Project Maths
Tionscadal Mata
Development Team

Module 3

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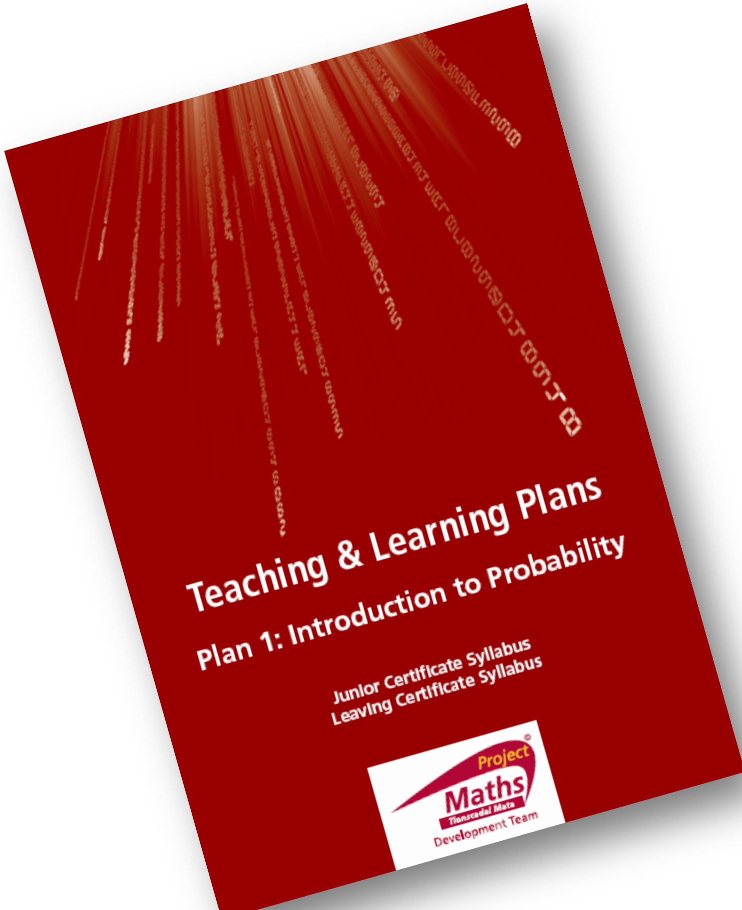
JUNIOR CERTIFICATE

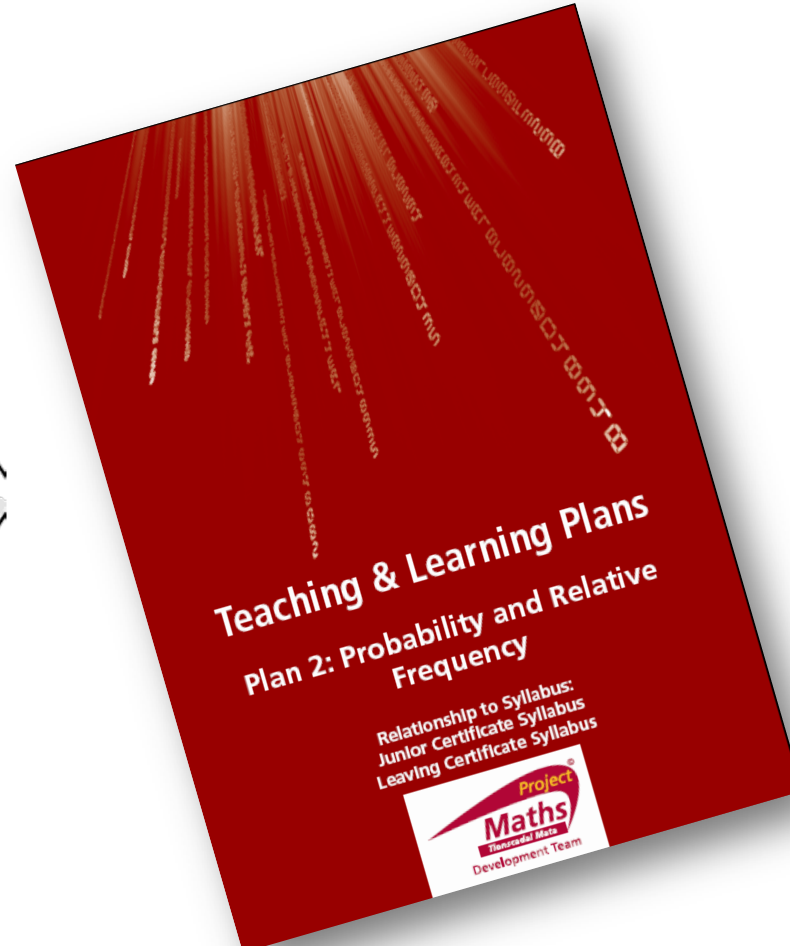
1. Probability Scale
2. Relative Frequency
3. Fundamental Principle of Counting
4. Outcomes of simple random processes
5. Basic set theory (HL)
6. Equally likely outcomes
7. Single Events questions
8. Multiple event questions
9. Tree Diagrams (HL)

LEAVING CERTIFICATE

1. All Junior Certificate Content
2. Arrangements and Selections
3. Set theory
4. Conditional Probability
5. Expected Value
6. Bernoulli Trials
7. Normal Distribution
8. Empirical Rule
9. Standard Normal Distribution (HL)
10. Standard Scores (z values) (HL)
11. Hypothesis Testing using Margin of Error (HL)

Junior Cert Probability





This introduces the Probability Scale

This makes the link between Relative Frequency and Probability

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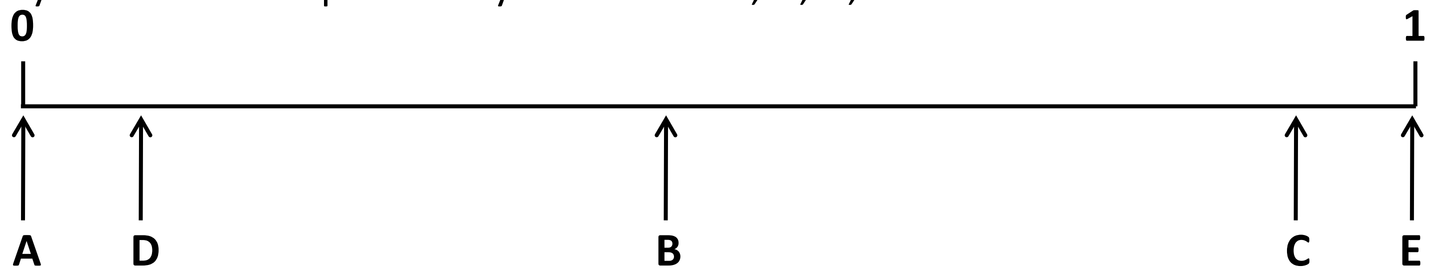
Concepts of Probability: Probability Scale

Example

The probability line shows the probability of 5 events A, B, C, D and E



Which event is:

(a) Certain to occur

Give an example of such an event

(b) Unlikely but possible

Give an example of such an event

(c) Impossible

Give an example of such an event

(d) Likely but not certain

Give an example of such an event

(e) Has a 50:50 chance of occuring

Give an example of such an event

Counting: Listing all Possible Outcomes of an Experiment

Example

A calculator can be used to generate random digits. Sandra generates 100 random digits with her calculator, she lists the results in the table below.

0	𝍸 I	5	𝍸 𝍸
1	𝍸 II	6	𝍸 𝍸 III
2	𝍸 I	7	𝍸 𝍸 II
3	𝍸 𝍸 II	8	𝍸 III
4	𝍸 𝍸 II	9	𝍸 𝍸 IIII

Based on Sandra's results, estimate the probability that the calculator produces

(a) 9 **(b)** 2 **(c)** a digit that is a multiple of 3 **(d)** a digit that is a prime.

Solution :

Counting: Listing all Possible Outcomes of an Experiment

Example

A calculator can be used to generate random digits. Sandra generates 100 random digits with her calculator, she lists the results in the table below.

0	𝍸 I	5	𝍸 𝍸
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2	𝍸 I	7	𝍸 𝍸 II
3	𝍸 𝍸 II	8	𝍸 III
4	𝍸 𝍸 II	9	𝍸 𝍸 IIII

Based on Sandra's results, estimate the probability that the calculator produces

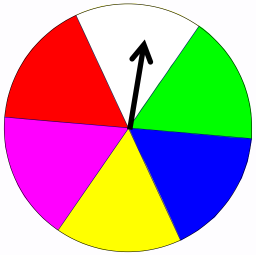
(a) 9 **(b)** 2 **(c)** a digit that is a multiple of 3 **(d)** a digit that is a prime.

Solution :

$$\text{Estimated Probability} = \frac{\text{Number of successful trials}}{\text{Total number of trials}}$$

(a) $\frac{14}{100} = \frac{7}{50}$ **(b)** $\frac{6}{100} = \frac{3}{50}$ **(c)** $\frac{39}{100}$ **(d)** $\frac{40}{100} = \frac{2}{5}$

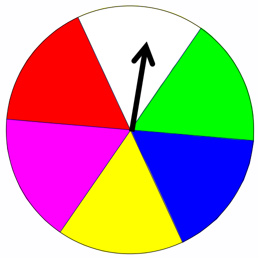
Outcomes of Simple Random Processes: Equally likely outcomes



Example

A spinner has 6 equal sectors coloured yellow, blue, green, white, red and purple. After spinning the spinner, what is the probability of landing on each colour?

Solution



Example

A spinner has 6 equal sectors coloured yellow, blue, green, white, red and purple. After spinning the spinner, what is the probability of landing on each colour?

Solution

We have a one in six chance of landing on any of the colours. Each colour has an equally likely chance, The outcomes are all equally likely.

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$\text{Probability of each colour} = \frac{1}{6}$$

Fundamental Principle of Counting

If one task can be accomplished in *x* different ways, and following this another task can be accomplished in *y* different ways, then the first task followed by the second can be accomplished in *x* by *y* different ways.

Example: A car manufacturer makes 3 models of cars, a mini, a saloon and an estate. These cars are all available in a choice of 5 colours; red, green, blue, brown and orange. How many different cars are available ?

Single Problem Events

Example A card is picked at random from a pack of 52 cards.

What is the probability that it is **(i)** an ace **(ii)** a spade?

Solution

Example A bag contains 8 blue marbles and 6 white marbles.

A marble is drawn at random from the bag. What is the probability that the marble is white?

Solution

Single Problem Events

Example A card is picked at random from a pack of 52 cards. What is the probability that it is **(i)** an ace **(ii)** a spade?

Solution

(i) $$P(\text{ace}) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{4}{52} = \frac{1}{13}$$

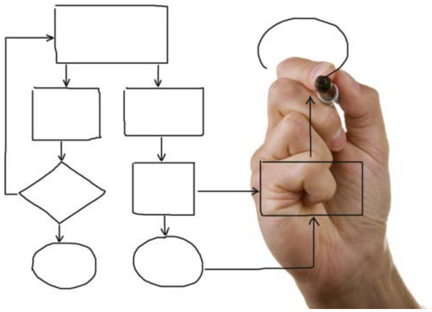
(ii) $$P(\text{spade}) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{13}{52} = \frac{1}{4}$$

Example A bag contains 8 blue marbles and 6 white marbles. A marble is drawn at random from the bag. What is the probability that the marble is white?

Solution $$P(\text{white}) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{6}{14} = \frac{3}{7}$$

Drawing a Picture

Drawing a Picture!



It is often said that the three basic strategies in probability are:

1. Draw a picture.

2. Draw a picture.

3. Draw a picture.

Some of the ways that this can be done are as follows:

(a) Writing out the sample space

(b) Make a simple visual of the sample space such as a two way table

(c) Draw a tree diagram (HL Junior Cert.)

(d) Use a Venn diagram

Multiple Event Problems

Example A die is thrown and a coin is tossed. What is the probability of getting a head and a 5?

Solution

Multiple Event Problems

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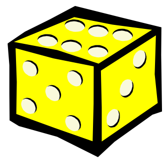
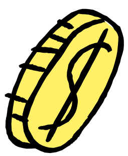
Solution





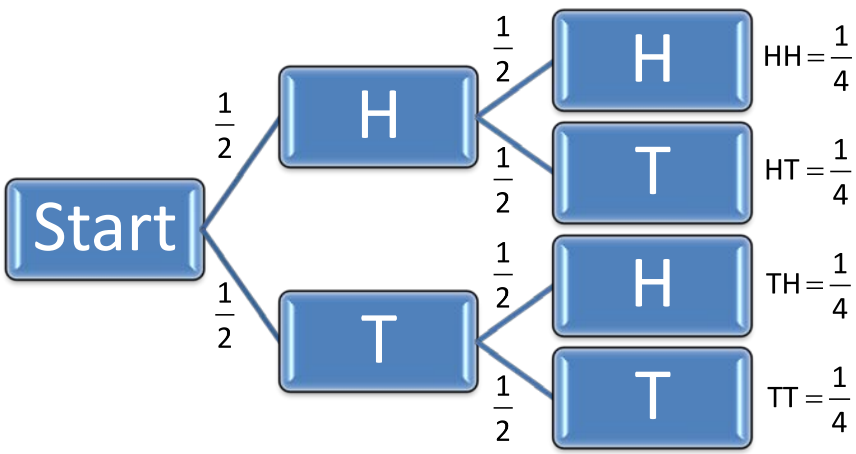
	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

$$\text{P(Head and 5)} = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{1}{12}$$

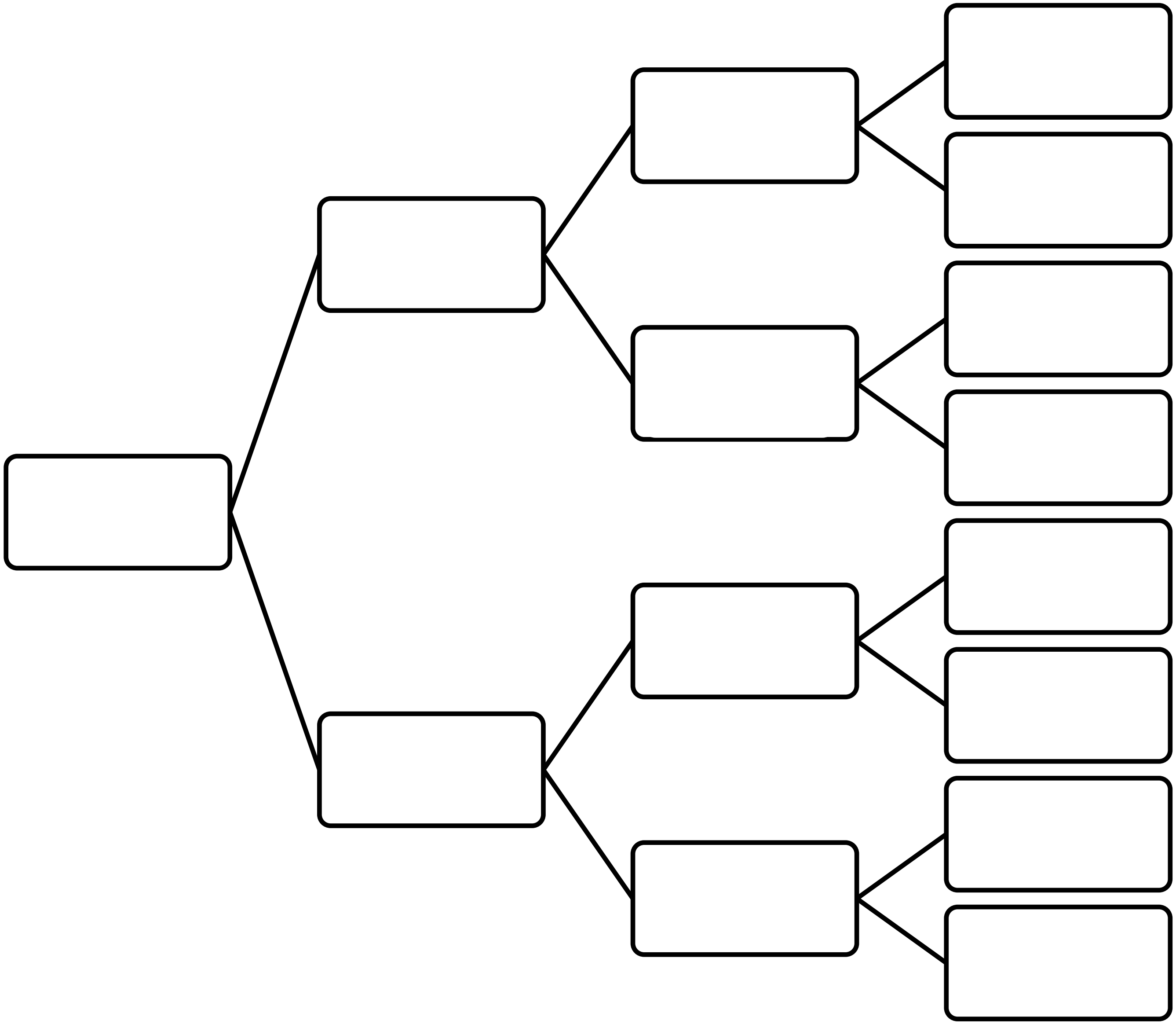
Tree Diagrams

Tree Diagram are very useful for
listing all possible outcomes (Sample Space).

Example Draw a tree diagram to show the number of possible outcomes when two coins are tossed.



$$P(\text{Head and Head}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$



Example Suppose that every child born has an equal chance of being a boy or a girl.

(i) Write out the sample space for the situtation in which a mother has two children.

(ii) What is the probability that a randomly chosen mother of two childern will have two girls?

(iii) What is the probability that this mother of two children would have two boys?

(iv) What is the probability that this mother of two children would have one boy and one girl?

Solution **(i)**

(ii)

(iii)

(iv)

Example Suppose that every child born has an equal chance of being a boy or a girl.

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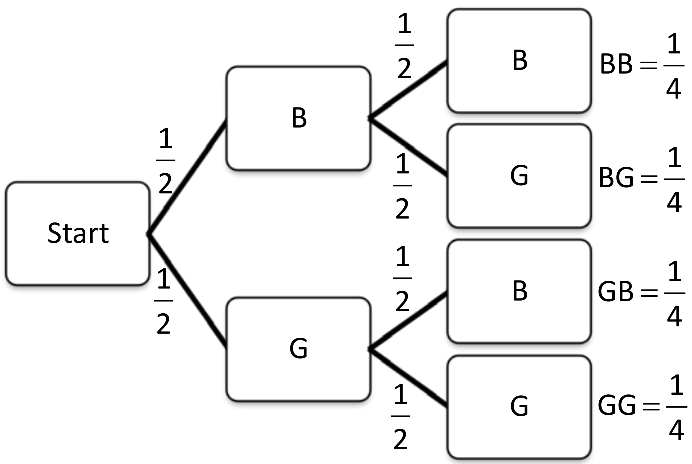
Solution **(i)**



(ii) P(G and G) = $\frac{1}{4}$

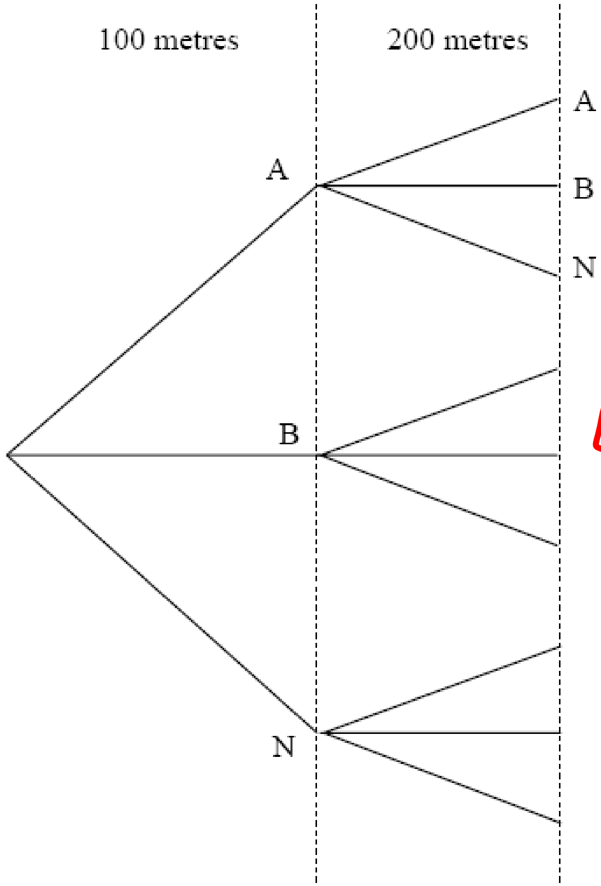
(iii) P(B and B) = $\frac{1}{4}$

(iv) P(1 B and 1 G) = $\frac{2}{4} = \frac{1}{2}$

Your turn!

3.1 & 3.2

	Alex	Bobby	Neither
100 metre race	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{7}{12}$
200 metre race	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$



Outcome	Probability
(A, A)	$\frac{1}{24}$
(A, B)	$\frac{1}{16}$
(A, N)	$\frac{1}{16}$
(B, A)	$\frac{1}{16}$
(B, B)	$\frac{3}{32}$
(B, N)	$\frac{3}{32}$
(N, A)	$\frac{7}{48}$
(N, B)	$\frac{7}{32}$
(N, N)	$\frac{7}{32}$

$$\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$$

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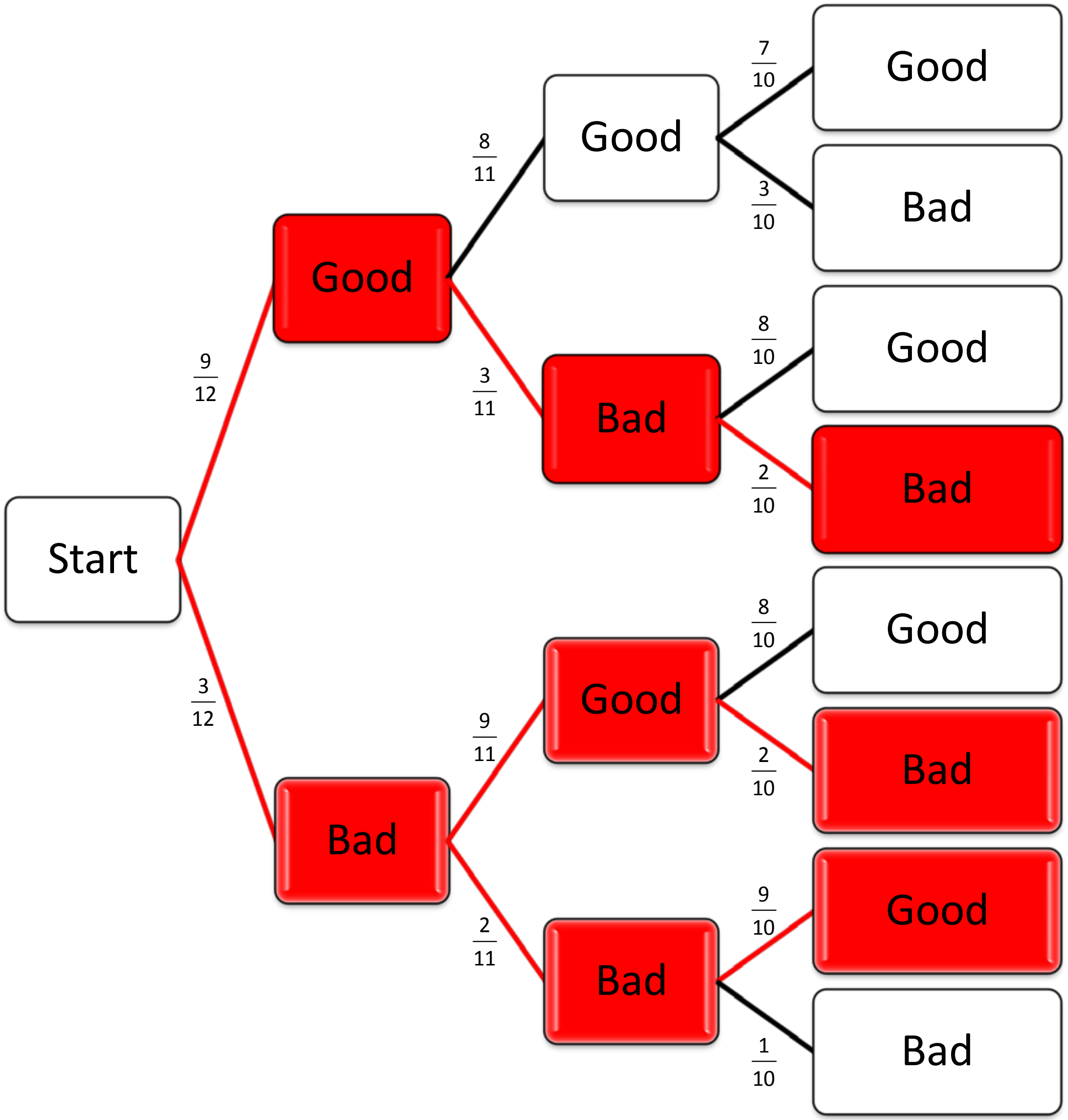

$$\left(\frac{9}{12}\times\frac{3}{11}\times\frac{2}{10}\right)+\left(\frac{3}{12}\times\frac{9}{11}\times\frac{2}{10}\right)+\left(\frac{3}{12}\times\frac{2}{11}\times\frac{9}{10}\right)=\frac{27}{220}$$

Example 1

In a class $\frac{1}{2}$ the students represent the school at Winter Sports and $\frac{1}{3}$ of the students represent the school at Summer Sports and $\frac{1}{10}$ at both.

Draw a Venn diagram to represent this.

If a student is chosen at random, what is the probability that someone who represents the school at sport will be selected?

Solution

Example 1

In a class $\frac{1}{2}$ the students represent the school at Winter Sports and $\frac{1}{3}$ of the students represent the school at Summer Sports and $\frac{1}{10}$ at both.

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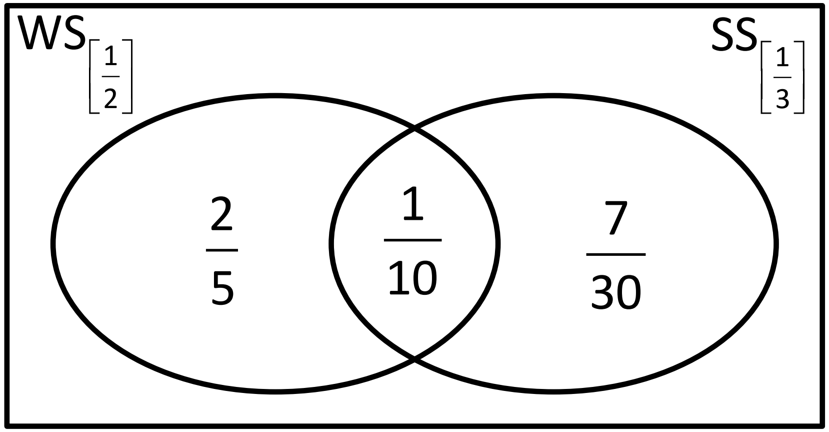

$$P(\text{Someone represents the school at sport}) = \frac{2}{5} + \frac{1}{10} + \frac{7}{30} = \frac{11}{15}$$

$\left[\Rightarrow \frac{4}{15}\right.$ is the probability than no one represents the school, which could be now entered on the venn diagram.$\left.\right]$

Example 2

The probability that John will be on the school football team is 0.6. The probability that he will be on the school hurling team is 0.5 and the probability that he will be on both the football and hurling teams is 0.3. His father says that he will buy him a bicycle if he represents the school in any of the above sports. What is the chance that he gets the bicycle from his father?

Solution:

Example 2

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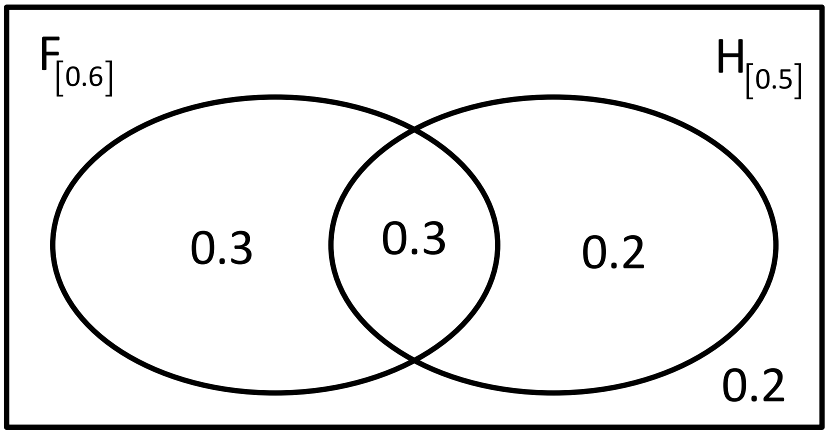

P(F or H) = 0.8

Example 3

There are 30 students in a class. 20 had been to France, 19 had been to Spain and 3 had been to neither country.

(a) Represent this on a Venn diagram and find how many students had been to both countries.

(b) If a student from the class is chosen at random, what is the probability that the student has:

- **(i)** visited France?
- **(ii)** visited France only?
- **(iii)** visited France or Spain?
- **(iv)** visited only one country?

Solution

(i)

(ii)

(iii)

(iv)

Example 3

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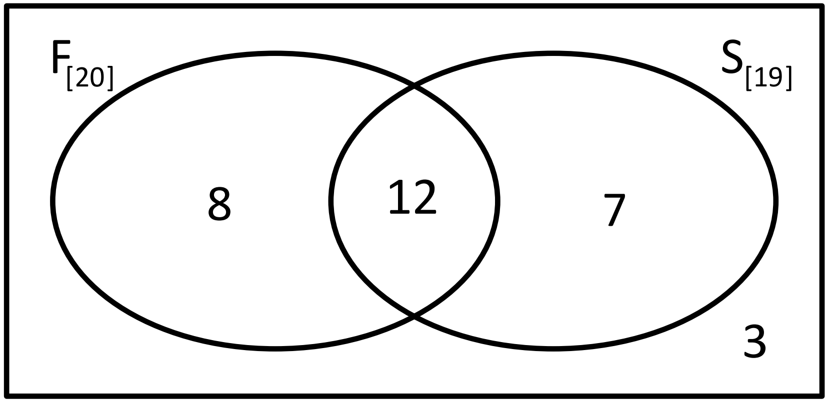
Solution



(i) $P(F) = \frac{20}{30} = \frac{2}{3}$

(ii) $P(F \text{ only}) = \frac{8}{30} = \frac{4}{15}$

(iii) $P(F \text{ or } S) = \frac{27}{30} = \frac{9}{10}$

(iv) $P(\text{one country only}) = \frac{15}{30} = \frac{1}{2}$

Your turn!

3.3 & 3.4

In a certain street $\frac{1}{5}$ of the households have no newspaper delivered, $\frac{1}{2}$ have a national paper delivered and $\frac{1}{3}$ have a local paper delivered. Draw a Venn diagram to represent this.

If a household is selected at random find the probability that it will have both papers delivered.

Solution

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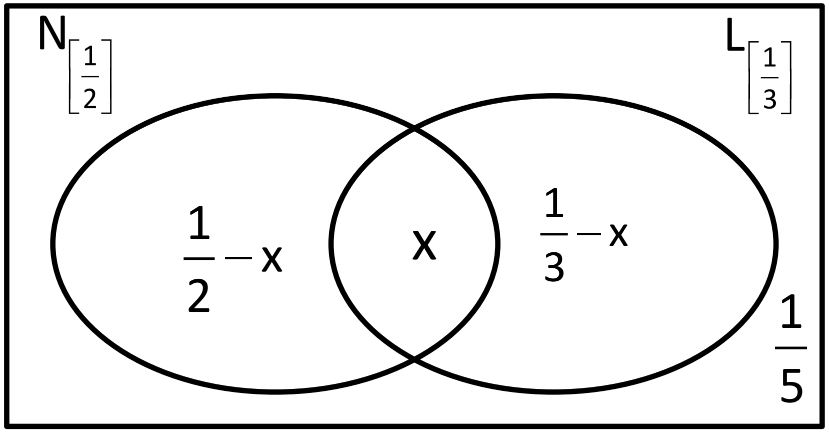
3.3

In a certain street $\frac{1}{5}$ of the households have no newspaper delivered, $\frac{1}{2}$ have a national paper delivered and $\frac{1}{3}$ have a local paper delivered. Draw a Venn diagram to represent this.

If a household is selected at random find the probability that it will have both papers delivered.

Solution



$$\left(\frac{1}{2}-x\right)+x+\left(\frac{1}{3}-x\right)=\frac{4}{5}$$

$$\frac{5}{6}-x=\frac{4}{5}$$

$$x=\frac{5}{6}-\frac{4}{5}$$

$$x=\frac{1}{30}$$

$$P(\text{both})=\frac{1}{30}$$

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3.4

In a class of 30 pupils, 12 got an A grade in Maths, 8 got an A grade in Biology and 8 got an A grade in French. 3 got A grades in Maths and Biology. 3 got A grades in Maths and French. 4 got A grades in Biology and French. 2 students got A grades in all three of the above subjects.

(i) Draw a Venn diagram to represent the above data.

(ii) One student is chosen at random. Find the probability that they have an A grade in

- **(a)** at least one of the three subjects
- **(b)** only one of the three subjects
- **(c)** French but not Biology.

(a)

(b)

(c)

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3.4

In a class of 30 pupils, 12 got an A grade in Maths, 8 got an A grade in Biology and 8 got an A grade in French. 3 got A grades in Maths and Biology. 3 got A grades in Maths and French. 4 got A grades in Biology and French. 2 students got A grades in all three of the above subjects.

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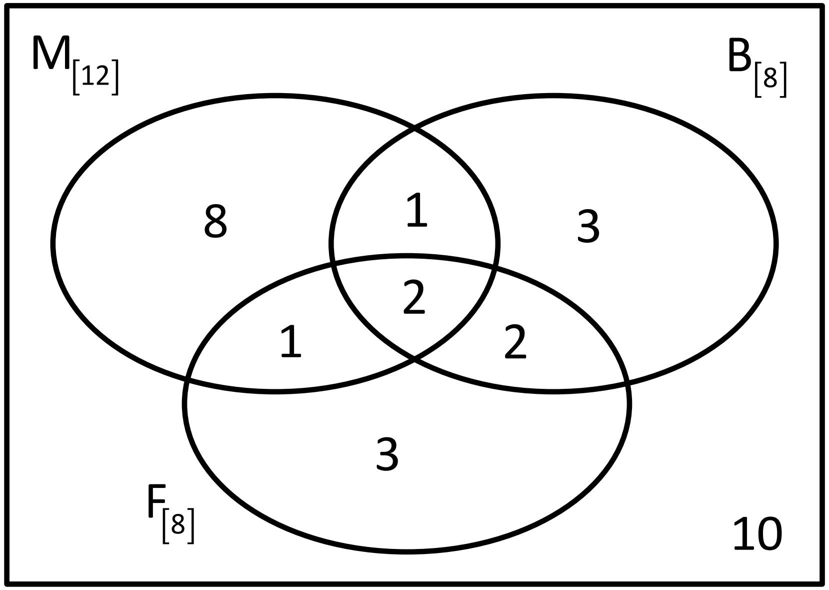
- **(b)** only one of the three subjects
- **(c)** French but not Biology.



(a) $P(\text{at least one of the three subjects}) = \frac{20}{30} = \frac{2}{3}$

(b) $P(\text{only one of the three subjects}) = \frac{2}{30} = \frac{1}{15}$

(c) $P(\text{French but not boilogy}) = \frac{4}{30} = \frac{2}{15}$

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Permutations (Arrangements)

Example 1

In how many ways can we arrange the letters a, b, c, d, e, taking the letters two at a time?

Solution

Method 1 :

Method 2:

Method 3:

Permutations (Arrangements)

Example 1

In how many ways can we arrange the letters a, b, c, d, e, taking the letters two at a time?

Solution

Method 1: Write out the arrangements

(a,b)	(a,c)	(a,d)	(a,e)	(b,c)	(b,d)	(b,e)	(c,d)	(c,e)	(d,e)
(b,a)	(c,a)	(d,a)	(e,a)	(c,b)	(d,b)	(e,b)	(d,c)	(e,c)	(e,d)

Total = 20 ways

Method 2: Box method

$$\boxed{} \times \boxed{}$$

$$5 \times 4 = 20 \text{ ways}$$

Method 3: Use the P notation

$${}^{5}P_{2} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5.4.3.2.1}{3.2.1} = 5.4 = 20 \text{ ways}$$

Example 2

(i) In how many ways can the letters of the word IRELAND be arranged if each letter is used exactly once in each arrangement?

(ii) In how many of these arrangements do the vowels come together?

(iii) In how many arrangements do the vowels not come together?

Solution

(i)

(ii)

(iii)

Example 2

(i) In how many ways can the letters of the word IRELAND be arranged if each letter is used exactly once in each arrangement?

(ii) In how many of these arrangements do the vowels come together?

(iii) In how many arrangements do the vowels not come together?

Solution

(i) $\square \times \square \times \square \times \square \times \square \times \square \times \square$

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7!$ ways $= 5040$ ways

(ii) Treat the three vowels as one unit, but don't forget to move the vowels within the unit.

$\square \times \square \times \square \times \boxed{\mathrm{I} \mid \mathrm{E} \mid \mathrm{A}} \times \square$

$5 \times 4 \times 3 \times 2 \times 1 \times 3! = 5! \times 3! = 120 \times 6 = 720$ ways

(iii) No. of ways in which the three vowels do not come together. This is a negative statement.

This is equal to:

Total $-$ number of ways they come together.

$5040 - 720 = 4320$ ways

Combinations (Selections)

Example 1

In how many ways can we select the letters a, b, c, d, e, taking the letters two at a time?

Solution

Method 1:

Method 2:

Combinations (Selections)

Example 1

In how many ways can we select the letters a, b, c, d, e, taking the letters two at a time?

Solution

Method 1: Write out the selections

(a,b) (a,c) (a,d) (a,e) (b,c) (b,d) (b,e) (c,d) (c,e) (d,e)

Note: (a,b) is the same selection as (b,a)

Total = 10 ways

Method 2: Use the C notation

Formal definition: $\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

$$\binom{5}{2} = {}^{5}C_{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\times 3!} = \frac{5.4.3.2.1}{2.1\times 3.2.1} = \frac{5.4}{2.1} = \frac{20}{2} = 10 \text{ ways}$$

Example 2

(i) How many groups of five people can be selected from ten people?

(ii) How many groups can be selected if two particular people from the ten cannot be in the same group?

Solution

(i) $^{10}C_5 = \dfrac{10.9.8.7.6}{1.2.3.4.5} = 252$ groups

(ii) This is a negative statement.

Total $-$ number of ways the two people can be together.

Total $= 252$ groups

Number of ways the two people can be together $= {}^{8}C_3 = \dfrac{8.7.6}{3.2.1} = 56$

$252 - 56 = 196$ groups

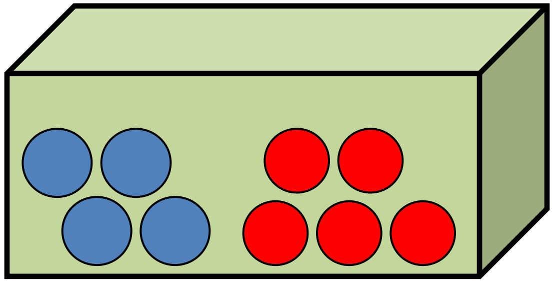
Two already picked need 3 more from the 8 to join them to form the committee

Probability Using the Counting Method

Example 1

A box contains four blue and five red discs. A disc is drawn at random from the box.

Find the probability that the disc is red.



Solution

Probability Using the Counting Method

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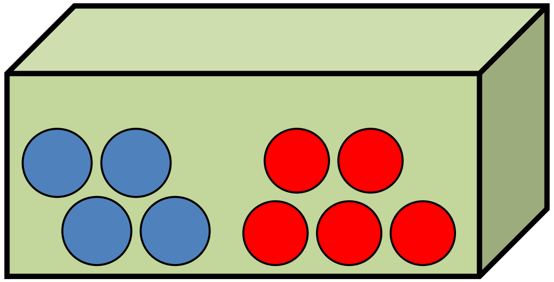


Solution

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

No. of favourable outcomes:

From 5 red choose $1 = {}^{5}C_{1}$

Total no. of outcomes:

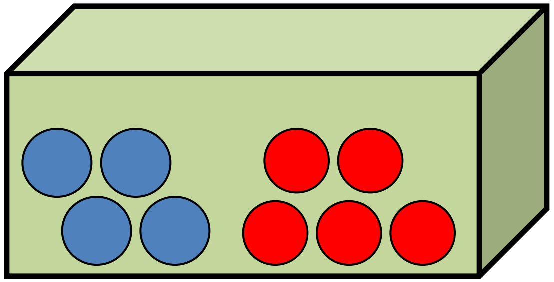
From 9 discs choose $1 = {}^{9}C_{1}$

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^{5}C_{1}}{{}^{9}C_{1}} = \frac{5}{9}$$

Example 2

A box contains four blue and five red discs. Two discs are drawn at random from the box.

Find the probability that both discs are blue.

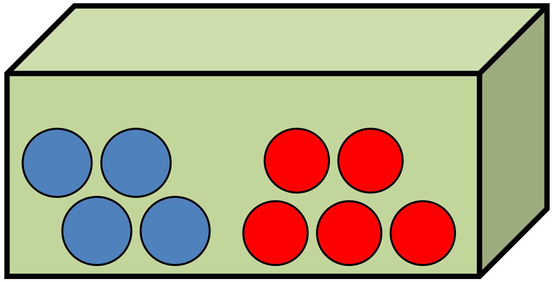


Solution

Example 2

A box contains four blue and five red discs. Two discs are drawn at random from the box.

Find the probability that both discs are blue.



Solution

No. of favourable outcomes:

From 4 blue choose 2 $= {}^{4}C_{2}$

Total no. of outcomes:

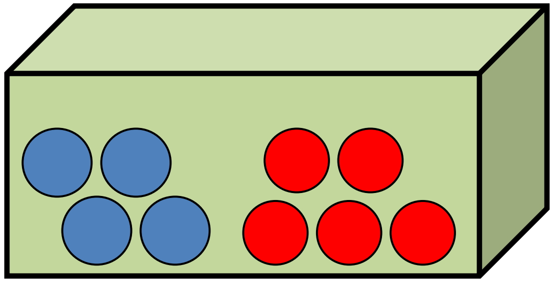
From 9 discs choose 2 $= {}^{9}C_{2}$

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^{4}C_{2}}{{}^{9}C_{2}} = \frac{6}{36} = \frac{1}{6}$$

Example 3

A box contains four blue and five red discs. Two discs are drawn at random from the box.

Find the probability that exactly one red disc is drawn.

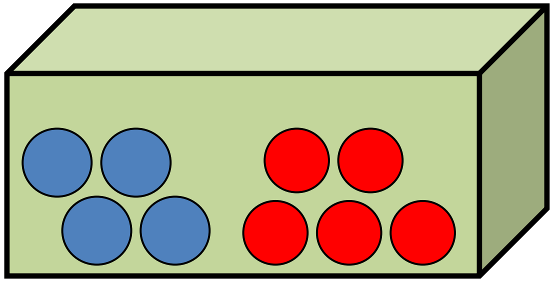


Solution

Example 3

A box contains four blue and five red discs. Two discs are drawn at random from the box.

Find the probability that exactly one red disc is drawn.



Solution

Note : This is the same as asking the probability of drawing one red disk and one blue disk.

No. of favourable outcomes:

From 5 red choose 1 and from 4 blue choose 1 $= {}^5C_1 \times {}^4C_1$

Total no. of outcomes:

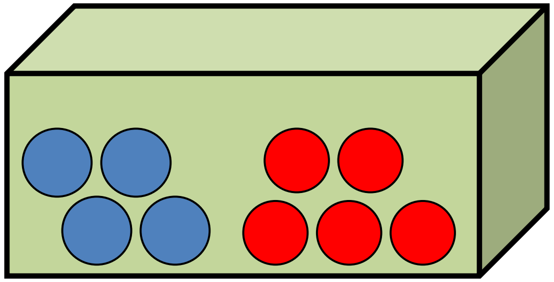
From 9 discs choose 2 $= {}^9C_2$

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^5C_1 \times {}^4C_1}{{}^9C_2} = \frac{20}{36} = \frac{5}{9}$$

Example 4

A box contains four blue and five red discs. Four discs are drawn at random from the box.

Find the probability that exactly three of the discs are red.

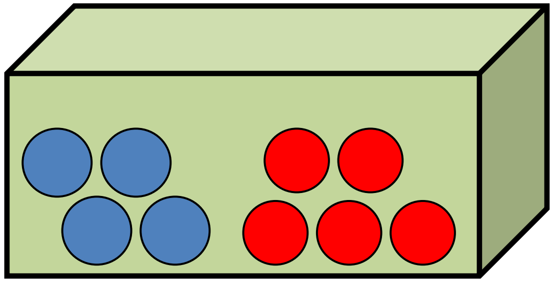


Solution

Example 4

A box contains four blue and five red discs. Four discs are drawn at random from the box.

Find the probability that exactly three of the discs are red.



Solution

No. of favourable outcomes:

From 5 red choose 3 and from 4 blue choose 1 $= {}^5C_3 \times {}^4C_1$

Total no. of outcomes:

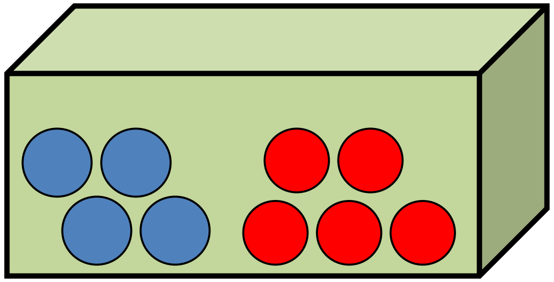
From 9 discs choose 4 $= {}^9C_4$

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^5C_3 \times {}^4C_1}{{}^9C_4} = \frac{40}{126} = \frac{20}{63}$$

Example 5

A box contains four blue and five red discs. Three discs are drawn at random from the box.

Find the probability that at least one red disc is drawn.

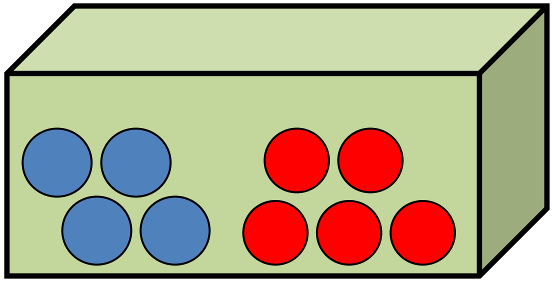


Solution

Example 5

A box contains four blue and five red discs. Three discs are drawn at random from the box.

Find the probability that at least one red disc is drawn.



Solution

Note: This is the negative of drawing three blue disks

$$\text{No. of Favourble outcomes} = \text{Total outcomes} - \text{unfavourable outcomes}$$

No. of unfavourable outcomes:

From 4 blue choose 3 $= {}^4C_3$

$$\text{No. of favourble outcomes} = {}^9C_3 - {}^4C_3 = 84 - 4 = 80$$

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{80}{{}^9C_3} = \frac{80}{84} = \frac{20}{21}$$

Replacement

Example 1

A card is picked from a pack of 52 and then replaced. Another card is then picked from the pack. What is the probability that the two cards picked are clubs?

Solution

Example 2

Two cards are picked from a pack of 52. Find the probability that both cards are clubs.

Solution

Replacement

Example 1

A card is picked from a pack of 52 and then replaced. Another card is then picked from the pack. What is the probability that the two cards picked are clubs?

Solution

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{^{13}C_1}{^{52}C_1} \times \frac{^{13}C_1}{^{52}C_1} = \frac{1}{16}$$

Example 2

Two cards are picked from a pack of 52. Find the probability that both cards are clubs.

Solution

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{^{13}C_2}{^{52}C_2} = \frac{12}{204} = \frac{1}{17}$$

Order

NOTE: By using the counting method you do not have to take order into account.

Example

Three cards are drawn from a pack of 52. Find the probability that the three cards drawn are the Jack of Hearts, the Queen of Diamonds and the King of Clubs.

Solution

Counting Method

Non Counting Method

Order

NOTE: By using the counting method you do not have to take order into account.

Example

Three cards are drawn from a pack of 52. Find the probability that the three cards drawn are the Jack of Hearts, the Queen of Diamonds and the King of Clubs.

Solution

Counting Method

Favourable outcomes:

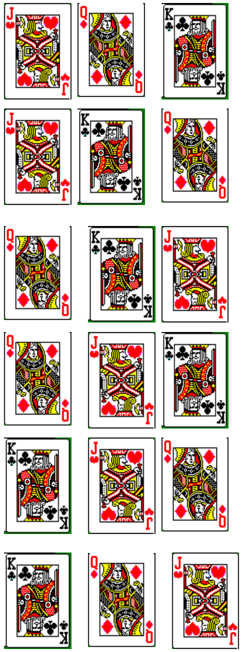
$3 \text{ cards choose } 3 = {}^{3}C_{3}$

Total outcomes:

$52 \text{ cards choose } 3 = {}^{52}C_{3}$

$$\text{Probability} = \frac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}} = \frac{{}^{3}C_{3}}{{}^{52}C_{3}} = \frac{1}{22100}$$

Non Counting Method



$$\frac{1}{52} \times \frac{1}{51} \times \frac{1}{50} \times 3! = \frac{1}{22100}$$

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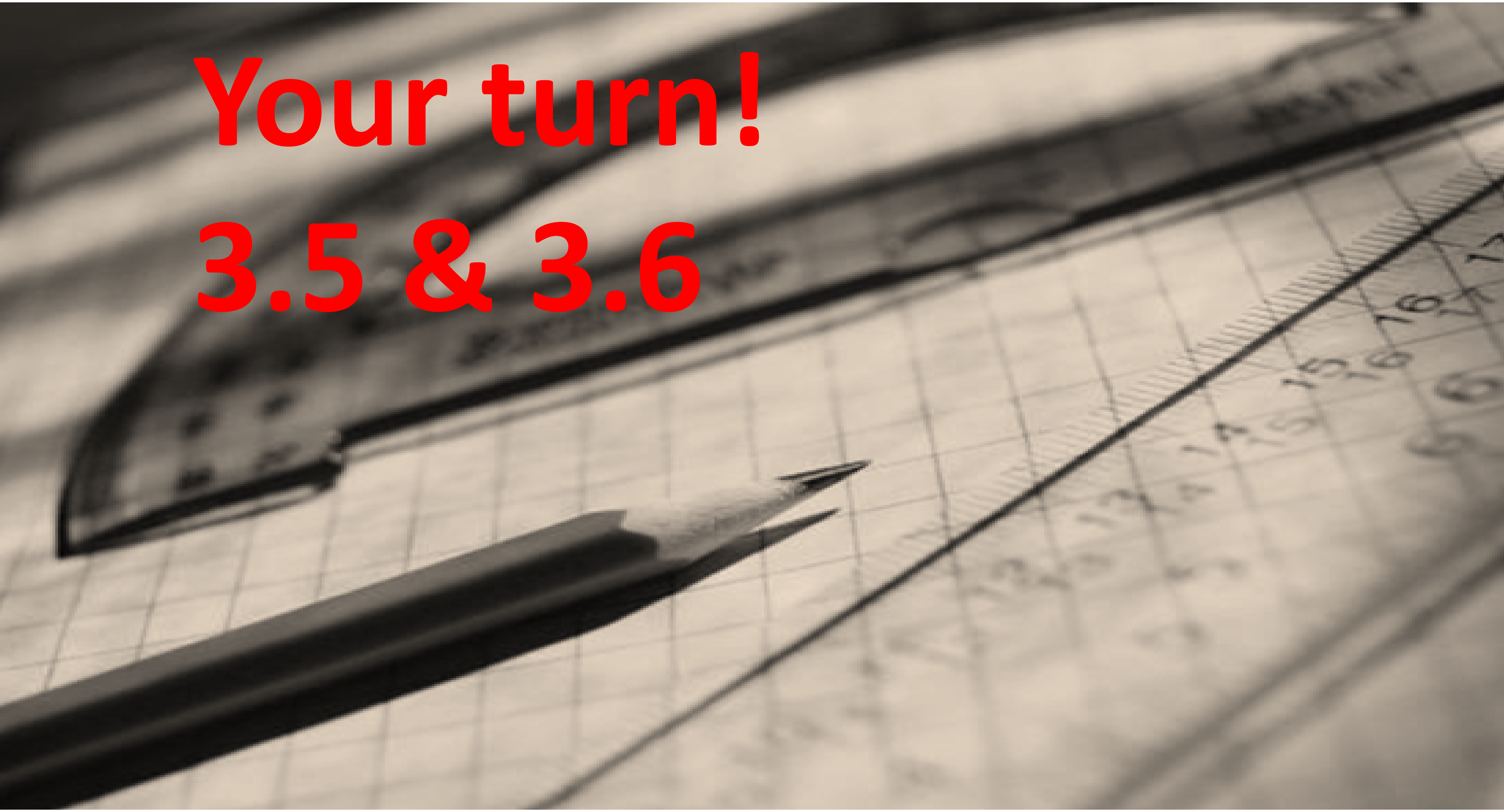
Your turn!
3.5 & 3.6

Counting Method

What is the probability of choosing 2 red cards and 1 black card from a pack of 52 cards?

Use the Counting Method and the Non Counting Method.

Use a tree diagram to show the sample space.

Solution

Counting Method :

Non Counting Method :

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3.5

Counting Method

What is the probability of choosing 2 red cards and 1 black card from a pack of 52 cards?

Use the Counting Method and the Non Counting Method.

Use a tree diagram to show the sample space.

Solution

Counting Method:

$$P(\text{2 red and 1 black}) = \frac{{}^{26}C_2 \times {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$$

Non Counting Method:

$$P(\text{2 red and 1 black}) = P(R).P(R).P(B) + P(R).P(B).P(R) + P(B).P(R).P(R)$$

$$P(\text{2 red and 1 black}) = \frac{26}{52}\cdot\frac{25}{51}\cdot\frac{26}{50} + \frac{26}{52}\cdot\frac{26}{51}\cdot\frac{25}{50} + \frac{26}{52}\cdot\frac{26}{51}\cdot\frac{25}{50} = \frac{13}{34}$$

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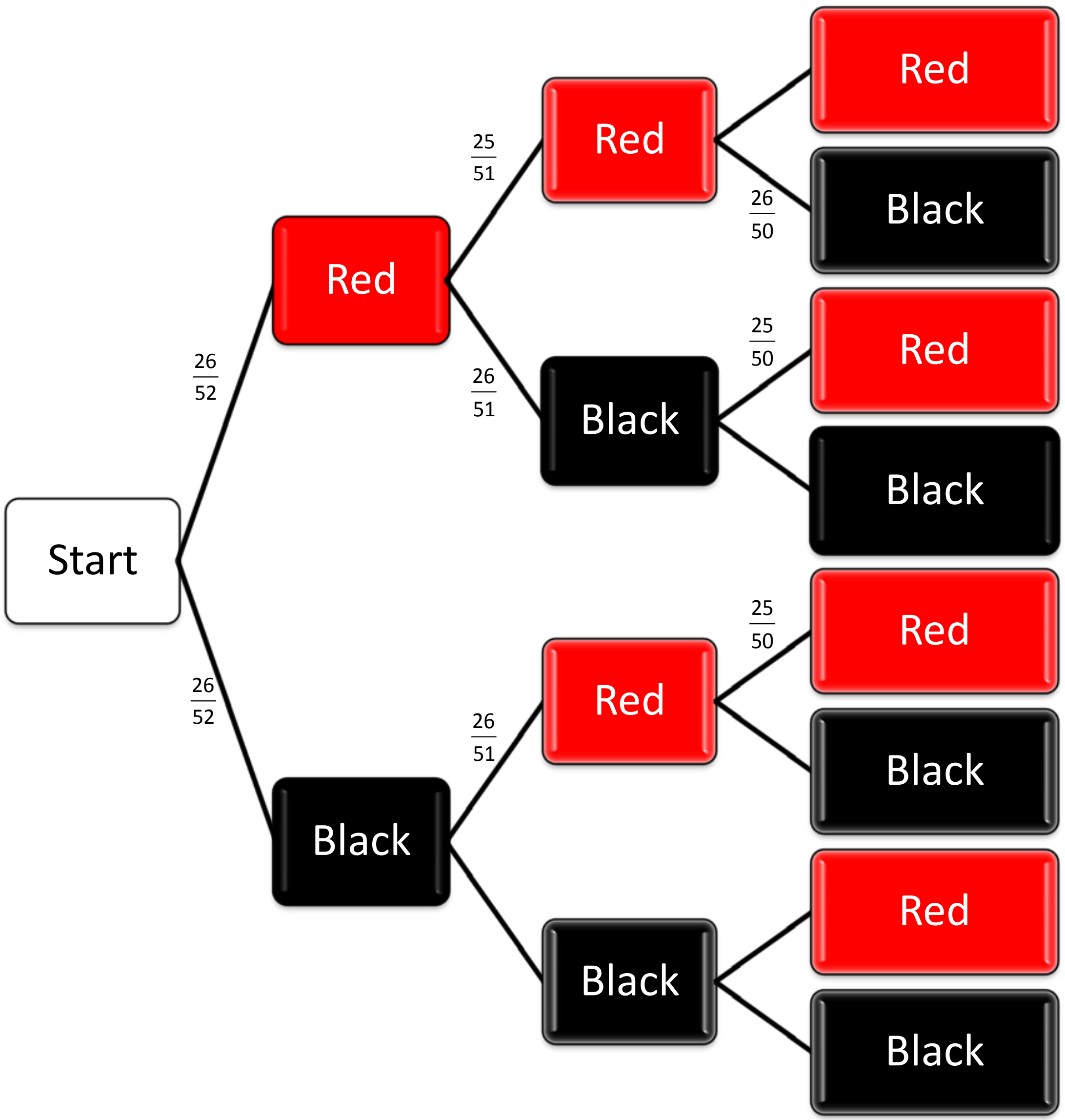
3.5 ctd.



Start
26/52
26/52
Red
Black
25/51
26/51
26/51
Red
Black
Red
Black
26/50
25/50
25/50
Red
Black
Red
Black
Red
Black
Red
Black

Counting Method

Probability of getting 4 kings and 1 Queen in a hand of 5 cards.

Use the Counting Method and the Non Counting Method.

Solution

Counting Method :

$$P(4\ \text{Kings and 1 Queen}) = \frac{{}^4C_4 \times {}^4C_1}{{}^{52}C_5} = \frac{1}{659740}$$

Non Counting Method :

$$P(4\ \text{Kings and 1 Queen}) = P(K).P(K).P(K).P(K).P(Q) + P(K).P(K).P(K).P(Q).P(K) + P(K).P(K).P(Q).P(K).P(K)$$
$$+P(K).P(Q).P(K).P(K).P(K) + P(Q).P(K).P(K).P(K).P(K)$$

$$P(4\ \text{Kings and 1 Queen}) = \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}\cdot\frac{1}{49}\cdot\frac{4}{48} + \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}\cdot\frac{4}{49}\cdot\frac{1}{48} + \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{4}{50}\cdot\frac{2}{49}\cdot\frac{1}{48}$$
$$+\frac{4}{52}\cdot\frac{4}{51}\cdot\frac{3}{50}\cdot\frac{2}{49}\cdot\frac{1}{48} + \frac{4}{52}\cdot\frac{4}{51}\cdot\frac{3}{50}\cdot\frac{2}{49}\cdot\frac{1}{48} = \frac{1}{649740}$$

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Lotto

Example

A lottery consists of drawing 6 balls and a bonus ball from a total of 45 balls. The balls are numbered from 1 to 45. If a person buys just one ticket:

(i) What is the probability of winning the lotto?

Solution

(i)

Example

A lottery consists of drawing 6 balls and a bonus ball from a total of 45 balls. The balls are numbered from 1 to 45. If a person buys just one ticket:

(i) What is the probability of winning the lotto?

Solution

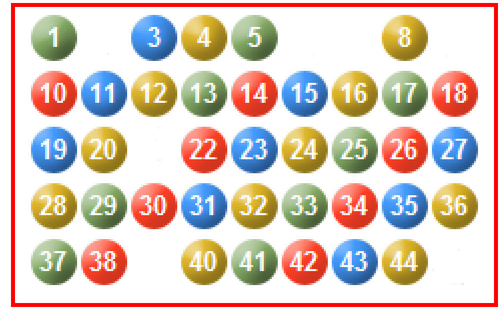
(i)

From 6 you want 6

Not interested in bonus ball

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^{6}C_{6}}{{}^{45}C_{6}} = \frac{1}{8145060}$$

(ii) What is the probability of getting 3 of the winning numbers plus the bonus ball?



(iii) What is the probability of getting 3 of the winning numbers?

(ii) What is the probability of getting 3 of the winning numbers plus the bonus ball?

From 6 you want 3

Want the bonus ball

From the remaining, non winning balls, you want 2

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^{6}C_3 \times {}^{1}C_1 \times {}^{38}C_2}{{}^{45}C_6} = \frac{14060}{8145060} = \frac{703}{407253}$$

(iii) What is the probability of getting 3 of the winning numbers?

From 6 you want 3

Not interested in bonus ball

From the remaining, non winning balls, you want 3

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^{6}C_3 \times {}^{1}C_0 \times {}^{38}C_3}{{}^{45}C_6} = \frac{168720}{8145060} = \frac{2812}{135751}$$

Your turn!

3.7

Lotto

What is the probability of getting 4 of the winning numbers plus the bonus ball in the National Lottery? Note there are 45 ball in the National Lotto.

Solution

From 6 you want 4

Want the bonus ball

From the remaining, non winning balls, you want 1

3.7

Lotto

What is the probability of getting 4 of the winning numbers plus the bonus ball in the National Lottery? Note there are 45 ball in the National Lotto.

Solution

From 6 you want 4

Want the bonus ball

From the remaining, non winning balls, you want 1

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{{}^{6}C_{4} \times {}^{1}C_{1} \times {}^{38}C_{1}}{{}^{45}C_{6}} = \frac{19}{271502}$$

Notes

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