Teaching & Learning Plans

Plan 1: Introduction to Probability

Junior Certificate Syllabus
Leaving Certificate Syllabus

Project Maths Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. Student Learning Tasks – Teacher Input: This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. Student Activities – Possible and Expected Responses: Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. Teacher's Support and Actions: Gives details of teacher actions designed to support and scaffold student learning.

iv. Checking Understanding: Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 1: Introduction to Probability

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Aims

- To familiarise students with the ways in which we talk about uncertainty and look at everyday situations in which probability arises
- To engage students in activities that will give them contact with the main ideas of probability
- To rehearse the language and patterns associated with probability

Prior Knowledge

Prior knowledge and experience of handling fractions and percentages is required.

Students have prior knowledge of some of the ideas and language patterns of the topic of probability from the primary school curriculum, third class upwards, but the topic may need to be revisited to ensure that all students know the basics. Students may have certain 'misconceptions' based on intuition and personal experience. Experimentation is required, where students count and analyse outcomes, thereby constructing their own meanings by connecting the new information to what they already believe. Students accept new ideas only when they see that their old ideas do not work: for example, finding out experimentally that 6 is not the hardest number to get when throwing a fair die, and that all outcomes of such a throw are equally likely.

When working together cooperatively in small groups, to test a hypothesis for example, students will be using the language of the topic thus improving their ability to communicate effectively using correct terminology. They can then move on from the experimental approach, where they calculate the relative frequency of an event, which tends towards the probability for an infinite sequence of trials, to the theoretical approach, which is based on logical reasoning.

Learning Outcomes

As a result of studying this topic, students will be able to

- distinguish certain from uncertain events
- describe events as being more or less likely from experience
- order events from least likely to most likely and justify their choice
- use a scale from 0 to 1 to informally place everyday chance-related events
- represent and interpret probabilities as fractions, decimals and percentages
- represent the probability of an event as a fraction or decimal between 0 and 1 or as a percentage
- list all possible outcomes for practical experiments such as rolling one die
- determine the probability of an event using the results of an experiment

Relationship to Junior Certificate Syllabus

Sub-topics	Ordinary Level
1.5 Counting	Listing outcomes of experiments in a systematic way
1.6 Concepts of probability	The probability of an event occurring: students progress from informal to formal descriptions of probability. Predicting and determining probabilities Decide whether an everyday event is likely or unlikely to occur.

Relationship to Leaving Certificate Syllabus

Sub-topics	Foundation Level	Ordinary Level
1.2 Concepts of probability	Decide whether an everyday event is likely or unlikely to occur. Recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur.	Estimate probabilities from experimental data. Associate the probability of an event with its long-run, relative frequency.
1.3 Outcomes of random processes	Apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes.	

Introducing the Topic

Students need to get into a frame of mind for learning probability by looking at the language of uncertainty and then trying to order phrases used to describe uncertainty, leading to being able to quantifying it.

The following examples could be used to explore misconceptions:

- What is the most difficult number to get when throwing a fair die?
- Random events should have outcomes which appear random; for example, in the lotto theory tells us that any of the six numbers is equally likely to turn up, yet more people choose randomly spaced numbers than numbers which form a pattern like 1,2,3,4,5,6 etc.
- The likelihood of 2 consecutive numbers appearing in any Lotto draw (which is > 50%) could easily be investigated by reference to a number of recent draws.

Real Life Context

The following examples could be used to explore real life contexts.

Looking at statistics from the Census, questions like:

- How long will I live?
- Will I get married?
- How many children will I have?

These questions can be answered with some degree of certainty based on population statistics.

Life assurance companies work out how much to charge for their premiums based on tables of life expectancy. Why are some premiums cheaper than others?

Lesson Interaction			
Student Learning Tasks: Teacher Input	**Student Activities: Possible and Expected Responses**	**Teacher's Support and Actions**	**Checking Understanding**
» In this lesson we will be investigating ideas about uncertainty or chance or probability.			
» When, in our everyday lives, would knowing the chance or likelihood of an event occurring affect our actions? Do you have any suggestions?	• Betting on a horse – knowing the odds. • Taking a new drug – what are the chances of it curing the disease? • Flying with a particular airline – what is its safety record to date? • Car insurance companies charge premiums based on the probability that you will have an accident. • Health insurance premiums are less if you don't smoke and have no diagnosed serious illnesses since the probability of you being sick is less.		» Did students come up with several varied suggestions?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» We will begin by looking at the language used in uncertainty and chance, and move on to how numbers are assigned to uncertainty.		» Distribute **Student Activity 1.** » Divide class into small groups. Each member of the group should have a clear task. Make sure there is a time limit on the activity.	
» **Student Activity 1A,** fill in: » 3 events which are certain to happen. » 3 events which are certain not to happen or to have happened. » (in the area marked uncertainty) 3 events which might or might not happen. » Depending on the event, place the uncertain ones nearer the "certain to happen" or "certain not to happen" side of the diagram.		» Walk around to see what students are writing down; if they are struggling, ask questions which will give them a hint of an example. » Take selections from each group and put these on the board.	» Did students come up with several varied suggestions?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
	• Certain to happen/have happened: 1. I have been born. 2. The sun rose this morning. 3. There is a Chinese Olympian who is 6' 7". • Certain not to happen/have happened/impossible: 1. Ireland will not host the next Olympics. 2. Finding a triangle with 4 sides. 3. Someone in our class was born during the First World War. 4. Michael Jackson is still alive. • Might/might not happen 1. Ireland will win a gold medal in the next Olympics. 2. It will rain tomorrow. 3. Roy Keane will manage the Irish soccer squad (students basing their convictions on experience or intuition).	» Walk around to see what students are writing down; if they are struggling, ask questions which will give them a hint of an example. » Take selections from each group and put these on the board.	» Did students come up with several varied suggestions?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Working in pairs, brainstorm phrases used to describe the probability or chance of an event occurring on a scale from absolute certainty to no chance at all. » Write down at least 5 phrases on the bottom of **Student Activity 1B.**	» Students will fill out phrases on **Student Activity 1B:** • a dead cert • a very good chance • likely • almost certain • no chance • more than likely • extremely likely • 50/50 • a small chance • extremely unlikely • never • not in a month of Sundays • fairly likely	» Check if everyone understands the task. » Walk around to see what students are writing down. If anyone is struggling, ask questions which will give them a hint of an example. » Take selections from each group and record them on the board/flipchart/laptop.	» Did students have lots of different suggestions once they understood the task?
» Take down the selection of class phrases from the board into the table on **Student Activity 2A.**	» Students copy from the board the selection of phrases (about 12) suggested by class.	» Distribute **Student Activity 2.**	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» From this list of phrases identify an event which can best be described by each of these terms.	» Students then fill in one event for each phrase on **Student Activity 2B.**	» Check if everyone understands the task. » Walk around to see what students are writing down; if they are struggling, ask questions which will give them a hint of an example. » Take selections from each group and put on the board. Invite students to agree or disagree, but explain that they must have a valid reason for doing so.	» Were there many and varied suggestions and were they appropriate to the phrases?
» Now think in terms of how you might interpret this lack of precision in different situations, for example the plane you're on probably won't crash vs. it probably won't rain tomorrow. Would you be happy with the phrase "probably won't" in both situations?	» Students will be looking for more precision in the case of a plane not crashing – in the form of a numeric representation of the phrase "probably won't".		» Do students recognise the need for a numeric representation of the phrase "probably won't"?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Let's try to get more precision by using percentages. » In terms of percentages, how would you describe a "dead cert", or something which was definitely going to happen?	• 100%	» Distribute **Student Activity 3.** » Ask the class and give them a moment to think before asking one student.	
» Write this in at the correct position on 'The Probability Scale (**Student Activity 3A**). » In terms of percentages, how would you describe something which had no chance of happening? Write this in on 'The Probability Scale' (**Student Activity 3A**).	• 0%	» Emphasise no chance.	» Are students getting the idea of limits of 0% and 100% for the range of probabilities of an event.
» Say that something probably will happen, that it is not a "dead cert" but that it has a very good chance; can you assign a percentage to this event? » What is the range of the answers?	• Maybe 70% , 80% but greater than 50% • >50%	» Emphasise the idea of a possibility of a range of answers but note that all answers are greater than 50%.	» Have students understood that phrases like "has a good chance" are imprecise but yet have a bias towards the upper end of the scale?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Assign an estimated percentage to the phrase "probably won't happen?"(snow on St. Patrick's day). » What is the range of the answers?	• Maybe 20%, 10% • <50%		» Again, have students understood the idea of a possibility of a range of answers but all less than 50%?
» What range of percentages can we use to represent the chance or likelihood of something happening, to cover all possibilities?	• 0% to 100%	» Ask the class; wait a short while and then ask an individual student. If students can't answer, take them back through the previous questions.	» Do students understand that chance can be represented by a range from 0% to 100%?
» Consider a whole bar of chocolate – what percentage of the bar are we looking at?	• 100%		
» If we toss a coin, what is the chance of getting a 'tail'? » What is another way of expressing this chance?	• 50% • ½	» Lead the class to the idea of a fraction as expressing the chance of something happening. » If students are comfortable with the idea of fractions, it may be possible to explore the rolling of a fair die.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Instead of giving 0% as the chance of something happening what number could we use? » Now write this in on 'The Probability Scale (**Student Activity 3A**).	• 0	» Ask the class; then select an individual student to answer.	» Are students associating a 0% chance with the number 0?
» Between what ranges of numbers can I represent the chance of something happening to cover all possibilities?	• 0 to 1	» Ask the class; then select an individual student to answer.	» Do students understand that probability can be represented on a scale of 0 to 1 as well as from 0% to 100%?
» So now you have two ways of representing a scale of probabilities. What are they?	• 0% to 100% or • 0 to 1	» Ask the class; then select an individual student to answer.	
» Give examples of numbers which can represent the possibility of something happening? » Position them on 'The Probability Scale1 on **Student Activity 3A**. » Show me where 5/8 would be placed. Has anyone anything different?	• 0.9, 1, 0, .5, .3, ¾, 0.75 • 1 = dead cert • 0 = never • 0.5 = evens • 0.8 = quite likely • 0.1 =very unlikely • 0.2 = quite unlikely • 0.4 = not a good chance	» Ask most students in the class, each time asking the class to verify if they are correct, and why. » Write up correct values on the board. **Note:** (Ensure that a good selection of proper fractions and decimals are included). Note that, by contrast with "likely", we can all agree with the placing of these values.	» Can students apply what they have learned about the probability scale and can students give a range of numbers including both decimals and fractions?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If I say that the chance of it raining tomorrow is 2.5 – is this possible? » If I say that the probability of one of you flying to Mars tomorrow is 3 – is this possible?	• No, because the chance of something happening must be a number between 0 and 1.	» Ask the class; then select an individual student to answer.	» Can students apply what they have learned about the probability scale?
» Can you give examples of numbers which cannot represent the chance of something happening?	• -1 • 5, 7, • 2000, • 23.6, • 9/8	» Ask most students in the class. Then ask the class to verify if the answers are correct, and why. » Suggest fractions and negative numbers.	» Can students give a range of numbers including negative numbers?
» Using a ruler, draw in your copy 'The Probability Scale' line segment from **Student Activity 3A**. » Label this "The Probability Scale". Mark in the numbers listed on the board.	» Students draw the line and fill in the numbers.	» Circulate, supporting students who have difficulty with the task.	» Have all students been successful in drawing and marking the scale?
» The line on **Student Activity 3B** represents a scale from 0 to 1. Working in pairs write in each item from Box A at the most appropriate position on the line. (Use arrows.)	» Students write in each phrase or number onto the appropriate spot on the scale. » Students might start with percentages, which they are most familiar with, and proceed to phrases and then fractions.	» Pin a large scale onto the board and have cards with the various options written on them (or draw in on the board). » Check if everyone understands the task.	» Are there many students having difficulty understanding percentages etc?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» To keep the diagram clear you could put all percentages in a line, fractions underneath on another line and then phrases on another line.	» Students may leave the phrases until last as they are imprecise.	» Walk around to see what students are writing down. Some students may have difficulty here changing percentages or decimals to fractions or vice versa, and as you walk around identify and guide those students.	» Has everyone completed the task?
» Attach the phrases to the large scale on the board. (or write in on the board). » Does everyone agree with these placements? » Does anyone think they should be changed? » Has anyone anything different? » Why do you think it should be changed?	» When everyone is finished a volunteer student fills in the phrases on the board.	» Thank student who does board work. » If a student disagrees they must give a reason. Brief class discussion to achieve consensus.	» Is the student body in agreement with the placement of the phrases?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» For each of the numerical representations of probability in **Box A** write it in the two other possible forms, for example as a fraction/decimal/ percentage.	• ¾= 75%=0.75 • 0.375=3/8=37.5% • 87.5%=7/8=0.875 • 0.125=1/8=12.5% • 0.25=1/4=25%	» Place students who can do this competently with students who have difficulty. The better student can have a supporting role when the other student is asked to explain his/her answer. » Ask a student who was previously unsure to call out and explain how he/ she did the conversions from one form to another.	
Reflection » Now think how you have represented probability numerically. » Give 3 numerical representations of a 50/50 chance? » Now write down three ideas you have learned about probability and at least one question.	• %, fractions and decimals between 0 and 1 • 0.5, 1/2, 50%		» Are students aware of these ideas? 1. Probability is about uncertainty and how to assign numbers to uncertainty (phrases being too imprecise) given some information about the particular situation. 2. The Probability scale is between 0 and 1. 3. Probability can be represented by a fraction or a decimal between 0 and 1 or by a percentage e.g.½, 0.5, 50%. 4. Not everything in mathematics is certain!

Student Activity 1

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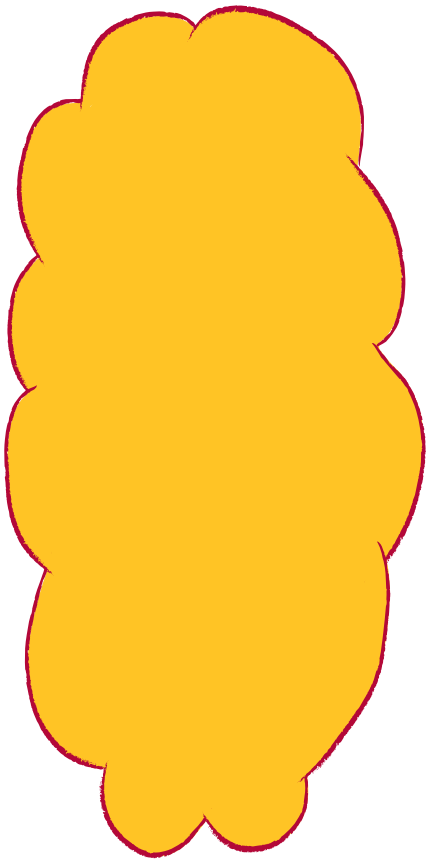
Student Activity 1A

Certain not to happen	Area of Uncertainty	Certain to happen
1.		1.
2.		2.
3.		3.

Student Activity 1B

Phrases used to describe uncertainty

1. _____
2. _____
3. _____
4. _____
5. _____

Student Activity 2

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Student Activity 2A

Phrases used to describe uncertainty
(examples from the class)

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____

Student Activity 2B

An event associated with each phrase
(examples from the class)

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____

Student Activity 2C

Order the above phrases on the scaled line segment below – from least likely to most likely.

Student Activity 3

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Student Activity 3A

The Probability Scale

Student Activity 3B

The Probability Scale

Box A

Extremely unlikely

1/4

3/8

1 in 4 chance

0.125

Extremely likely

75%

87.5%

Probability of getting an odd number when rolling a die

0

0.25

¾

Certain

50:50

Equally likely

1/2

1

Impossible

Place the above phrases, numbers and percentages at the correct position on the probability scale.
Find and write down instances from TV, radio, or in the newspaper which illustrate how probability affects people's lives.

