



***Module 2***

# Leaving Certificate Synthetic Geometry

## Terms in Logic and Deductive Reasoning

### Ordinary Level:

Theorem, Proof, Axiom, Corollary, Converse, Implies.

### Higher Level:

Is equivalent to, If and only if, Proof by contradiction.

## Constructions

Foundation Level: 18,19,20

Ordinary Level: 16,17,21

Higher Level: 22

## Theorems

Application of theorems, converses and corollaries to solve problems.

### Ordinary Level:

Theorems: 7, 8, 11,12, 13, 16, 17, 18, 20, 21

Corollary: 6

### Higher Level:

Theorem Proofs: 11, 12, 13

# Terms in Logic and Deductive Reasoning – OL

**Theorem:** A theorem is a statement which has been proved to be true.

**Proof:** A proof is a sequence of statements (made up of axioms, assumptions and arguments) leading to the establishment of the truth of one final statement.

**Axiom:** An axiom is a statement which is assumed to be true and is used as a basis for developing a system.  
Example: Axiom 1 - There is exactly one line through any two given points.

**Corollary:** A corollary follows after a theorem and is a proposition which must be true because of that theorem.  
Example: Corollary 6 - This corollary follows Theorem 20 and states: "If two circles share a common tangent line at one point, then the centres and that point are collinear".

**Converse:** The converse of a theorem is formed by taking the conclusion as the starting point and having the starting point as the conclusion.  
Example: The converse of Theorem 2 states "If two angles are equal, then the triangle is isosceles".  
See activity on Theorem 7

**Implies:** Implies indicates a logical relationship between two statements, such that if the first is true then the second must be true.

## Terms in Logic and Deductive Reasoning – HL

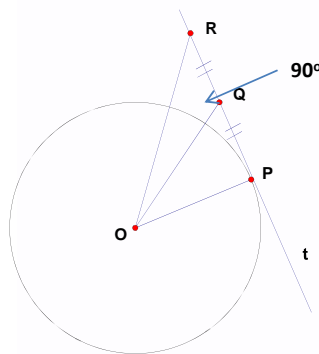
**Is equivalent to:** Two things are said to be equivalent if they have the same value but different forms.

**If and only if:** Often shortened to “iff”.

**Proof by contradiction:** A proof by contradiction is a proof in which one assumption is made. Then, by using valid arguments a statement is arrived at which is clearly false, so the original assumption must have been false.

## Theorem 20: Proof by Contradiction

(i) Each tangent is perpendicular to the radius that goes to the point of contact.



Suppose the point of contact is  $P$  and the tangent  $t$  is **not** on the perpendicular to  $OP$

Let the perpendicular to the tangent from  $O$  meet it at  $Q$ .

Pick  $R$  on  $PQ$ , on the other side of  $Q$  from  $P$ , with  $|QR| = |PQ|$

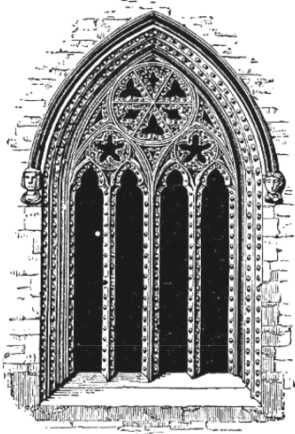
Then triangle  $OQR$  is congruent to triangle  $OQP$

$|OR| = |OP|$ , so  $R$  is a second point where  $t$  meets the circle.

This contradicts the given fact that  $t$  is a tangent.

Thus  $t$  must be a perpendicular to  $OP$ , as required.

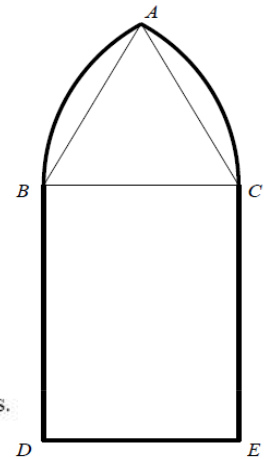
# Constructions – OL 2010



Windows are sometimes in the shape of a pointed arch, like the one shown in the picture.

A person is designing such an arched window. The outline is shown in the diagram below the picture.

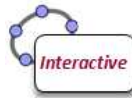
The centre for the arc  $AB$  is  $C$  and the centre for the arc  $AC$  is  $B$ .  $|BD| = 2.4$  metres and  $|DE| = 1.8$  metres.



- Show that  $|\angle ABC| = 60^\circ$ .
  - Find the length of the arc  $AB$ .  
Give your answer in metres, correct to three decimal places.
  - Find the length of the perimeter of the window.  
Give your answer in metres, correct to two decimal places.
  - Find the height of the window.  
Give your answer in metres, correct to two decimal places.
- (e) Make an accurate scaled drawing below of the outline of the window, using the scale 1:30. That is, 1 cm on your diagram should represent 30 cm in reality.

## Exercises

- Construct an angle of  $60^\circ$
- Complete the question, part (e)

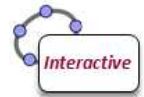


# Applications of Constructions

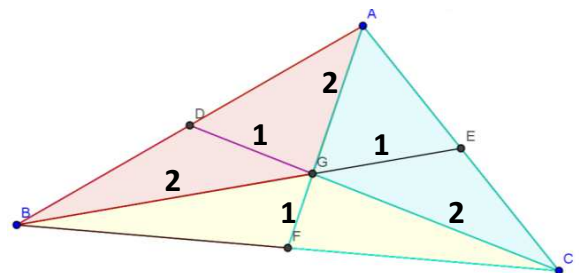
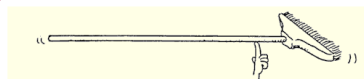
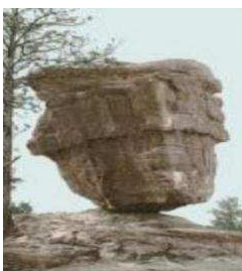
## Rotary Engine



Rotary Engine Mazda RX-8



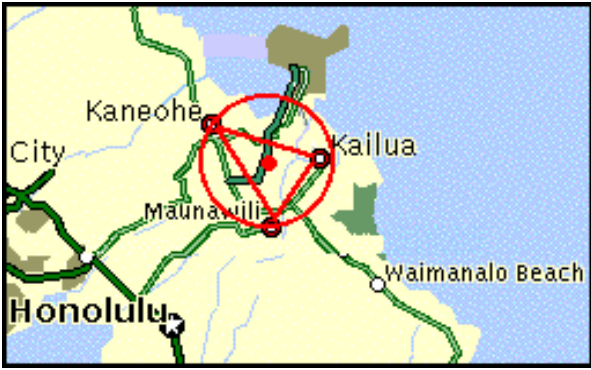
## Centre of Gravity



**Medians divided in the ratio 1:2 at the centroid.**

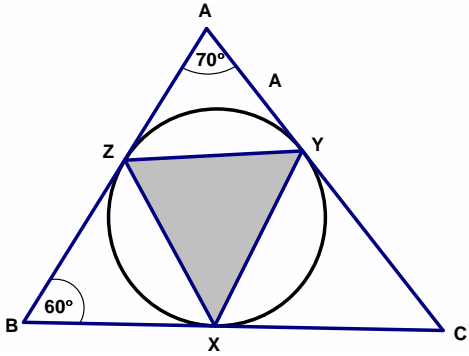


**Fire Station**



**Leaving Certificate**

The diagram shows the incircle of the triangle ABC. Given the measures of the angles of the triangle ABC as in the diagram, find the measures of the angles of the triangle XYZ.



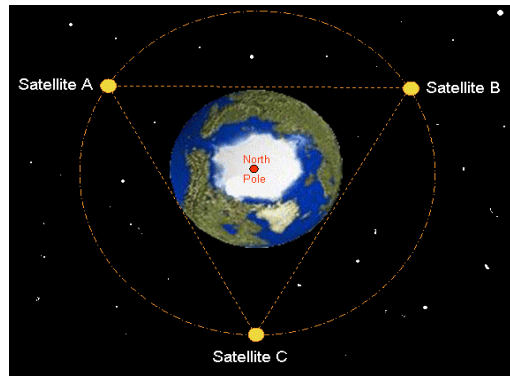
**Euler Line**



H: Orthocentre  
G: Centroid  
I: Circumcentre  
K: Incentre

- perpendiculars from the vertices
- medians
- mediators - perpendicular bisectors of the sides
- angle bisectors
- Euler line

## Satellites



There are three satellites orbiting the Earth over the equator. They are equally spaced around a circular orbit. In the diagram above, you are looking down on the north pole of the Earth, with the equator running around the edge of the earth's disk. In order that any point on the equator is visible by at least one satellite, their positions form an equilateral triangle. If the satellites are in the lowest orbit possible, the Earth is the incircle of the triangle, and the orbit is the circumcircle. Assume the Earth has a circumference of 40000 km:

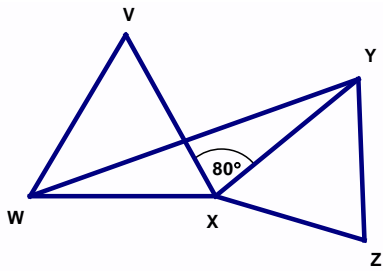
- (i) What is the radius of the Earth, to nearest kilometre?
- (ii) What is the size of the equilateral triangle linking the three spacecraft?
- (iii) What is the radius of the orbit?
- (iv) How high above the Earth's surface are the spacecraft flying?
- (v) From the above results, can you see a connection between the answers to (i) and (iv)?  
What does this mean in mathematical terms?

## Choice

- In the examination, candidates will have the option of answering a question on the synthetic geometry set out here, or answering a problem – solving question based on the geometrical results from the corresponding syllabus level at Junior Cert.
- This option will apply for a three year period only for candidates sitting the Leaving Certificate examination in 2012, 2013 and 2014.

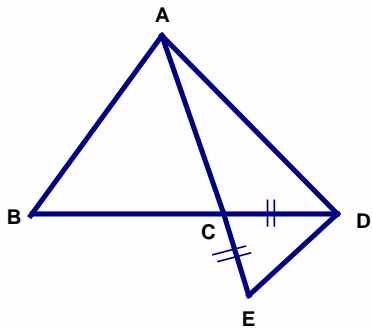
## Questions

1. In the diagram,  $VWX$  and  $XYZ$  are congruent equilateral triangles and the angle  $VXY = 80^\circ$ . What is the size of the angle  $VWY$ ?



2. Investigate the following statements. Give a reason for your conclusion.  
 (i) A triangle always has just one perpendicular height (altitude).  
 (ii) An altitude always has to be inside a triangle.

3. In the diagram,  $DCE$  is an isosceles triangle. Prove that  $[AE]$  is greater than  $[AD]$ .



4.  $XYZ$  is an isosceles triangle where  $|XY| = |XZ|$ .  $P$  is a point on  $XY$  and  $XQ$  is parallel to  $YZ$ . Prove that  $QX$  bisects the angle  $PXZ$ .

