tr></table>
• Multiply and divide both sides of the inequality by a negative number.</td><td>» Complete the titles and first column of the table, opposite.

» Through discussion with the students complete the table.</td><td>» Do students understand that when one multiplies or divides both sides of the inequality by a negative number, the direction of the inequality is reversed?</td></tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section F: Solving inequalities of the form –ax+b<c.			
» What is the first action you would perform to solve the inequality $-x + 2 > 5$, $x \in R$?	• Subtract 2 from each side.	» Write each step involved in solving the inequality on the board.	» Do students recognise that this inequality is different from what they have met to date?
» What will the inequality now look like?	• $-x > 3$		
» What will the next step be?	• Divide both sides by -1.		» Do students recognise that the direction of the inequality had to reverse because the solution required division of both sides by -1?
» What happens when you divide both sides of an inequality by any negative number?	• > becomes < and < becomes >.		
» So what is the solution of the inequality?	• $x < -3$, $x \in R$		

Section G: Solving inequalities graphically.

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» Complete question 1 in **Section G: Student Activity 1**.	<table><tr><th>x</th><th>$f(x) = 3x + 6$</th></tr><tr><td>-3</td><td>-3</td></tr><tr><td>-2</td><td>0</td></tr><tr><td>-1</td><td>3</td></tr><tr><td>0</td><td>6</td></tr><tr><td>1</td><td>9</td></tr></table>	» Distribute **Section G: Student Activity 1**.	
		» Allow students time to engage with this question.	» Do students understand that when the line is above the x axis the values of $f(x)$ are positive and when it is below the x axis they are negative?
» Where does the line cut the x axis?	• $x = -2$	» Encourage the more able students to draw the function by locating the x and y intercepts only and then checking that other points in the table satisfy the equation of the line.	» Can students solve linear inequalities graphically?
» When is $f(x) > 0$?	• It is greater than zero for x values greater than minus two.	» After the students have had an opportunity to produce a table and graph themselves through discussion draw the table and graph on the board.	» Do students know that the solutions are the values on the x axis that make the inequality true?
» When is $f(x) < 0$?	• It is less than zero for x values less than minus two.		
» In solving an equation we find the x values that make it true and in the same way when we solve an inequality we find the values of x that make it true.			
» So what x values make the inequality $3x + 6 \geq 0$ true?	• $x \geq -2$	» Encourage the students to check if this is true using other values of x.	

Lesson Interaction			
Student Learning Tasks: Teacher Input	**Student Activities: Possible and Expected Responses**	**Teacher's Supports and Actions**	**Checking Understanding**
» What x values make the inequality $3x + 6 \leq 0$ true? » What x values make the inequality $3x + 6 > 0$ true? » What x values make the inequality $3x + 6 < 0$ true?	• $x \leq -2$ • $x > -2$ • $x < 2$		
» Complete the exercises in **Section G: Student Activity 1**.	• Students complete the exercises in **Section G: Student Activity 1**.	» Circulate the room and offer assistance when required.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section H: To investigate the rules of inequalities.			
Note: in this exercise a and b are Real Numbers unless it states otherwise. » Working in groups, use suitable values for a, b, c to investigate the activities contained in **Section H: Student Activity 1**.	• Working in pairs, students discuss these inequalities.	» Distribute **Section H: Student Activity 1**. **Note:** Solutions are contained in this Teaching and Learning plan on page 52. » Encourage students to replace a, b and c with positive and negative whole numbers and fractions to help them investigate the relationships. » Circulate the room and offer assistance when required.	» Can students replace a, b and c with numerical values to investigate the relationships? » Do students understand that these inequalities have to be true for all values? » Do students understand the impact of $c \in R^+$ and $c \in R^-$? » Can students justify their answers?
» Now I want the selected students to present their solutions on the board. **Note:** I am particularly interested in the reasons behind the solutions.	• Selected students present their solutions on the board.	» Select students to present their findings to the class. » Encourage the students who are presenting to reinforce the validity of their answers by substituting with positive and negative whole numbers and fractions.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
Section I: Double inequalities.			
» Angus wants to buy a present for a friend and he wants to spend at least €24 and no more than €30.			
» Does this include €24 and €30?	• Yes		
» What is the least he can spend?	• €24		
» What is the most he can spend?	• €30		
» Write this problem as a set of inequalities?	• $x \geq 24$ and $x \leq 30$?		
» What is another way of writing the inequality $x \geq 24$?	• $24 \leq x$		
» Now we can put the two inequalities together as $24 \leq x \leq 30$.	• x is any Real Number between 2 and 5, but cannot be equal to 2 or 5.	» Write the individual and combined inequalities on the board.	» Do students understand that $x \geq 24$ is equivalent to $24 \leq x$?
» What does the following inequality mean $2 < x < 5$, $x \in \mathbb{R}$?			

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» How could we represent $-2 < x < 5$, $x \in R$ on the number line?		» Draw the number line on the board and write the inequality it represents beside it.	» Do students understand how to represent inequalities of the form $a < x < b$, for different number systems on the number line?
» Why did you use open circles at -2 and at 5?	• Because those points are not included, it is between -2 and 5.	» Draw a diagram on the board to illustrate this inequality.	
» If the inequality had been $\leq$ instead of $<$, how would this affect the representation of the inequality on the number line?	• It would have been closed circles as -2 and 5 would have been included.	» Distribute **Section I: Student Activity 1**.	
» If the inequality had been $-2 < x < 5$, $x \in Z$, what effect would this have had?	• You would just have put closed circles at the relevant points -1, 0, 1, 2, 3 and 4		
» Do the exercises 1-5 contained in **Section I: Student Activity 1**.			

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Supports and Actions	Checking Understanding
» How would one solve the inequality $3 \leq -2x + 1 \leq 7$, $x \in R$? » Now write this inequality as two inequalities. » Can any of these inequalities be simplified? » Now draw this on the number line. » So what is another way of writing the solution? » Complete the remaining exercises in **Section I: Student Activity 1**.	-1 ÷2 | $3 \leq -2x + 1 \leq 7$ | -1 ÷2 $2 \leq -2x \leq 6$ $1 \leq -x \leq 3$ Divide by -1 gives $-1 \geq x \geq -3$. • $-1 \geq x$ and $x \geq -3$ • Yes. The first can be written as $x \leq -1$.  • Another way of writing this inequality is $-3 \leq x \leq -1$.	» On the board, draw a diagram to illustrate that the solution is more correctly represented as $-3 < -1$.	» Can students solve and represent the solutions of inequalities of the form $a < x < b$ on the number line?
» Working in pairs complete the Tarsia that has been distributed. It is a domino jigsaw. You will need to find the piece containing the word **Start**. It also contains an algebraic expression; you must find the equivalent expression in another piece and you continue matching until you arrive at **Finish**.	• In pairs students complete the Tarsia.	» Distribute **Section I: Student Activity 2**.	» Can students complete the Tarsia?
» Match the appropriate words, sample sentences, equivalent forms and translations.	• In pairs students complete the graph matching exercise.	» Distribute **Section I: Student Activity 3**.	

Section A: Student Activity 1

Project Maths Tionscadal Mata Development Team

Revision of < and > symbols.

1. The table below contains a number of inequalities. In the space provided, indicate which are true and which are false

$2 < 3$		$4 \leq 5$	
$-1 > 4$		$1 > -4$	
$-2 > -1$		$3 > 4$	
$-1 \leq 4$		$-1 \geq 4$	
$1.2 < 4$		$-1.8 > 4$	
$^1/_2 < {}^3/_4$		$^1/_2 > {}^1/_4$	
$-^1/_2 < {}^3/_4$		$^1/_2 > -^1/_4$	

2. Insert the appropriate symbol between these numbers.

	Insert < or > between these numbers	
6		10
-6		-10
5		-4
1.5		3.5
-6		-4
$^1/_2$		$^1/_4$
$-^1/_2$		$-^1/_4$
20%		0.02

3. In each case, below, circle the algebraic expression which represents the statement given.

a	is less than 5	$x > 5$	$x < 5$	$x \leq 5$	$x \geq 5$
b	is more than 8	$x > 8$	$x < 8$	$x \leq 8$	$x \geq 8$
c	is less than or equal to 4	$x > 4$	$x < 4$	$x \leq 4$	$x \geq 4$
d	is greater than or equal to 10	$x > 10$	$x < 10$	$x \leq 10$	$x \geq 10$
e	is at least 10	$x < 10$	$x > 10$	$x \geq 10$	$x \leq 10$
f	is at most 10	$x < 10$	$x > 10$	$x \leq 10$	$x \geq 10$
g	Let r be the amount of rain (in mm) which falls each day. More than 23mm of rain fell yesterday.	$r < 23$	$r > 23$	$r \leq 23$	$r \geq 23$
h	p is no more than 9	$p < 9$	$p > 9$	$p \geq 9$	$p \leq 9$

Section A: Student Activity 1

Project Maths Tionscadal Mata Development Team

4. For each of the statements below, circle the inequality which represents the given statement.

a	The speed limit on a certain road is 60 km. Does this mean each driver has to drive at this speed or less? s= speed in km/h. Represent the speed limit using the variable s. $s < 60$ $s \leq 60$ $s > 60$ $s \geq 60$
b	In order to be able to go on a school trip Kelly needs to have saved €40 or more. Kelly has saved €d and is not yet able to go on the trip. Which of the following is true? $d \leq 40$ $d \geq 40$ $d > 40$ $d < 40$
c	To enter a particular art competition you must be at least 12 years old. Tom is n years old and he can enter the competition. Which of the following is true? $n < 12$ $n \leq 12$ $n \geq 12$ $n > 12$
d	To enter an art competition you must be over 12 years old. Tom is n years old and he can enter the competition. Which of the following is true? $n < 12$ $n \leq 12$ $n \geq 12$ $n > 12$
e	The maximum number of people allowed in a cinema is 130. If there are b people in the cinema. Which of the following is true? $b > 130$ $b \geq 130$ $b \leq 130$ $b < 130$
f	The best paid workers in a business earn €40 per hour. Mark earns €m per hour and he is not one of the best paid employees in the business. Which of the following is true? $m \geq 40$ $m < 40$ $m > 40$ $m \leq 40$
g	Emma's mother says that when she reaches the age of 17 she will get an increase in her pocket money. Emma is r years old and has not yet received that increase. Which of the following is true? $r < 17$ $r > 17$ $r \leq 17$ $r \geq 17$
h	There are at least 200 animals in a zoo. If there are h animals in the zoo, which of the following is true? $h > 200$ $h < 200$ $h \geq 200$ $h \leq 2\,00$
i	At most 10 people can fit in a bus and there are k people in the bus. Which of the following is true? $k < 10$ $k > 10$ $k \leq 10$ $k \geq 10$
j	A film is given an age 18 certificate. Let a = age of a student. To watch the film which statement is true? $a < 18$ $a > 18$ $a \leq 18$ $a \geq 18$
k	In Ireland you have to be at least 18 years old in order to be able to vote. Conor is w years old and he can vote, Which of the following is true? $w \leq 18$ $w < 18$ $w > 18$ $w \geq 18$
l	In order to get to the next stage of the competition a team must have at least 20 points. Given x = the number of points scored by a team and they qualify for the next stage of the competition, which of the following is true? $x < 20$ $x \leq 20$ $x > 20$ $x \geq 20$
m	The temperature in Dublin on a particular day is 20°C and it is warmer in Cork on the same day. Given x°C is the temperature in Cork, which of the following is true? $x \leq 20$ $x < 20$ $x > 20$ $x \geq 20$

Section B: Student Activity 1

Project Maths Tionscadal Mata Development Team

Revision of Number Systems.

1. Write the relevant numbers from below into each of the boxes A, B, C, E, F, G and H. **Note:** Numbers may be used more than once.

   -3, 0·5, 6, 8, 10, -4, 20, -5, 3½, -6·2, 1, 11, 2, 3, 9, 5, 5·2, -2, 7·9, 12, 11·3.

A: A Natural Number greater than 2	B: A Real Number greater than 2
C: A Natural Number less than 9	D: An Integer less than 9
E: A Real Number greater than 7	F: A Real Number greater than or equal to 9
G: A Real Number bigger than -4 and less than or equal to 5	H: An Integer greater than -2

2. Write down one number which you included in box B, but did not include in box A. Give a reason for your choice._____

3. $x > 5$, $x \in N$ means: (**Note:** There may be more than one correct answer.)

   (i) a number less than 5 is a possible solution

   (ii) 5 is a possible solution

   (iii) 6 is a possible solution

   (iv) 5 is the only solution

   (v) natural numbers should be Natural Numbers

   (vi) Every natural number greater than 5 represents a general solution

4. $x > 5$, $x \in N$ means: (**Note:** There may be more than one correct answer.)

   (i) a number less than 4 is possible solution

   (ii) 8 is a possible solution

   (iii) 6 is a possible solution

   (iv) 5 is the only solution

Section B: Student Activity 1

Project Maths Development Team

5. What is the general solution to the inequality in Q4?
6. $x \leq 6, x \in N$ State giving a reason which of the following are true or false.
   i. 6 is a possible solution _____
   ii. 7·8 is a possible solution _____
   iii. -4 is a possible solution _____
   iv. 9 is a possible solution _____
   v. The solution set contains 6 elements
7. Given $x \leq 9, x \in Z$, write "True" or "False" beside each of the following.
   i. 6 is a possible solution _____
   ii. 10·8 is a possible solution _____
   iii. -4 is a possible solution _____
   iv. 9 is a possible solution _____
8. Given $x < 8, x \in Z$, write "True" or "False" beside each of the following.
   i. x is less than or equal to 8 _____
   ii. x is greater than 8 _____
   iii. x is less than 8 _____
   iv. x is 8 _____
   v. x can be a fraction _____
9. Write the sentence: "x is a Natural Number less than 4.", in algebraic form (using symbols).
10. Write the sentence: "x is a Natural Number greater than 3.", in algebraic form.
11. Write the sentence: "y is a Rational Number greater than or equal to 12.", in algebraic form.
12. Write the sentence: "p is an Integer less than 7.", in algebraic form.
13. Which of the statements below represents the pattern 14, 15, 16, 17, 18, 19, ...

    (a) $x < 14, x \in Z$ (b) $x > 14, x \in R$ (c) $x \leq 14, x \in R$
    (d) $x \geq 14, x \in N$ (e) $x > 14, x \in N$
14. Which of the statements below represents the set A? A={-10,-9,-8,-7,-6,-5,…}

    (a) $x < -10, x \in Z$ (b) $x \geq -10, x \in Z$ (c) $x \leq -10, x \in R$
    (d) $x > -10, x \in Z$ (e) $x \geq -10, x \in R$.

Section C: Student Activity 1

Project Maths Tionscadal Mata Development Team

Number Systems and the Number Line.

A class set of the following should be laminated and cut up prior to the beginning of the lesson.

$x < 3,\ x \in N$	x is less than 3 and x is an element of N	
$x < 3,\ x \in Z$	x is less than 3 and x is an element of Z	
$x < 3, x \in R$	x is less than 3 and x is an element of R	
$x > 3,\ x \in N$	x is greater than 3 and x is an element of N	
$x > 3,\ x \in Z$	x is greater than 3 and x is an element of Z	
$x > 3,\ x \in R$	x is greater than 3 and x is an element of R	
$x \leq 3,\ x \in N$	x is less than or equal to 3 and x is an element of N	
$x \leq 3,\ x \in Z$	x is less than or equal to 3 and x is an element of Z	
$x \leq 3,\ x \in R$	x is less than or equal to 3 and x is an element of R	

Section C: Student Activity 1

$x \geq 3,\ x \in N$	x is greater than or equal to 3 and x is an element of N	
$x \geq 3,\ x \in Z$	x is greater than or equal to 3 and x is an element of Z	
$x \geq 3,\ x \in R$	x is greater than or equal to 3 and x is an element of R	
$x < -3,\ x \in Z$	x is less than -3 and x is an element of Z	
$x < -3,\ x \in R$	x is less than -3 and x is an element of R	
$x \leq -3,\ x \in Z$	x is less than or equal to -3 and x is an element of Z	
$x \leq -3,\ x \in R$	x is less than or equal to -3 and x is an element of R	
$x \geq -3,\ x \in Z$	x is greater than or equal to -3 and x is an element of Z	
$x \geq -3,\ x \in R$	x is greater than or equal to -3 and x is an element of R	

Section D: Student Activity 1

Project Maths Tionscadal Mata Development Team

Solving inequalities of the form $ax + b < k$.

1. Annika has seen three pairs of trainers she likes. They cost €50, €55 and €68. She already has saved €20 and gets €4 pocket money per week at the end of each week. Annika is wondering how soon she can buy one of these pairs of trainers. What different methods can you use to help her solve this problem?
2. Which of the following is an element of the solution set of the inequality $4x + 5 > 29$?

   i. 4 ii. -5 iii. 6 iv. 7.
3. Solve the following inequalities for $x \in R$ and represent their solutions on the number line:

   i. $x + 3 < 5$

   ii. $x - 6 < 5$

   iii. $2x + 3 < 5$

   iv. $2x + 3 < 15$

   v. $12x + 3 \geq 111$
4. Complete the following two tasks for each of the problems below:

   i. Solve the problem using a table, graph or trial and error. Show your calculations and explain your reasoning in all cases.

   ii. Form an inequality to represent the problem and solve it algebraically.

a. Darren worked a six hour shift in his local restaurant and got €5 in tips. His total take home pay that evening was at least €69, find the minimum amount he was paid per hour.

b. A farmer wants to buy some cows and a tractor. The tractor costs €20,000 and the maximum he can spend is €60,000. Given that the average price of a cow is €900, find the maximum number of cows he can buy.

c. Ronan buys a tomato plant which is 5 centimetres in height. The tomato plant grows 3 centimetres every day. After how many days will the tomato plant reach the top of the glass house in which it is growing, given the glass house is 2 metres high.

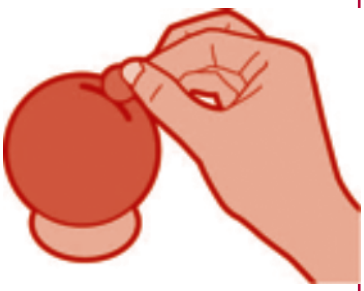
d. Declan is saving for a birthday present for his girlfriend and he already has €6. Given that he plans to spend at least €40 on the present and the birthday is five weeks hence, what is the least amount he should save per week?

e A bridge across the river Geo can support a maximum weight of 20 tonnes. The company's lorry weighs 8 tonnes and draws a trailer of 3 tonnes in weight. The company wants to find the maximum cargo they can take across this bridge using their lorry. Represent the problem, as an inequality and hence find the maximum weight of the cargo the lorry could carry cross the bridge.

Section D: Student Activity 2

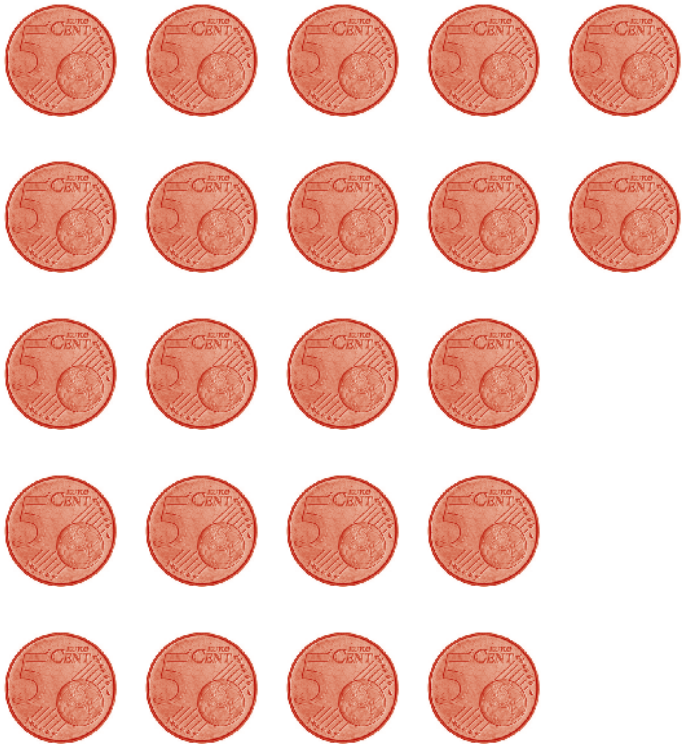
Project Maths Tionscadal Mata Development Team

Solving inequalities of the form $ax + b < cx + d$.



1. John has 18 ten-cent coins in his wallet and Owen has 22 five-cent coins in his wallet. Each day, they decide to take one coin from their wallets and put it into a money box, until one of them has no more coins left in their wallet.

   When does Owen have more money than John in his wallet?

John	Owen
	

Section D: Student Activity 2

Project Maths Tionscadal Mata Development Team

2. Declan is having a party and is buying pizzas and chips. He has at most €20 to spend on food and his budget for chips is exactly €4.60. What is the maximum number of pizzas he can buy, if each pizza costs €3.50? The shop only sells whole pizzas.

3. John and Michael go running in order to keep fit. John runs each day from Monday to Friday and then runs 2 kilometres each Saturday.

   Michael goes running on Mondays, Tuesdays and Wednesdays and also runs 9 kilometres each Sunday.

   They each run the same fixed distance on the weekdays on which they run.

   i. John runs further each week than Michael. Write an inequality to represent this situation.

   ii. Investigate what is the minimum number of kilometres for which this inequality is true using a table, graph and algebra.

4. When on holiday in France for a week, Emma's dad hired a car. He paid a fixed rental of €200 per week and €0.15 per kilometre for each journey undertaken. He has at most €300 to spend on car hire. What is the maximum number of kilometres he can drive in the hired car?

5. The length of Lisa's rectangular dining room is 4 metres. If the area of the room is at least 12 square metres, what is the smallest width the room could have?

6. Tell the story of possible shopping trips that could be represented by the following inequalities:

   i. $3x + 7 < 40$

   ii. $5x + 30 > 50$

Section D: Student Activity 3

Project Maths Tionscadal Mata Development Team

Tarsia

Cut along the line that separates each row and then cut each section in half again. This will form a set of dominos. Instruct the students to find the domino with **Start** on it and then search for the matching section that accompanies the Start domino. Continue this until **Finish** is reached.

$2x > 10$	$3x + 4 < 2x + 3$	$4x - 6 > 24$	$x < 12$
$3(x - 2) > 18$	$4 < -5$	$x < -1$	x is greater than or equal to 5
$3x - 4 < 5$	$x > 5$	$x + 7 < 8$	$-3 < -2$
$x - 5 < 7$	x is less than 7	$x < -4$	$x > 8$

Section D: Student Activity 3 (continued)

$x \geq 5$	$4x - 10 < -26$	True	$3x < 15$
$x > 2$	x is at least 7	$x < 7$	$x < 1$
False	$x > 4.5$	$x \geq 7$	Finish
$x < 5$	$3x - 8 < -2$	Start	$x < 3$

Note: The solution is on the page 43.

Solution to Tarsia

Project Maths Tionscadal Mata Development Team

					Start
					$x < 3$
x is greater than or equal to 5	$x < -1$	$3x + 4 < 2x + 3$	$2x > 10$	$x > 5$	$3x - 4 < 5$
$x \geq 5$					
$4x - 10 < -26$					
$x < -4$	$x > 8$	$3(x - 2) > 18$	$4 < -5$	False	$x > 4.5$
					$4x - 6 > 24$
					$x < 12$
$-3 < -2$	$x + 7 < 8$	$x < 1$	$x < 7$	x is less than 7	$x - 5 < 7$
True					
$3x < 15$					
$x < 5$	$3x - 8 < -2$	$x > 2$	x is at least 7	$x \geq 7$	Finish

Section E: Student Activity 1

Multiplying and Dividing an inequality by a Negative Number.

1a. Circle the numbers - 3 and 5 on the number line below.

b. Which of these two numbers is smaller? Explain your answer referring to the number line.

c. Insert the correct symbol (<, >, ≤, ≥) into the box -3 ☐ 5.

d. By multiplying the -3 and the 5 by -1 fill in the boxes below and represent the answer on the number line in part a.

-3 multiplied by -1 = ☐

5 multiplied by -1 = ☐

e Which of the numbers from part d. is the smaller?

Explain your answer referring to the number line.

2

	Original inequality	Multiply by -1	Resulting Inequality
i	$4 > 1$	Multiply by -1	
ii	$3 < 5$	Multiply by -1	
iii	$6 > -4$	Multiply by -1	
iv	$5 \geq 4$	Multiply by -1	
v	$-5 \leq 2$	Multiply by -2	
vi	$-1 \geq -6$	Multiply by -3	
vii	$-8 \leq -3$	Multiply by -1	
viii	$7 \geq 7$	Multiply by -1	
ix	$5 > 4$	Multiply by -2	
x	$-6 < -5$	Multiply by -5	
xi	$-4 < 3$	Multiply by -1	
xii	$\frac{1}{4} < \frac{1}{2}$	Multiply by -1	

3. From your experience of the exercise above, can you conclude what happens to an inequality when you multiply both sides by -1?

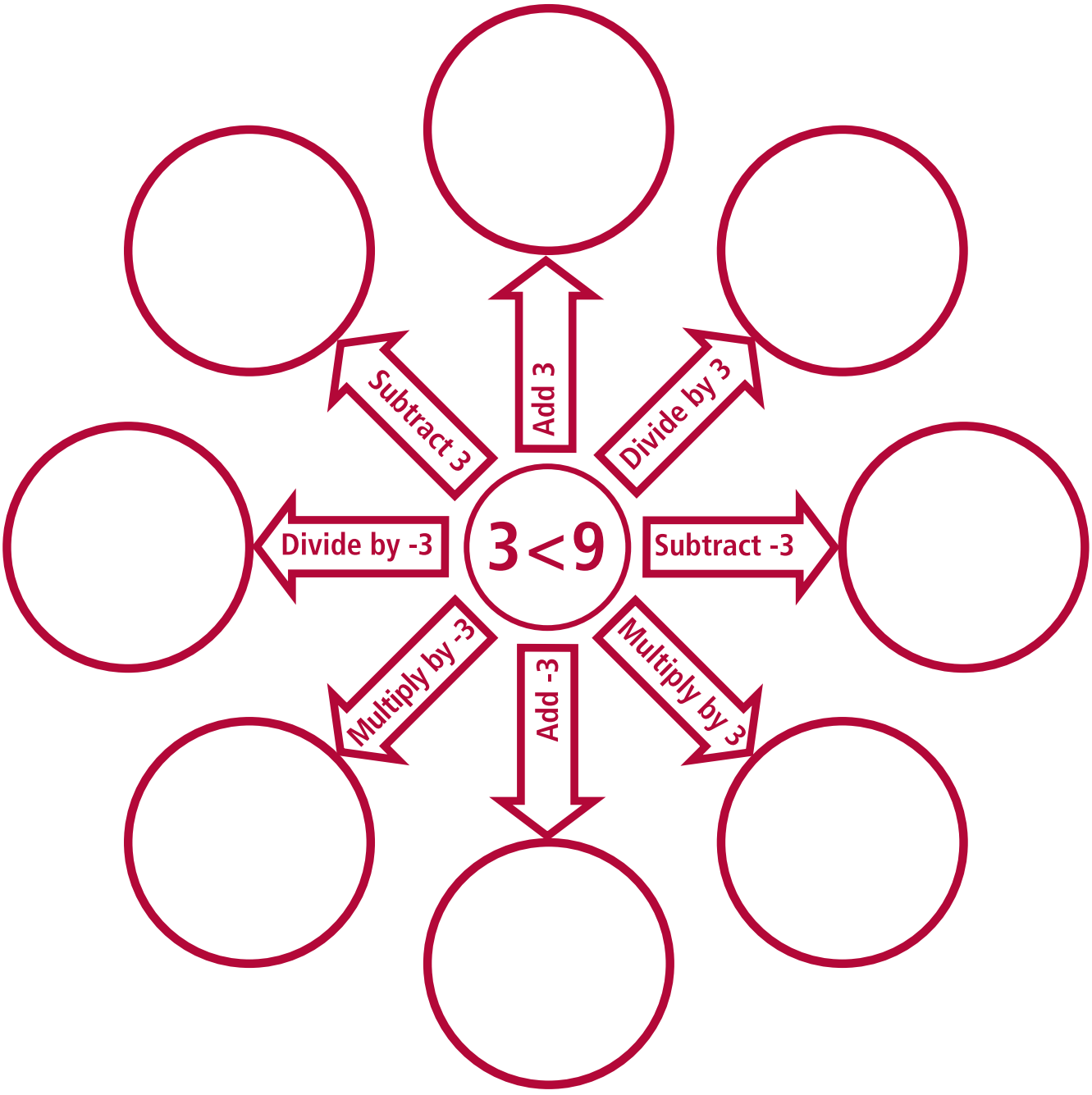
4. What conclusion do you arrive at when you multiply both sides of an inequality by the same negative number?

Section E: Student Activity 2

Project Maths Tionscadal Mata Development Team

Perform the action contained in the arrow *to each side of* the given inequality in the centre.

In which of the new inequalities did the direction of the inequality differ from the original inequality?



Section E: Student Activity 3

Project Maths Tionscadal Mata Development Team

1a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$2 < 3$	Add 4		
$6 > 4$	Add 1		
$2 = 2$	Add 5		
$a < b$	Add a positive number		
$a > b$	Add a positive number		

b If you add the same positive number to each side of the inequality do you reverse the direction of the inequality? Give examples. _____

_____

_____

2a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$2 < 5$	Add -5		
$6 > 2$	Add -3		
$7 = 7$	Add -4		
$a < b$	Add a negative number		
$a > b$	Add a negative number		

b If you add the same negative number to each side of an inequality do you reverse the direction of the inequality? Give examples. _____

_____

_____

3a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$3 < 5$	Subtract 2		
$8 > 2$	Subtract 4		
$7 = 7$	Subtract 6		
$a < b$	Subtract a positive number		
$a > b$	Subtract a positive number		

b If you subtract the same positive number from each side of an inequality do you reverse the direction of the inequality? Give examples. _____

_____

_____

Section E: Student Activity 3

Project Maths Tionscadal Mata Development Team

4a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$1 < 6$	Subtract -5		
$5 > 3$	Subtract -3		
$4 = 4$	Subtract -4		
$a < b$	Subtract a negative number		
$a > b$	Subtract a negative number		

b If you subtract the same negative number from each side of an inequality do you reverse the direction of the inequality? Give examples. _____

_____

_____

5a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$3 < 8$	Multiply by 2		
$9 > 4$	Multiply by 3		
$6 = 6$	Multiply by 5		
$a < b$	Multiply by a positive number		
$a > b$	Multiply by a positive number		

b If you multiply each side of an inequality by the same positive number do you reverse the direction of the inequality? _____

_____

_____

3a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$4 < 8$	Multiply by -2		
$9 > 3$	Multiply by -4		
$10 = 10$	Multiply by -5		
$a < b$	Multiply by a negative number		
$a > b$	Multiply by a negative number		

b If you multiply each side of an inequality by the same negative number do you reverse the direction of the inequality? _____

_____

_____

Section E: Student Activity 3

Project Maths Tionscadal Mata Development Team

7a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$6 < 18$	Divide by 6		
$12 > 6$	Divide by 3		
$20 = 20$	Divide by 5		
$a < b$	Divide by a positive number		
$a > b$	Divide by a positive number		

b If you divide each side of an inequality by the same positive number do you reverse the direction of the inequality? Give examples. _____

_____

_____

8a

Inequality	Action to each side of the inequality	Result	Did you have to reverse the direction of the inequality?
$6 < 10$	Divide by -2		
$12 > 4$	Divide by -4		
$15 = 15$	Divide by -5		
$a < b$	Divide by a negative number		
$a > b$	Divide by a negative number		

b If you divide each side of an inequality by the same negative number do you reverse the direction of the inequality? Give examples. _____

_____

_____

Section F: Student Activity 1

Project Maths Tionscadal Mata Development Team

To solve inequalities of the form $-ax + b < c$.

1 Solve the following inequalities where $x \in R$:

i. $-x + 2 > 5$

ii. $-x + 2 > -5$

iii. $-x - 6 < 5$

iv. $-2x + 3 < 5$

v. $-2x + 3 < 27$

vi. $-12x + 3 \geq 111$

vii. $4 - 3x \geq 46$

viii. $-2x + 3 \geq 10$

ix. $-3x + 3 \geq 17$

2 Fiona has €500 in her bank account. The only payment she makes from this account is to cover her mobile phone bill of €40 per month. This money is paid from the bank account at the end of each month. The bank stated they will warn her when the account goes below €25. Write an inequality to represent this problem and solve the inequality.

Section G: Student Activity 1

Project Maths Tionscadal Mata Development Team

Solving inequalities graphically.

1. Complete a table and draw the graph of the function $f(x) = 3x + 6$, $x \in R$, in the domain $-3 \leq x \leq 1$ and graphically determine the solution set to each of the following inequalities:

   i. $3x + 6 \geq 0$

   ii. $3x + 6 \leq 0$

   iii. $3x + 6 > 0$

   iv. $3x + 6 < 0$.

2. Complete a table and draw the graph of the function $f(x) = x + 4$, $x \in R$, in the domain $-3 \leq x \leq 3$ and graphically determine solution set to each of the following inequalities:

   i. $x + 4 \geq 0$

   ii. $x + 4 \leq 0$

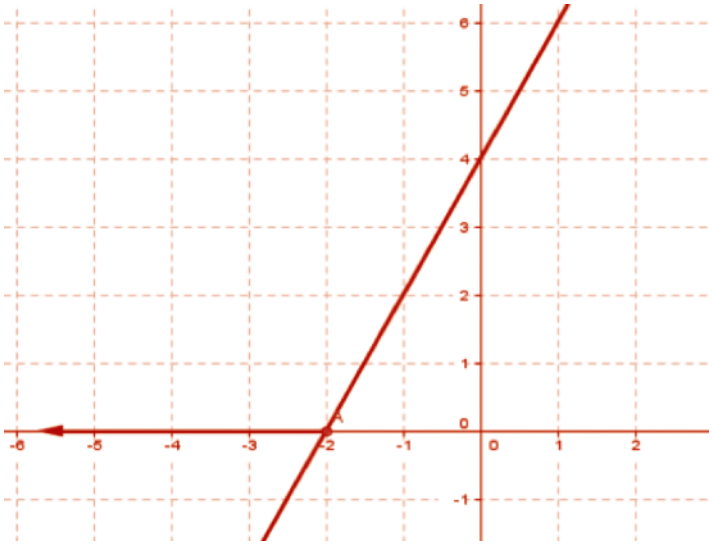
   iii. $x + 4 > 0$

   iv. $x + 4 < 0$.

3. Plot the function $f(x) = 2x + 5$ and the function $g(x) = 5$ where $x \in R$ on the same axes and scale. Find the point of intersection of the two graphs and hence find the solution set of $2x + 3 \geq 5$.

4. Draw the function $f(x) = x + 4$ and the function $g(x) = 2x + 1$ where $x \in R$ on the same graph. By examining the graph determine which values of x satisfy the inequality $2x + 1 < x + 4$.

5. Find the equation of the line represented in the diagram below and hence find an inequality of the form $ax + b < k$, whose solution is represented by the arrow on the x axis.



Section H: Student Activity 1

Project Maths Tionscadal Mata Development Team

To investigate the rules of inequalities.

Note in this exercise a and b are Real numbers unless it states otherwise.

1 a If $a<b$, what do we know about $a+c$ and $b+c$,if $c \in R$?

b If $a>b$, what do we know about $a+c$ and $b+c$,if $c \in R$?

c If $a<b$, what do we know about $a-c$ and $b-c$,if $c \in R$?

d If $a>b$, what do we know about $a-c$ and $b-c$,if $c \in R$?

2 a If $a<b$, what do we know about ac and bc,if c $\in R^+$ (Positive Reals)?

b If $a>b$, what do we know about ac and bc,if c $\in R^+$ (Positive Reals)?

c If $a<b$, what do we know about ac and bc,if $c \in R^-$ (Negative Reals)?

d If $a>b$, what do we know about ac and bc,if $c \in R^-$ (Negative Reals)?

3 a If $a<b$, what do we know about a/c and b/c,if $c \in R^+$ (Positive Reals)?

b If $a>b$, what do we know about a/c and b/c,if $c \in R^+$ (Positive Reals)?

c If $a<b$, what do we know about a/c and b/c,if $c \in R^-$ (Negative Reals)?

d If $a>b$, what do we know about a/c and b/c,if $c \in R^-$ (Negative Reals)?

4 a Can one have an expression such as $-b>0$? $b \in R^+$ Why?

b Can one have an expression such as $-b>0$? $b \in R^-$ Why?

c If $a<b$ then what is the relationship between $1/a$ and $1/b$?

d If $a>b$, how are $-a$ and $-b$ related?

5 a If $a<b$ and $b<c$, what do we know about a and c? (Transitive Property)

b If $a>b$ and $b>c$, what do we know about a and c? (Transitive Property)

c If Eoin is younger than John and Jean is younger than Eoin, what is the relationship between John and Jean's age? (Transitive property)

6 a If $a<0$, create an inequality relating a and a^2?

b If $a<0$, create an inequality relating a and a^3?

c If $a<0$, create an inequality relating a and a^4?

d If $0<a<1$, create an inequality relating a and a^2?

e If $0<a<1$, create an inequality relating a and a^3?

f If $0<a<1$, create an inequality relating a and a^4?

g If $a>1$, create an inequality relating a and a^2?

h If $a>1$, create an inequality relating a and a^3?

i If $a>1$, create an inequality relating a and a^4?

Section H: Student Activity 1

Project Maths Tionscadal Mata Development Team

Encourage students to support their answers with numerical values in all cases.

1a	If $a<b$, what do we know about $a+c$ and $b+c$, if $c \in R$?	$a + c < b + c$
1b	If $a>b$, what do we know about $a+c$ and $b+c$, if $c \in R$?	$a + b > b + c$
1c	If $a<b$, what do we know about $a-c$ and $b-c$, if $c \in R$?	$a - c < b - c$
1d	If $a>b$, what do we know about $a-c$ and $b-c$ if $c \in R$?	$a - c > b - c$
2a	If $a<b$, what do we know about ac and bc, if a and $b \in R$ and $c \in R^+$(Positive Reals)?	$ac < bc$
2b	If $a>b$, what do we know about ac and bc, if $c \in R^+$ (Positive Reals)?	$ac > bc$
2c	If $a<b$, what do we know about ac and bc, if $c \in R^-$ (Negative Reals)?	$ac > bc$
2d	If $a>b$, what do we know about ac and bc, if $c \in R^-$ (Negative Reals)?	$ac < bc$
3a	If $a<b$, what do we know about a/c and b/c, if $c \in R^+$ (Positive Reals)?	$a/c < b/c$
3b	If $a>b$, what do we know about a/c and b/c, if $c \in R^+$ (Positive Reals)?	$a/c > b/c$
3c	If $a<b$, what do we know about a/c and b/c, if $c \in R^-$ (Negative Reals)?	$a/c > b/c$
3d	If $a>b$, what do we know about a/c and b/c, if $c \in R^-$ (Negative Reals)?	$a/c < b/c$
4a	Can one have an expression such as $-b > 0$? $b \in R^+$ Why?	No. Negative numbers are always less than zero.
4b	Can one have an expression such as $-b>0$? $b \in R^-$ Why?	Yes. Minus a negative number is always positive.
4c	If $a<b$ then what is the relationship between $1/a$ and $1/b$?	$1/a>1/b$
4d	If $a>b$, how are $-a$ and $-b$ related?	$-a < -b$
5a	If $<b$ and $b<c$, what do we know about a and c?	$a < c$ This is known as the Transitive Property.
5b	If $a>b$ and $b>c$, what do we know about a and c?	$a > c$ This is also the Transitive Property.
5c	If Eoin is younger than John and Jean is younger than Eoin, what is the relationship between John and Jean's age? (Transitive property)	Jean is younger than John. John is older than Jean
6a	If $a<0$, create an inequality relating a and a^2?	$a < a^2$
6b	If $a<0$, create an inequality relating a and a^3?	$a > a^3$
6c	If $a<0$, create an inequality relating a and a^4?	$a < a^4$
6d	If $0<a<1$, create an inequality relating a and a^2?	$a > a^2$
6e	If $0<a<1$, create an inequality relating a and a^3?	$a > a^3$
6f	If $0<a<1$, create an inequality relating a and a^4?	$a > a^4$
6g	If $a>1$, create an inequality relating a and a^2?	$a < a^2$
6h	If $a>1$, create an inequality relating a and a^3?	$a < a^3$
6i	If $a>1$, create an inequality relating a and a^4?	$a < a^4$

Section I: Student Activity 1

Project Maths Tionscadal Mata Development Team

Double Inequalities

1 (Revision)

a Represent $x \leq 6$, $x \in R$ on the number line.

b Represent $x \geq 2$, $x \in R$ on the number line.

c Represent the intersection of the above two inequalities above on the number line.

d Rewrite the inequality $x \geq 2$ in the format $a \leq x$.

e Now write the intersection of the two inequalities in the format $a \leq x \leq b$.

2 Maureen takes between two and three hours to do her homework. Write this as an inequality.

3 A sweet company produces bags of sweets that will contain between 18 and 22 sweets. If there are less than 18 sweets or more than 22 sweets in a bag then their machine is faulty. Write an inequality to represent the number of sweets in an acceptable bag of sweets.

4 A farmer says the minimum number of cows he can keep on his farm is 22 and the maximum is 35, write this as an inequality.

5 Represent the following on the number lines provided:

a $1 \leq x \leq 4$, $x \in N$

b $-3 \leq x \leq 4$, $x \in Z$

c $-3 \leq x \leq 4$, $x \in R$

d $-3 < x \leq 4$, $x \in Z$

e $-3 < x < 4$, $x \in Z$

Section I: Student Activity 1

f $-3 < x < 4,\ x \in R$

g $-3 \leq x < 4,\ x \in R$

6 Solve the following inequalities and represent the solution set on the number lines:

a $-3 < x + 2 < 5,\ x \in R$

b $-3 \leq x + 2 \leq 5,\ x \in Z$

c $-2 \leq x + 3 \leq 5,\ x \in R$

d $-2 < x + 3 < 4,\ x \in Z$

e $-2 \leq x - 1 \leq 4,\ x \in R$

7 Solve the following inequalities and represent the solution set on the number line:

a $-1 \leq 2x + 3 \leq 7,\ x \in R$

b $-3 \leq 2x + 4 \leq 3,\ x \in R$

c $4 < 2x + 1 < 5,\ x \in R$

d $4 < 2x + 1 < 5,\ x \in Z$

e $4 < 2x + 1 < 5,\ x \in N$

f $3 < -2x + 1 < 5,\ x \in R$

Section I: Student Activity 1

Project Maths Tionscadal Mata Development Team

8 The temperature in a certain town ranged between 18°C and 22°C on a particular day. Represent this as an inequality in the form $a < t < b$, where t represents temperature.

9 Ryan wants to spend between €45 and €67 on a present. If €a is the amount he wishes to spend, represent his situation as an inequality.

10 The number of matches in a box is between 95 and 103. Represent this statement as an inequality.

11 Brendan has €x and we know that if he had 3 less it, he would have between €2 and €10. Represent this as an inequality and solve it.

12 A student wants to keep the cost of his phone calls per week between €5 and €7 per week. Calls cost €0.20 per call. Write an inequality to represent this and solve the inequality to determine the maximum number of calls this student can make while remaining within his budget.

13 A household wants to keep its electricity charges between €30 and €35 per week. Assuming that there is a standing charge of €5 per week and each unit of electricity costs €0.10 per unit. Write an inequality to represent the range of units that this family can use per week.

14 The average number of sweets in a box of a particular brand of sweets is 150, but the number can vary by ±5. Write an inequality to represent the number of sweets in the box.

15 A dog food company has a tolerance level of 0.8 Kg on an 8Kg bag of dog food. Write an inequality that represents an acceptable weight for a bag of this dog food.

16 Solve $2(x + 3) > x + 3$.

17 Solve $-8 < -3x + 1 < 4$, $x \in R$ and show the solution on the number line.

18 Solve $11 < -4x + 3 < 15$, $x \in R$ and show the solution on the number line.

Section I: Student Activity 2

Project Maths Tionscadal Mata Development Team

Tarsia

Cut along the line that separates each row and then cut each section a half again. This will form a set of dominos. Instruct the students to find the domino with **Start** on it and then search for the matching section that accompanies the Start domino. Continue this until **Finish** is reached. **Note:** The solution is on the page 57.

$x \leq -4$	$4 < 2x < 6$	$-3x < 6$	$x < 3$
$x > 8$	$x < 2$	$-2 < x < 3,\ x \in Z$	x is at least 4
$x \geq 4$	Finish	$-1 < x + 2 < 4$	$-x < -8$
$-x > 4$	$x > -2$	$2x + 3 > 3x + 1$	$x < -4$
$2 < x < 3$	$-3 < x < 2$	$4 \leq 2x \leq 8$	$\{-1, 0, 1, 2\}$
$x + 3 < 6$	$2 \leq x \leq 4$	Start	$-x + 1 \geq 5$

Solution to Tarsia

Project Maths Tionscadal Mata Development Team

Solution to Tarsia contained in Section I: Student Activity 2 page 56

					Start
					$-x + 1 \geq 5$
$-x < -8$	$-1 < x + 2 < 4$	$-3 < x < 2$	$2 < x < 3$	$4 < 2x < 6$	$x \leq -4$
$x > 8$					
$x < 2$					
$2x + 3 > 3x + 1$	$x < -4$	$-x > 4$	$x > -2$	$-3x < 6$	$x < 3$
					$x + 3 < 6$
					$2 \leq x \leq 4$
Finish	$x \geq 4$	x is at least 4	$-2 < x < 3$, $x \in Z$	{-1, 0, 1, 2}	$4 \leq 2x \leq 8$

Section I: Student Activity 3

Project Maths Tionscadal Mata Development Team

Graph Matching Exercise.

These need to be laminated and cut into separate pieces before the class. Students will then be asked to match them. **Note** every Important Word does not have an Equivalent Form to match with.

Important Words	Sample Sentence	Equivalent Form	Translation
at least	Brian is at least 40 years old.	Brian's age is greater than or equal to 40.	$x \geq 40, x \in R$
at most	At most 40 people attended a function.	40 or fewer people attended the function.	$x \leq 40, x \in \mathrm{N} \cup \{0\}$
must exceed	Earnings must exceed €40.	Earnings must be greater than €40 per hour.	$x > 40, x \in Q$
must not exceed	The speed must not exceed 40km per hour.	The speed is not greater than 40km per hour.	$x \leq 40, x \in R^+$
less than	Spot's weight is less than 40kg.	Spot's weight is below 40kg.	$x < 40, x \in R^+$
more than	Dublin is more than 40 miles away.	is greater than	$x>40, x \in R$
between	The film is between 40 and 50 minutes long.	The film is greater than or equal to 40 minutes or less than or equal to 50 minutes.	$40 \leq x \leq 50, x \in R$
more than	Mary spent €40 or more.		$x \geq 40, x \in Q$
Not less than	Galway is not less than 40km away.	Galway is more than or equal to 40km.	$x \geq 40, x \in R$
Less than	Damien is paid less than €40 per day.		$x < 40, x \in R^+$
Not more than	There are some animals on the farm but not more than 40.	There are less than or equal to 40 animals on the farm, given that there are some animals on the farm.	$x \leq 40, x \in N$

