Teaching & Learning Plans

Plan 2: Probability and Relative Frequency

Junior Certificate Syllabus
Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

Project Maths Tionscadal Mata Development Team

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible and Expected Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Checking Understanding:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 2: Probability and Relative Frequency

Project Maths
Tionscadal Mata
Development Team

Aims

- To introduce the concept of 'outcomes' of an 'event'
- To estimate likelihood of occurrence of events
- To establish the 'Sample Space' as the set of all possible outcomes

Prior Knowledge

Students should have prior knowledge (from T and L Plan 1 and/or from primary school) of some terms associated with chance and uncertainty. They should be familiar with probability expressed as a fraction or decimal in the range 0 to 1, or as a percentage in the range 0% to 100%.

Learning Outcomes

As a result of studying this topic, students will be able to

- understand and use the following terminology: trial, outcome, set of all possible outcomes, relative frequency, event, theoretical probability, equally likely outcomes
- list all the possible outcomes when rolling a fair die
- recognise that the outcomes on successive throws of a die are independent of each other
- calculate, from experimental results, the relative frequency for each outcome and note how it approaches the theoretical probability as the number of trials increases
- understand the concept of a fair die
- distinguish equally likely outcomes from those which are not
- associate probability with long-run relative frequency

Relationship to Junior Certificate Syllabus

Sub-topics	Ordinary Level	Higher Level (Inc.OL)
1.5 Counting	Listing outcomes of experiments in a systematic way	
1.6 Concepts of probability	Recognise that probability is a measure on a scale of 0-1 (and 0-100%) of how likely an event is to occur. Estimate probabilities from experimental data. Associate the probability of an event with its long-run, relative frequency.	
1.7 Outcomes of simple random processes		Apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes.

Project Maths
Tionscadal Mata
Development Team

Relationship to Leaving Certificate Syllabus

Sub-topics	Foundation Level	Ordinary Level
1.1 Counting	List outcomes of an experiment. Apply the fundamental principle of counting.	
1.2 Concepts of probability	Recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur.	Estimate probabilities from experimental data. Associate the probability of an event with its long-run, relative frequency.
1.3 Outcomes of random processes	Apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes.	

Resources Required

A die for each pair of students

Project Maths
Tionscadal Mata
Development Team

Introducing the Topic

An "Event"

Following from our last lesson we know that, directly or indirectly, probability or chance plays a role in a wide range of activities. We often make statements which involve terms such as the likelihood or the chance of occurrence of an event – what do we mean by an 'event'?

- 'It will probably rain today'. The event is 'It will rain today.'
- 'Though we are sending the national team to the Olympics, we cannot confidently predict that we shall win a gold medal'. The event is 'We shall win a gold medal'.
- 'There is a chance that Roy Keane will manage the Irish Football team.' The event is 'Roy Keane will manage the Irish football team.

Each statement above suggests an event whose occurrence or non-occurrence involves an element of uncertainty.

Estimating the chance of an 'event' occurring?

Because of past information or currently available statistics for an event, we can predict, with some degree of confidence, what the outcome of the event will be.

The past performance of Brazil's football team in the World Cup can help us to estimate the probability of the team winning the next World Cup (they have won 5 out of the last 12 world cups: 1958, 1962, 1970, 1994 and 2002).

Thus, for example, we may make the statement (a) above if most of the days we have observed recently were rainy days.

Associating numbers with phrases like "very likely" and "probably"

In conversation we might say that it was 'very warm' yesterday. Would this have the same meaning for a person living in central Australia as for someone living in Birr? How can we be clearer about what "very warm" means? The expert from the Meteorological Office would state the maximum temperature in degrees C, thus quantifying the situation.

Similarly a person might describe himself as 'having big feet', but when it comes to buying shoes, a more exact description, i.e. shoe size, is needed.

Terms like 'most likely' and 'probably' are too vague for many purposes; so, ways of measuring probability have been devised.

Real Life Context

Apart from gambling, (mention Las Vegas) the theory of probability can help us make relatively reasonable choices in our daily lives.

Students choose college courses which have a high probability of leading to employment after college.

Many people have left New Orleans for good because there is a high probability of devastating hurricanes in that region.

People don't smoke because there is a high probability of their developing cancer as a result of smoking.

Probability has widespread use in business, science and industry. Its uses range from the determination of life insurance premiums to the description of the behaviour of molecules in a gas and also the prediction of the outcomes of an election.

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» When you toss a die what are the possible outcomes?	• 1, 2, 3, 4, 5, 6	» Ask an individual student. Tell the class that we call this set of all possible outcomes "the sample space".	
» When you toss a die which number is most likely to appear? » Which is least likely? » Write down your answer individually on the top of **Student Activity 1A.** » Working in pairs, roll a die 30 times (i.e. 30 trials) and fill in columns 2 and 3 on **Student Activity 1B.**	• Most likely__________ • Least likely__________	» Distribute **Student Activity 1**. » By walking around ensure that everyone has made a guess. » Give a die to each pair.	» Do the students' answers show misconceptions? This may influence your decision on the next part – whether to ask students to do 30 or 50 rolls of the die. » Do the students know how to do tally marks? » Are students using correct terminology, e.g. what was the outcome for that trial?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Fill your results onto the Master Table A on the board when you have completed columns 2 and 3.	» As students finish they fill results into the **Master Table A** on the board i.e. the frequencies of 1's, 2's, etc. for each group.	» Put an A3 size copy of **Master Table A** (as in **Student Activity 2A**) on the board. » Collect all dice. While collecting dice note what the last column adds up to for each group.	
» Look at the **Master Table A**, did every group get the same number of 1's or 2's etc. for 30 trials? » Is there a trend appearing?	» Students can see that their results are not the same but similar. » Students might suggest calculating the average number of 1's etc. » Students can calculate the average numbers of 1's etc. and inform the class.	» Give groups a minute to look at the results from the class and compare the class results to their own.	» Do students notice a trend in the outcomes?
» Calculate the relative frequency for each outcome (column 4) and the corresponding percentages in column 5 (**Student Activity 1B**)	» Students calculate the relative frequencies and should see that they are all almost equal.	» Circulate checking students' calculations asking questions where necessary.	» Are students beginning to see that all outcomes are equally likely?

Project Maths Tionscadal Mata Development Team

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If you add up all the values in the last 2 columns in your own table (**Student Activity 1B**) what do they add up to? » Fill these answers into **Student Activity 1C**. » There is a connection between probability and relative frequency which we will come back to later in the lesson.	• 1 and 100%	» Give students a few moments to consider their answers and then ask different groups for their answers, so that the class realises everyone got 1. » Ask those who didn't get 1 to recheck their answers. Circulate and help with queries.	» Have all students realised that the sum of all the probabilities for a sample space is 1?
» From the last lesson, what did 1 mean on the probability scale? » What outcomes are you certain to get with this die?	• "1" meant certainty. • 1 or 2 or 3 or 4 or 5 or 6. The probability of getting one of these numbers is 1.	» While walking around check that everyone has added up last 2 columns and has got 1 and 100%.	
» What do you notice about your results? Do they confirm /refute your prediction? » Write down a conclusion (**Student Activity 1D**) and refer back to your prediction and why you made that prediction.	» Students write up their conclusions. » Different groups read out their conclusion and if it is in agreement with their prediction, give reasons why they made that prediction.	» Circulate giving advice where necessary, asking some groups to state their conclusions to the class while checking that each group has a written conclusion.	» Was each group able to make a written conclusion?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» How might you modify this experiment so that you could have more confidence in your conclusion?	» Students may suggest more trials.		
» How might we look at the results for a larger number of trials?	» Students suggest using the set of class results from: **Master Table A** on the board.		» Do students expect less variation and clearer trends with more trials?
» Copy **Master Table A** from the board and fill in class results. » Calculate the relative frequency for each outcome (column 4) and the corresponding percentages (column 5) for each outcome. » Now that you have over 300 trials can you see any pattern emerging? Write a conclusion, referring again to your prediction.	» Students make out their own **Master Table** and do the calculations (**Student Activity 2B**) and write a conclusion (**Student Activity 2C**) comparing it with their conclusion from 30 trials. » This time different students read out their conclusions noting any differences with the last time – trend clearer with more trials. » Students note that the relative frequency of each outcome is the same i.e. 1/6. approx.	» Walk around and check the prediction against the experimental results and ask students why they made that prediction. » Ask those who thought 6 was the hardest to get and why they thought this. (Experience from board games, biggest number?)	» Did students see a clearer pattern of almost equally likely outcomes from the larger number of trials? » Did students who had thought that 6 was harder to get acknowledge their misconception?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Individually draw a graph on graph paper using the data from the **Master Table**. » Write a conclusion.	» Students choose the type of graph to draw, for example a bar chart. » Students may draw different graphs. Students comment on the effectiveness of their graph in illustrating a trend. Students could also draw a graph of their own results comparing it to the class **Master Table** results. » Conclusions from graphs should be consistent with those from tables.	» Circulate giving hints /help where necessary. » Circulate reading the conclusions, asking a few students to read out theirs to the class. **Note:** Possible use of Autograph –simulating throwing a die thousands of times.	
» Because all outcomes are equally likely can you think of a word to describe a die like this if it were used in a game which gave everyone the same chance?	• Some students may come up with the word "fair".	» Give the word 'unbiased', if the students don't suggest it and reiterate that what is meant by a fair die is that all outcomes are equally likely.	
» The menu choices for today's lunch are pizza, roast beef and salad. Is the probability that I will choose pizza = 1/3?	• Yes • I am allergic to cheese so the probability that I would choose pizza is zero. • I am a vegetarian and would never choose beef.		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What is different about this and the outcomes for the fair die? » How might we find out the probability?	• They are not all equally likely. • We might do a survey but could only get estimated values of the probability as outcomes are not equally likely.	» Give students a minute to discuss this in their group and then ask different groups. If someone answers 'yes', ask if anyone else has a suggestion.	» Can students distinguish between 'equally likely 'and 'not equally likely'?
» Can you think of other examples of situations where the outcomes are not equally likely?	• In horse racing – depends on form, jockey, etc. • 5 people going for an interview do not all have an equal chance of getting the job. • Loaded die , die with, say, two 1's and two 2's, etc.	» Pose the question to the class and take answers as students put up their hands.	» Did students actively participate in the discussion, giving lots of ideas and showing understanding?
» The last approach is known as the 'experimental 'or 'empirical approach' to calculating probabilities. However, we did not calculate probabilities but instead calculated the relative frequencies – so what is the connection with probability? Has anyone got any ideas on this? **Note:** (Students who may see the connection need to be allowed say so). » What do you think would be the value of the relative frequencies if we did more and more trials?	• Experiments give estimates of theoretical probability based on the relative frequency of each outcome. Relative frequency tends towards the probability as the number of trials gets very large. • They would get closer and closer to 1/6 and hence would all be the same.		

Project Maths Tionscadal Mata Development Team

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Based on what you now know about probability and its relationship to long term relative frequency, and also based on the bar chart representing the frequency of each outcome, can you now fill in the last column 6 in the **Master Table A**?	» Students fill in 1/6 for the probability of each outcome.	» If the number of trials has not been sufficiently large students may not see that the outcomes are equally likely so either repeat with a larger number of trials or use of a computer generated simulation. This should convince students that all outcomes are equally likely.	» Have all students been able to complete the probability column correctly?
Reflection » Write down 3 items you learned about probability today. » Write down anything you found difficult. » Write down any question you may have.	1. All outcomes are equally likely when tossing a fair die. 2. Relative frequency = frequency of outcome divided by the number of trials. 3. Equally likely outcomes all have an equal probability of occurring. 4. Relative frequency = probability for an infinite number of trials where outcomes are equally likely.	• Circulate and take note particularly of any questions students have and help them to answer these.	

Student Activity 1

Project Maths
Tionscadal Mata
Development Team

Student Activity 1A
My prediction:

Which number is most likely to appear________________?

Which number is least likely to appear________________?

Student Activity 1B

Number Which Appears on Die (Outcome of Trial)	How Many Times Did This Happen? Use Tally Marks to Help You Count	Total (Frequency)	Fraction of Total Scores (Relative Frequency) $\frac{Frequency}{no.\ of\ trials}$	Percentages of Total Scores $\frac{Frequency}{no.\ of\ trials}$ x100
1				
2				
3				
4				
5				
6				
Totals				

Student Activity 1C

The sum of all the relative frequencies is ________________?

The sum of all the percentages is ________________?

Student Activity 1D
Conclusion: (Refer to prediction)

Project Maths
Tionscadal Mata
Development Team

Student Activity 2

Student Activity 2A Master Table

Number Which Appears on Die (Outcome of Trial)	How Many Times Did This Happen? (Frequency) Each Class Group Writes in its Result	Total of Frequencies	Fraction of Total Scores (Relative Frequency) $\frac{total}{300} = \frac{Frequency}{sample\ size}$	Percentage of Total Scores $\frac{Frequency}{no.\ of\ trials}$ x100	Probability of Each Outcome
1	E.g. 5+6+5+....				
2					
3					
4					
5					
6					

Student Activity 2B

The sum of all the relative frequencies is _________________?

The sum of all the percentages is _________________?

Student Activity 2C
Conclusion: (Refer to prediction)

____

____

____

____

____

____

____