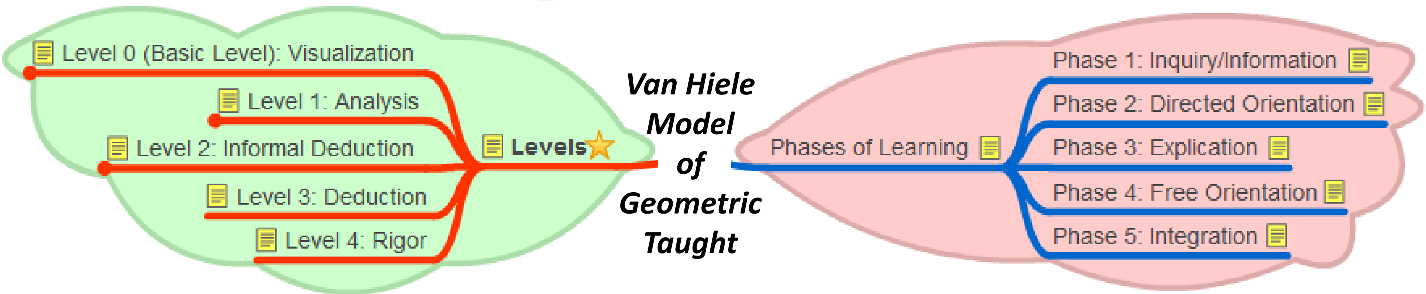
Project Maths
Tionscadal Mata
Development Team

Module 1

Theorems: A Discovery Approach

"Theorems are full of potential for surprise and delight. Every theorem can be taught by considering the unexpected matter which theorems claim to be true. Rather than simply telling students what the theorem claims, it would be helpful if we assumed we didn't know it… it is the mathematics teacher's responsibility to recover the surprise embedded in the theorem and convey it to the pupils. The method is simple: just imagine you do not know the fact. This is where the teacher meets the students".



Definition 25

Given two lines AB and CD, and a transversal AE of them, as in fig (a), the angles ∠EAB and ∠ACD are called corresponding angles, with respect to the two lines and the given transversal.

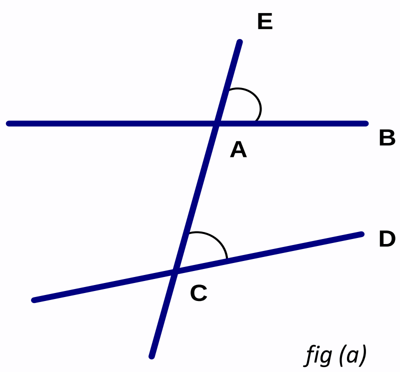


fig (a)

Theorem 5 *(Corresponding Angles)*

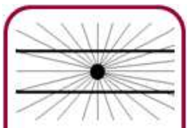
Two lines are parallel if and only if for any transversal, corresponding angles are equal.

Approaches:

1. Drawing

Students discover the theorem using ruler and protractor.

Activity Sheet



Interactive

2. Geostrips

Let's examine the use of Geostrips for theorems in the classroom

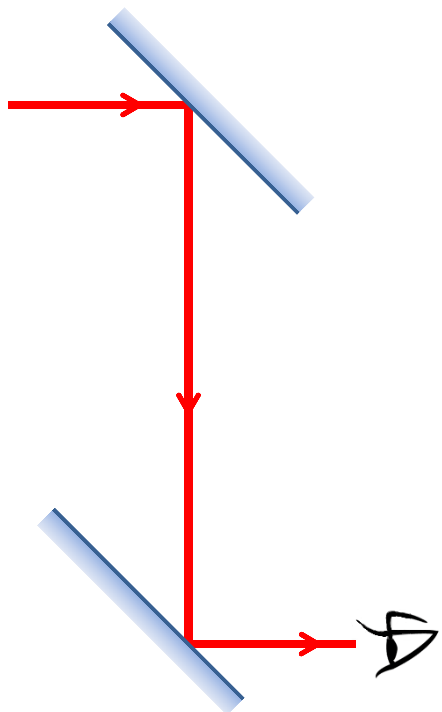
3. Student CD

How can the Student CD and Student Activity Sheet help?

Interactive *Student Activity*

Application of Theorem 5

Periscope

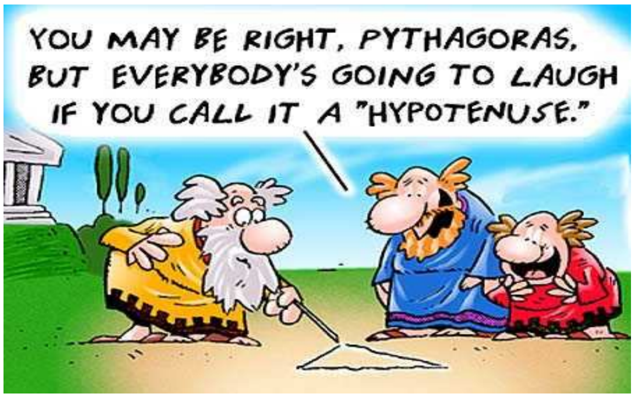


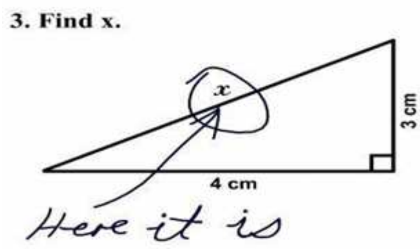
Interactive

Theorem 14 (Pythagoras)

In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.

Monument to Pythagoras in his birthplace, Samos.





Theorem 14 (Pythagoras)

In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.

Approaches:

1. **Drawing**
   Construction on squared paper. Activity Sheet

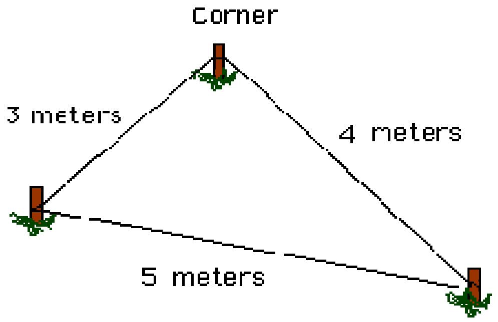
2. **Cut - out**
   Two activity sheets. Activity Sheet Activity Sheet

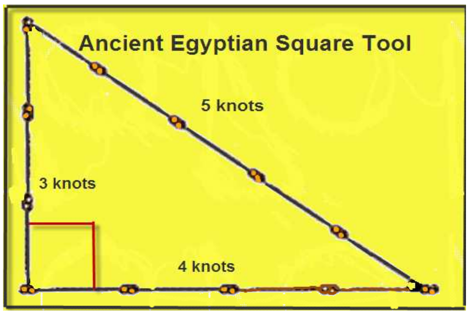
3. **Perigal's Dissection**
   One of many illustrations of Pythagoras Theorem. Interactive

4. **Student CD.**
   How can the Student CD and Student Activity Sheet help? Interactive Student Activity

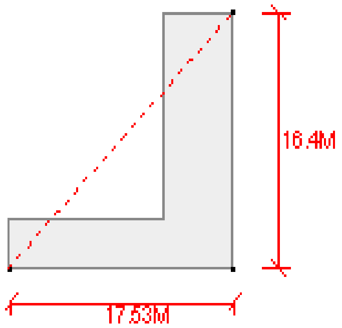
Applications of Theorem 5 and its Converse

Marking out a Site





Checking for Square

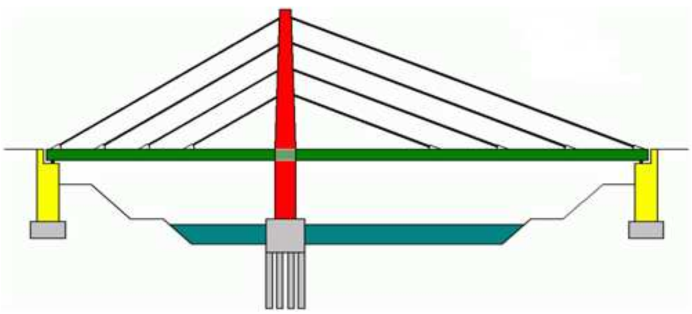


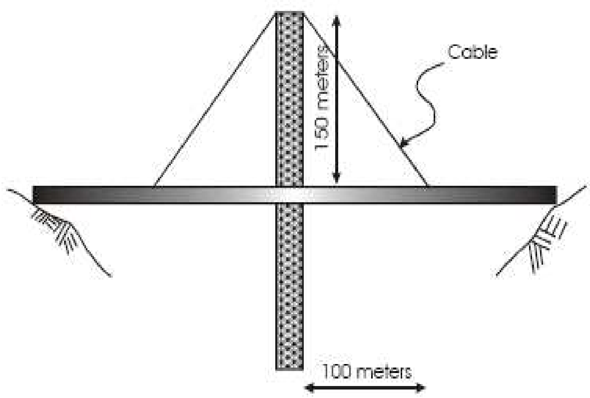
On the left we have a sketch of say an "L" shaped building.

When builders are setting it out, after putting pegs in the three main corners, they want to check for square before getting the rest of the profiles fixed.

They work out the length of the red dotted line to check for a right angle.

Cable Stay Bridges





Problem

Suppose an engineer has to design the cable supporting the bridge deck.

If the concrete tower is 150m above the road deck and the support point for the main cable is 100 m away from the tower, how long does the cable have to be?

Solution

$c^2 = a^2 + b^2$

$a = 100 \text{ m}$

$b = 150 \text{ m}$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{100^2 + 150^2}$

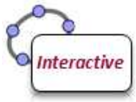
$c = \sqrt{10000 + 22500}$

$c = \sqrt{32500}$

$c = 180.28\text{m}$

Practical Problem 1

Given that the second story window is 4.5 m above the ground and the only sure footing for the ladder is at least 2m from the house, can a window cleaner who has a ladder of length 5m reach the window to clean it?

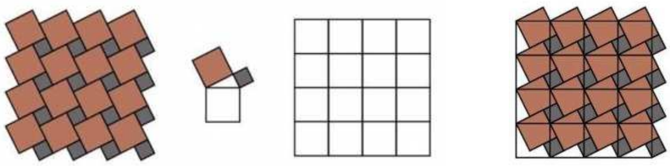


Practical Problem 2

The dimensions of your TV cabinet are 18" by 24".
You want to buy a TV with a 27" screen in the 16 : 9 format.
Check to see if it will fit in your cabinet before you go and purchase it?

A Greek in an Italian Restaurant

Can you see Pythagoras in the floor of this Italian restaurant?



http://www.qedcat.com/archive/32.html

Some More Applications

Triangulation helps to locate a cell phone making an emergency call. It might use the Sine Rule to find the phone. The law of cosines can also reduce to the Pythagorean theorem.

Pythagoras' theorem is used in ***fractal geometry*** e.g. Movie and Video game environments are drawn in 3-D using triangles.

Towards Geometric Proofs

Consider a geometry wall: the foundations of the wall consist of the early axioms and definitions.
Each conclusion builds upon previous knowledge.
The cement holding the wall together is the deductive logic that is used to prove the next theorem.

Definition 26: Exterior Angle					
Definition 25: Corresponding	Theorem 5: Corresponding				
Theorem 2: Isosceles triangles	Theorem 3: Alternate Angles	Theorem 4: Angle sum 180°			
Definition 21 Isosceles	Definition 22 - 23	Definition 24: Alternate	Axiom of parallels Axiom 5		
Definitions 4 - 17	Definition 18: Complementary Angles	Theorem 1: Vertically opposite Angles	Congruency Axiom 4	Definition 19 - 20	
Two Points Axiom. Axiom 1	Ruler axiom 2. Axiom 2	Protractor axiom 3 Axiom 3	Definition 1: Line segment	Definition 2: Collinear	Definition 3: Triangle

Theorem 6

The exterior angle of a triangle is equal to the sum of the interior remote angles.

Approaches:

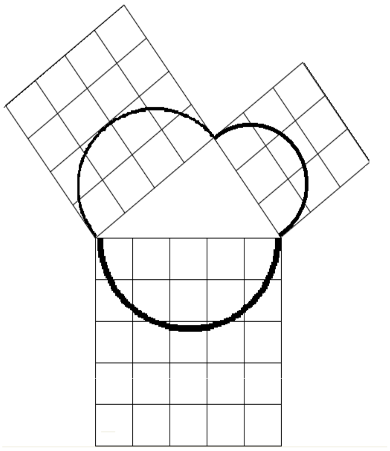
1. Proof

Interactive Proof. Interactive

2. Student CD.

How can the Student CD and Student Activity Sheet help? Interactive Student Activity

Other Shapes



Interactive

Pythagorean Triples

Take any two whole numbers and form:

1. The difference between their squares.
2. Twice their product.
3. The sum of their squares.

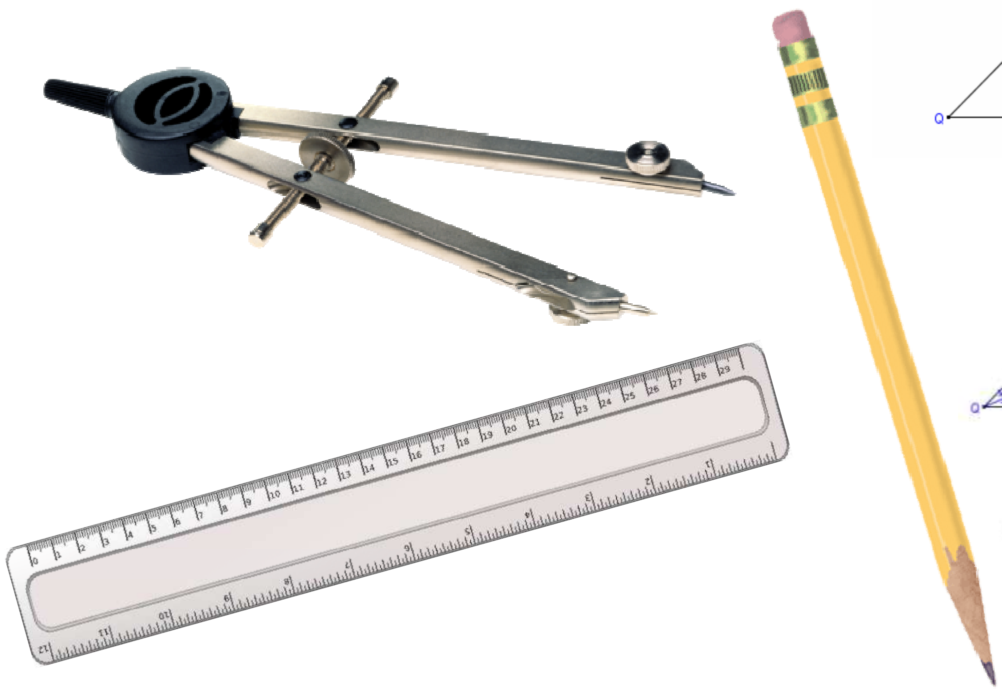
Proof :

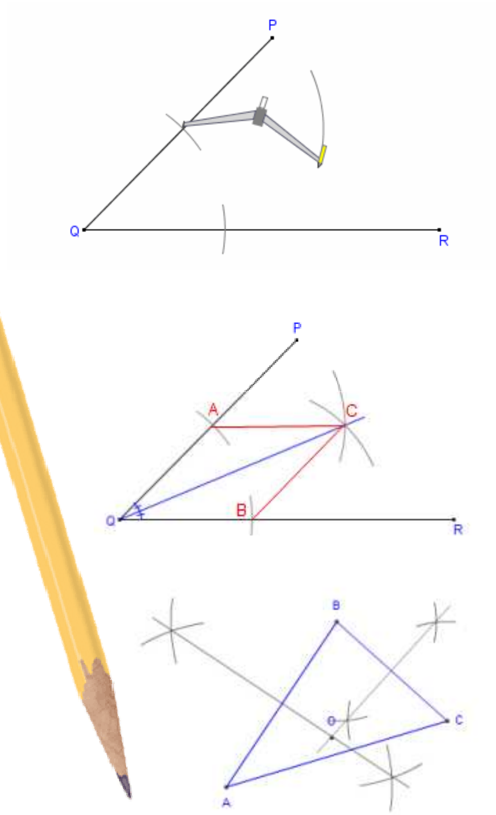
Let $a = n^2 - m^2$, $b = 2nm$ and $c = n^2 + m^2$, where $n > m$

$$\begin{aligned}(n^2 - m^2)^2 + (2mn)^2 &= n^4 - 2n^2m^2 + m^4 + 4n^2m^2 \\ &= n^4 + 2n^2m^2 + m^4 \\ &= (n^2 + m^2)^2\end{aligned}$$

Constructions

Ordinary Level: ***13 Constructions.***
Higher Level: ***15 Constructions***





Ready Reckoner

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	Junior Certificate			Leaving Certificate		
		CIC	OL	HL	FL	OL	HL
1	Bisector of an angle, using only compass and straight edge.	●	●	●	●	●	●
2	Perpendicular bisector of a segment, using only compass and straight edge.	●	●	●	●	●	●
3	Line perpendicular to a given line l, passing through a given point not on l.		●	●			●
4	Line perpendicular to a given line l, passing through a given point on l.	●	●	●	●	●	●
5	Line parallel to given line, through a given point.	●	●	●	●	●	●
6	Division of a line segment into 2 or 3 equal segments without measuring it.	●	●	●	●	●	●
7	Division of a line segment into any number of equal segments, without measuring it.			●			●
8	Line segment of a given length on a given ray.	●	●	●	●	●	●
9	Angle of a given number of degrees with a given ray as one arm.		●	●	●	●	●
10	Triangle, given lengths of 3 sides.		●	●	●	●	●
11	Triangle, given SAS data.		●	●	●	●	●

Construction 5

Line parallel to given line, through a given point.

Approaches:

1. Instructions

Use any method, animated or otherwise, to show the steps of the construction.

Interactive

2. Students using their mathematical instruments

Have instructions on board as they are working out their construction.

3. Student CD.

How can the Students CD and Student Activity Sheet help?

Interactive

Student Activity

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Axioms and Theorems (Supported by 46 definitions, 20 propositions) * proof required for JCHL and LCHL ** proof required for LCHL only		Junior Certificate			Leaving Certificate		
		CIC	OL	HL	FL	OL	HL
	Axiom 1: There is exactly one line through any two given points	●	●	●	●	●	●
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	●	●	●	●	●	●
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	●	●	●	●	●	●
1	Vertically opposite angles are equal in measure.	●	●	●	●	●	●
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	●	●	●	●	●	●
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	●	●	●	●	●	●
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	●	●	●	●	●	●
3	If a transversal makes equal alternate angles on two lines then the lines are parallel (and converse).	●	●	●	●	●	●
4*	The angles in any triangle add to 180°.	●	●	●	●	●	●
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	●	●	●	●	●	●
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	●	●	●	●	●	●
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					●	●
8	Two sides of a triangle are together greater than the third.					●	●
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal.		●	●	●	●	●
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			●			●
10	The diagonals of a parallelogram bisect each other.		●	●	●	●	●
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			●		●	●
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio.			●		●	●
13**	If two triangles are similar, then their sides are proportional, in order.		●	●	●	●	●
14*	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		●	●	●	●	●
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		●	●	●	●	●
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		●	●	●	●	●
16	For a triangle, base x height does not depend on the choice of base.					●	●

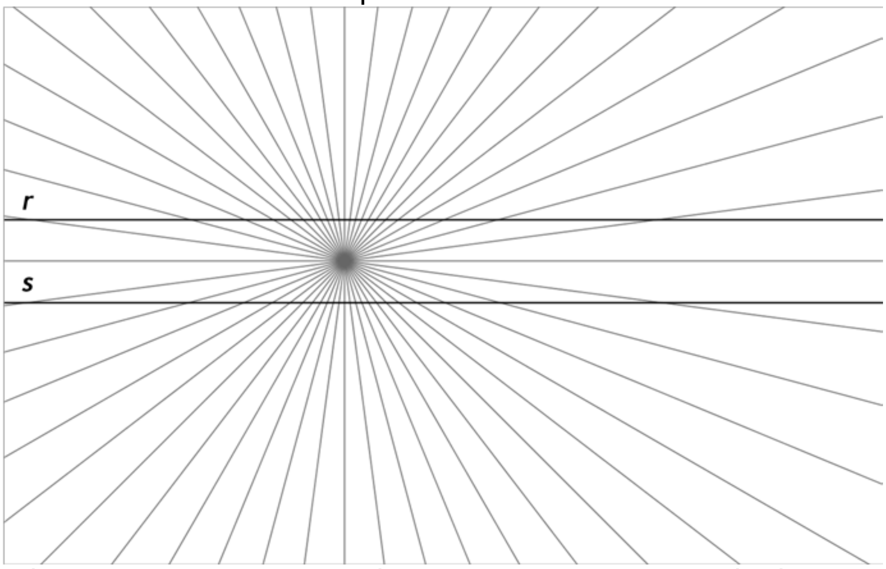
	Definition 38: The area of a triangle is half the base by the height.					●	●
17	A diagonal of a parallelogram bisects the area.					●	●
18	The area of a parallelogram is the base x height.					●	●
19*	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			●			●
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal			●			●
	Corollary 3: Each angle in a semi-circle is a right angle.		●	●	●	●	●
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		●	●	●	●	●
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180°.			●			●
20	**(i)** Each tangent is perpendicular to the radius that goes to the point of contact. **(ii)** If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					●	●
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					●	●
21	**(i)** The perpendicular from the centre to a chord bisects the chord. **(ii)** The perpendicular bisector of a chord passes through the centre.					●	●

† The corollaries are numbered as in the appendix; corollary 2 is the first one relating to theorem 9

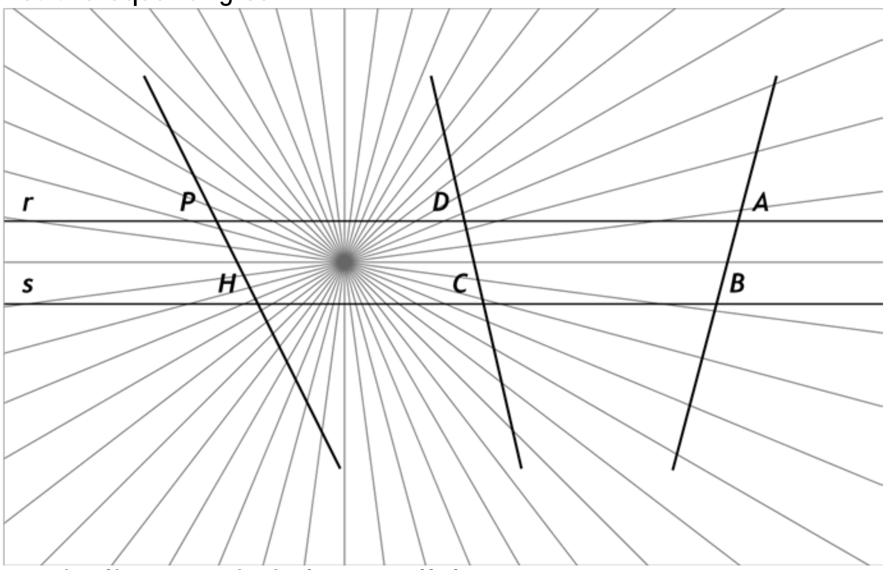
	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	**Junior Certificate**			**Leaving Certificate**		
		CIC	**OL**	**HL**	**FL**	**OL**	**HL**
1	Bisector of an angle, using only compass and straight edge.	●	●	●	●	●	●
2	Perpendicular bisector of a segment, using only compass and straight edge.	●	●	●	●	●	●
3	Line perpendicular to a given line l, passing through a given point not on l.		●	●			●
4	Line perpendicular to a given line l, passing through a given point on l.	●	●	●	●	●	●
5	Line parallel to given line, through a given point.	●	●	●	●	●	●
6	Division of a line segment into 2 or 3 equal segments without measuring it.	●	●	●	●	●	●
7	Division of a line segment into any number of equal segments, without measuring it.			●			●
8	Line segment of a given length on a given ray.	●	●	●	●	●	●
9	Angle of a given number of degrees with a given ray as one arm.		●	●	●	●	●
10	Triangle, given lengths of 3 sides.		●	●	●	●	●
11	Triangle, given SAS data.		●	●	●	●	●
12	Triangle, given ASA data		●	●	●	●	●
13	Right-angled triangle, given length of hypotenuse and one other side		●	●	●	●	●
14	Right-angled triangle, given one side and one of the acute angles.		●	●	●	●	●
15	Rectangle given side lengths.			●	●	●	●
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					●	●
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					●	●
18	Angle of 60° without using a protractor or set square.				●	●	●
19	Tangent to a given circle at a given point on it.				●	●	●
20	Parallelogram, given the length of the sides and the measure of the angles.				●	●	●
21	Centroid of a triangle.					●	●
22	Orthocentre of a triangle.						●

Theorem 5 Activity

1. Are the lines *r* and *s* below parallel?



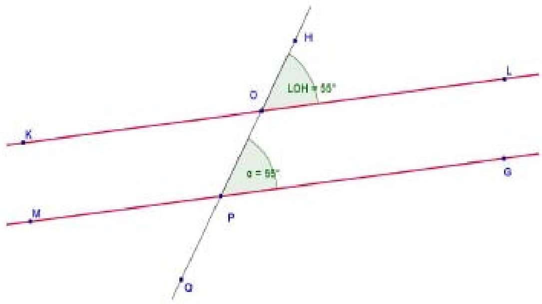
2. Using a protractor measure the angles, A, B, C, D, P and H below. List the equal angles.



3. Are the lines *r* and *s* below parallel?

Student Activity Theorem 5

Use in connection with interactive file Theorem 5 on the Student's CD.



1. What do you notice about the measure of the angles LOH and GPO?

____

Drag the point H to make the measure of the angle LOH = 30^0.
Write down the measure of the angle GPO. ________________

Are the measures of the two angles LOH and GPO equal in measure? ________________

2. Drag the point H to make the measure of the angle LOH = 100^o.
What is the measure of the angle GPO? ________________

Are the measures of the two angles LOH and GPO equal? ________________

3. The angles LOH and GOP are called CORRESPONDING angles. Drag the point H to various positions. Are these angles LOH and GOP always equal? ________________

4. Click on Tick Box to show the wording of this theorem.
Are the lines *a* and *b* parallel? ________________

5. Name another pair of corresponding angles in the diagram.

(i) ________________ **(ii)** ________________

Write down the measure of these angles **(i)** ________________ **(ii)** ________________

Are the measures of these angles equal? ________________

6. If you were told that the lines a and b are parallel what can we say about the measures of the following pairs of angles:

HOL and OPG ________________

QPG and POL ________________

QPM and POK ________________

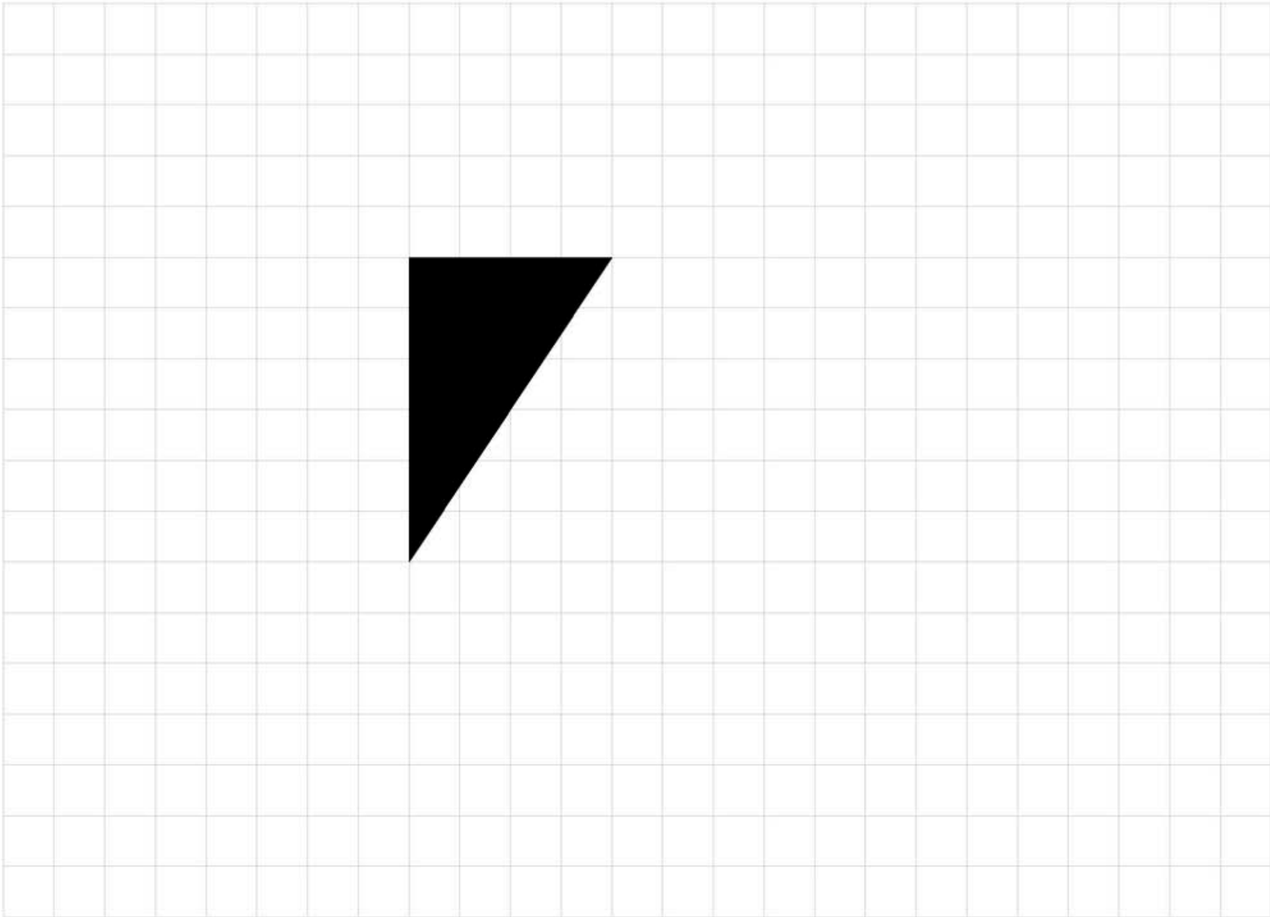
Drag the point H to make the angle OPG equal to 90^o and then write down the measures of the following angles.

(i) KOH ________________

(ii) MPO ________________

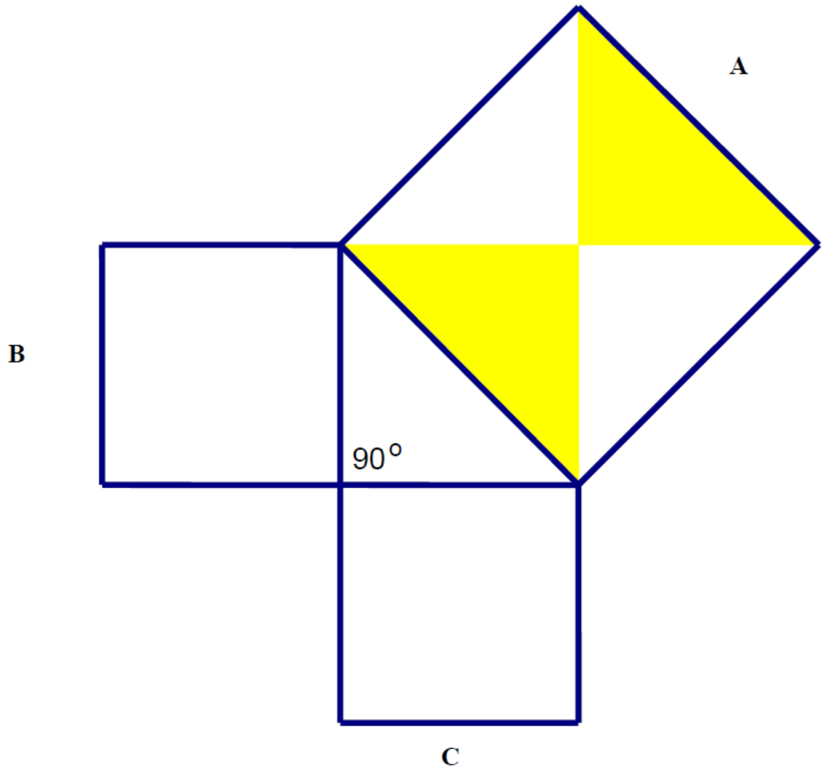
(iii) QPG ________________

Squared Paper Triangle



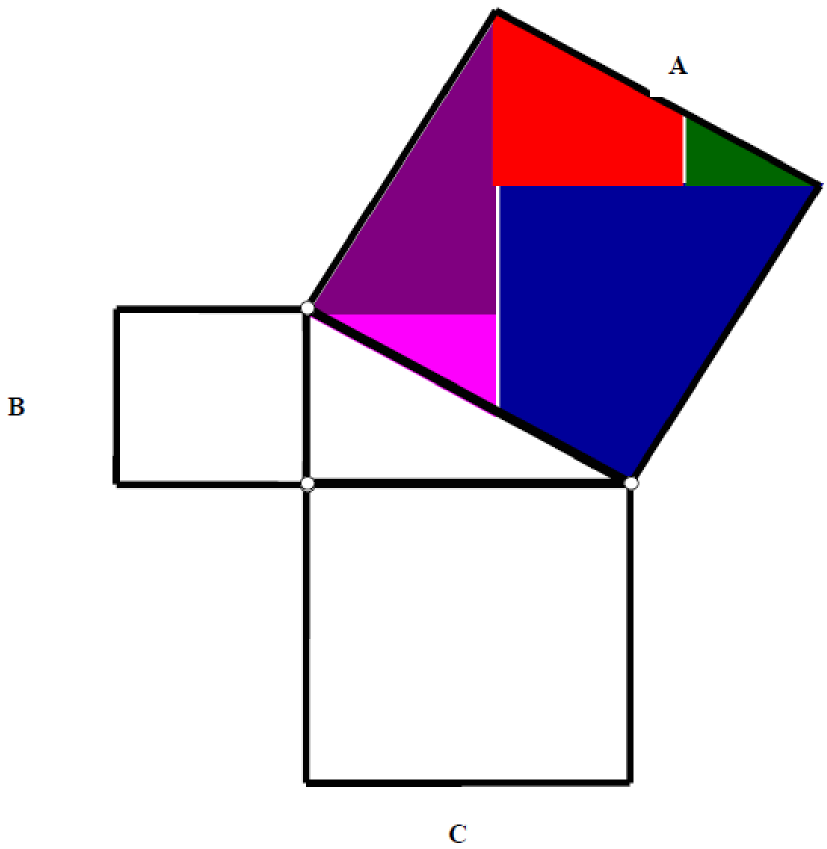
Pythagoras Theorem Cut out 1

Cut out the triangles in square A and fit them into the other squares.



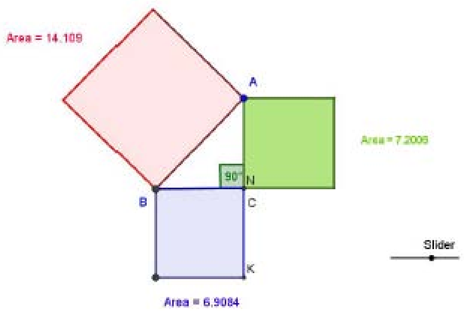
Pythagoras Theorem Cut out 2

Cut out the triangles in square A and fit them into the other squares.



Student Activity Theorem 14

Use in connection with interactive file Theorem 14 on the Student's CD.



1. There are three squares built on the sides of the right angled triangle in the diagram. Write down the areas of the three squares.

Red Square ______________________________

Blue Square ______________________________

Green Square ______________________________

Add the area of the Blue Square to the area of the Green Square

Area of Blue Square + Area of Green Square = ______________________

Does this total Area equal the Area of the red Square? ______________________

2. Drag the slider to the left.
Now write down the areas of the three squares.

Red Square ______________________

Blue Square ______________________

Green Square ______________________

Add the area of the Blue Square to the area of the Green Square

Area of Blue Square + Area of Green Square = ______________________

Does this total Area equal the Area of the red Square? ______________________

3. Drag the slider to the right.
Now write down the areas of the three squares.

Red Square ______________________

Blue Square ______________________

Green Square ______________________

Add the area of the Blue Square to the area of the Green Square

Area of Blue Square + Area of Green Square = ______________________

Does this total Area equal the Area of the red Square? ______________________

4. Write down in your own words what conclusion can be drawn from the answers to questions 1, 2 and 3.

____

5. Click on the Tick Box on the interactive file to reveal the wording of this theorem.
Did you come to this conclusion?

6. If the Area of the Red Square is a^2, the Area of the Blue Square is b^2 and the Area of the Green Square is c^2 can we conclude that:

$a^2 = b^2 + c^2$ ______________________

7. If the Area of the Red Square is a^2, the Area of the Blue Square is b^2 and the Area of the Green Square is c^2 can we conclude that:

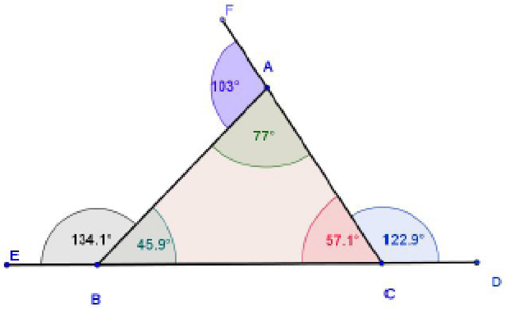
$b^2 = a^2 + c^2$ ______________________

8. If the Area of the Red Square is r^2, the Area of the Blue Square is b^2 and the Area of the Green Square is g^2 can we conclude that:

$r^2 = b^2 + g^2$ ______________________

Student Activity Theorem 6

Use in connection with interactive file Theorem 6 on the Student's CD.
Give all answers correct to the nearest degree.



1. Drag the point A to make the measure of the angle EBA = 130°.

What is the measure of the angle BAC? ____________________

What is the measure of the angle BCA? ____________________
What is the sum of the measures of the angles BAC and BCA?

Measure of the angle BAC + Measure of BCA = ____________________

Is this sum equal to the measure of the angle EBA? ____________________

2. Drag the point A to make the measure of the angle DCA = 100°.

What is the measure of the angle CBA? ____________________

What is the measure of the angle CAB? ____________________
What is the sum of the measures of the angles CBA and CAB?

Measure of the angle CBA + Measure of CAB = ____________________

Is this sum equal to the measure of the angle DCA? ____________________

3. Drag the point A to make the measure of the angle EBA = 110°.

What is the measure of the angle ABC? ____________________

What is the measure of the angle ACB? ____________________
What is the sum of the measures of the angles ABC and ACB?

Measure of the angle ABC + Measure of ACB = ____________________

Is this sum equal to the measure of the angle FAB? ____________________

4. Drag the point A to make the measure of the angle DCA = 84°.

What is the measure of the angle CBA? ____________________

What is the measure of the angle CAB? ____________________
What is the sum of the measures of the angles CBA and CAB?

Measure of the angle CBA + Measure of CAB = ____________________

Is this sum equal to the measure of the angle DCA? ____________________

5. What conclusion can you deduce from the measurements in Q 1, Q2, Q3, and Q4.

____

____

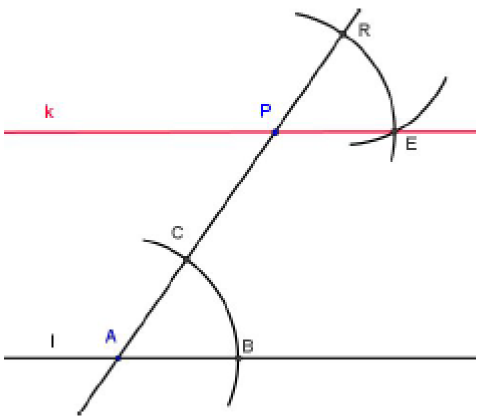
6. Click on the Tick Box on the interactive file to reveal the wording of this theorem. Did you come to this conclusion?

____

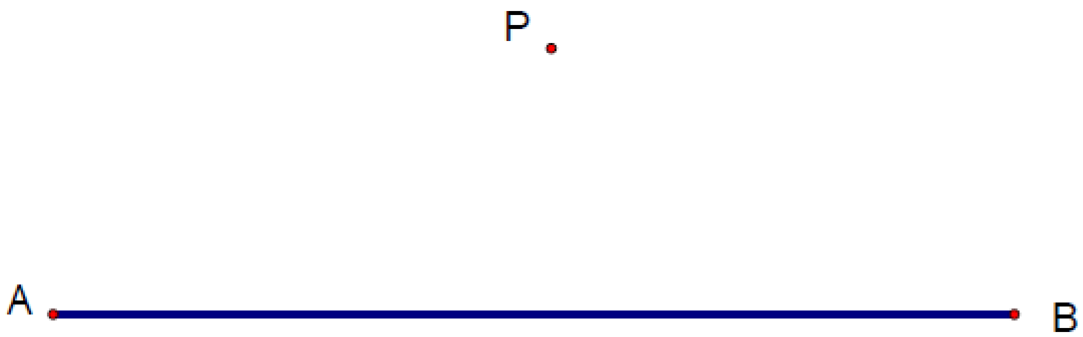
Student Activity Construction 5

Use in connection with interactive file Construction 6 on the Student's CD.

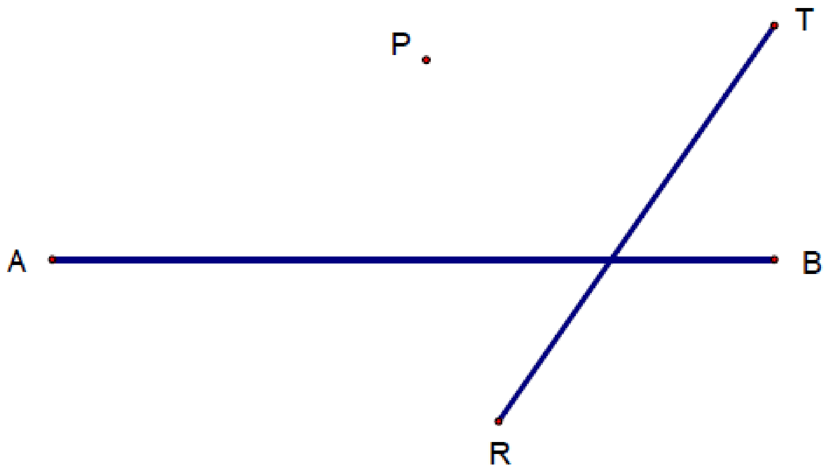
Construction 5: To construct a line parallel to a given line, through given point.



1. Construct a line through P parallel to [AB].



2. Construct a line through P parallel to [AB] and construct a line through P parallel to [RT]. Extend these lines to form a four sided figure.



What do you call the resulting four sided figure? ______________________________

Measure the lengths of the sides of the four sided figure.

What can you conclude? ______________________________

Measure the internal angles of the four sided figure

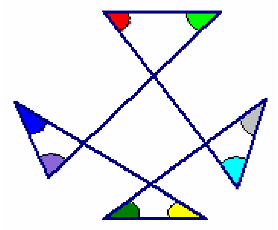
What conclusions can you make when you measure these angles.

(i) ______________________________

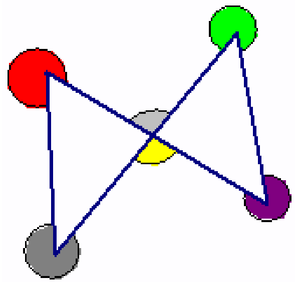
(ii) ______________________________

Geometry Questions

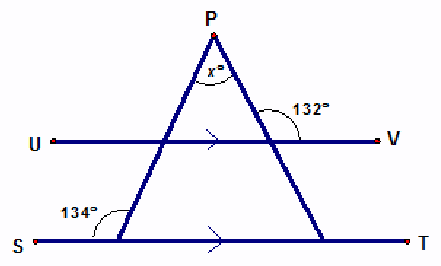
1. In the diagram, what is the sum of the marked angles? Explain your solution.



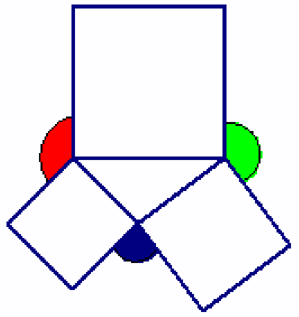
2. In the diagram, what is the sum of the marked angles? Explain your solution.



3. In the diagram, ST is parallel to UV
What is the value of *x*?



4. The diagram shows three squares drawn on the sides of a triangle.
What is the sum of the three marked angles?



5. State whether or not the following line segments will produce a triangle.
Give a reason for each set of lengths.

Lengths	Will these produce a triangle?	Reason
3 cm, 5 cm , 6 cm		
6.2 cm, 2.8 cm , 3.1 cm		
5.4 cm, 7.6 cm , 8.3 cm		
10.9 cm, 6.5 cm , 7.2 cm		
4.7 cm, 8.6 cm , 3.1 cm		