Number & Algebra: Strands 3 & 4

#1
A Relations Approach to Algebra: Linear Functions
#2
A Relations Approach to Algebra: Quadratic, Cubic & Exponential Functions
#3
Applications of Sequences & Series
#4
Applications of Sequences & Series

Project Maths
Tionscadal Mata
Development Team

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Name: ______________________________

School: ______________________________

Linear Patterns

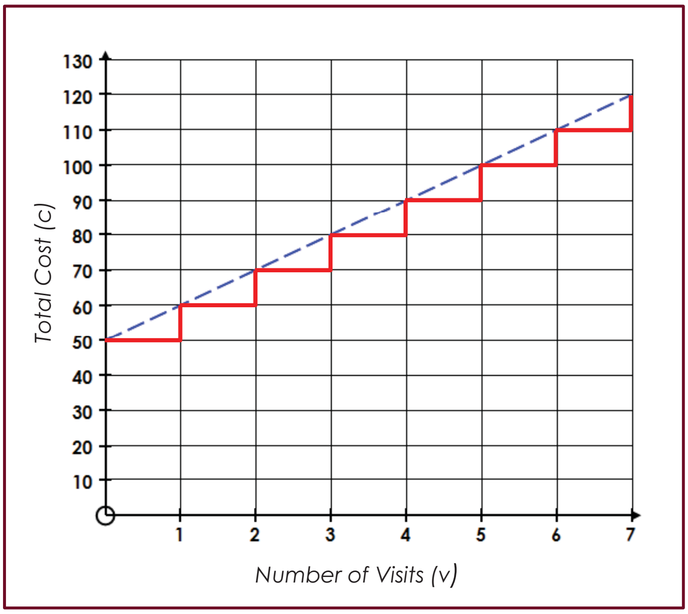
ARITHMETIC SEQUENCES AND SERIES

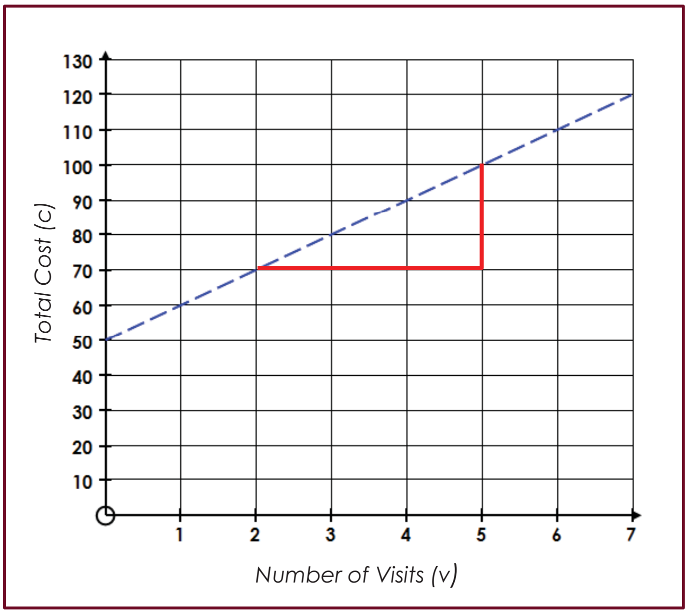
Linear Patterns

Linear Patterns
Junior Certificate 4.1 to 4.4

Arithmetic Sequences and Series
Leaving Certificate Section 3.1

For Linear Patterns Slope is the Same in each Interval

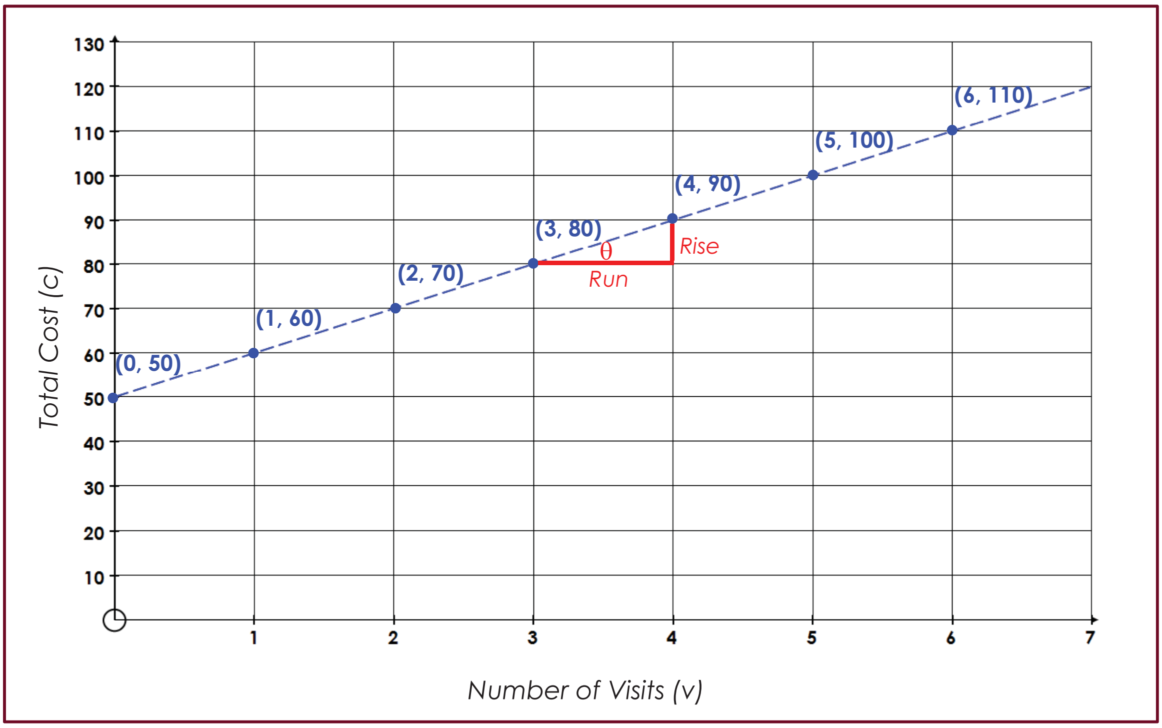




$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{10}{1} = 10$$

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{30}{3} = 10$$

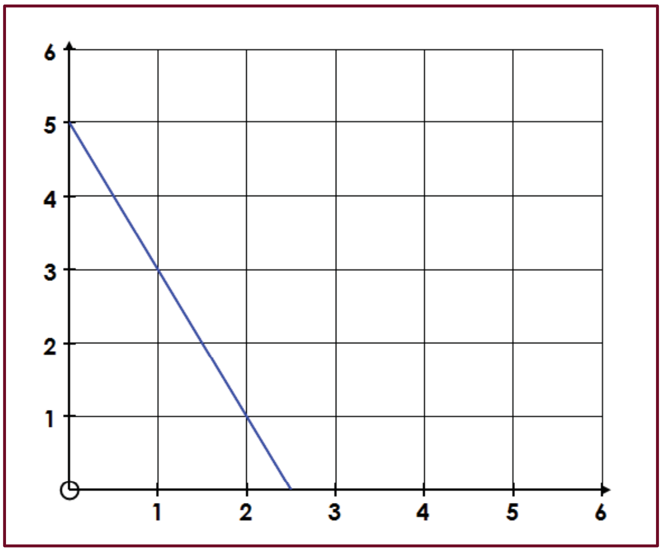
Alternative Ways of Finding the Slope or Rate of Change

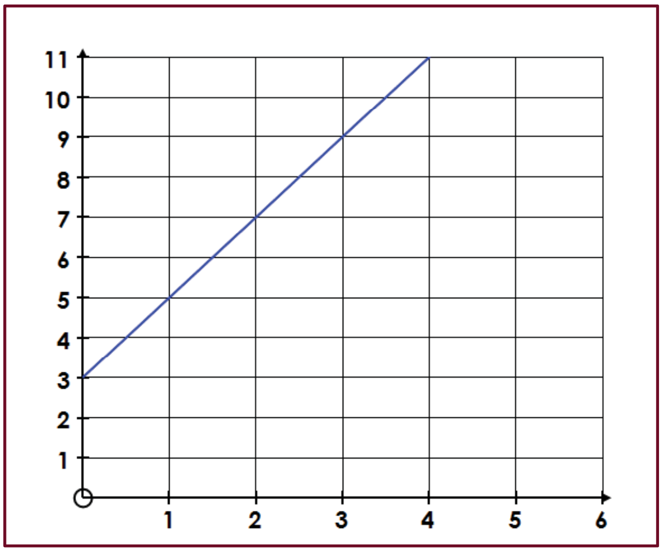


$$\text{Slope} = \text{Rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90-80}{4-3} = 10$$

$$\text{Slope} = \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{Rise}}{\text{Run}} = \frac{10}{1}$$

Give the Equation of the Following Lines





$y = 5 - 2x$

$y = -2x + 5$

$2x + y = 5$

$f(x) = 5 - 2x$

$y = 3 + 2x$

$y = 2x + 3$

$2x - y = -3$

$f(x) = 2x + 3$

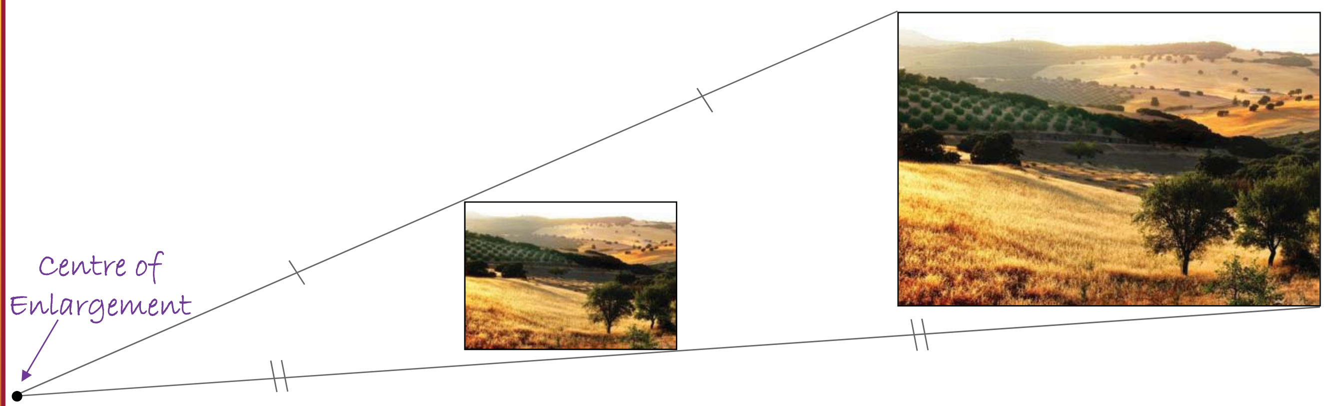
$f(x) = ax + b$

x	**$f(x) = ax + b$**	***Change***
0	$f(0) = b$	
		a
1	$f(1) = a + b$	
		a
2	$f(2) = 2a + b$	
		a
3	$f(3) = 3a + b$	
		a
4	$f(4) = 4a + b$	
		a
5	$f(5) = 5a + b$	

If $f(x) = ax + b$, the rate of change(slope) is always equal to a.

Examples of Linear Patterns

- Constant savings patterns with no interest
- Constant spending patterns
- ESB bills
- Mobile phone standing charge + charge per texts
- Call out charge for a workman + hourly rates
- Distance travelled by a car travelling at a constant speed
- Plant growing a constant amount per day
- Level of water in a tank as water is pumped from the tank at a constant pace
- Level of water in a tank when water enters the tank at a constant pace
- Vehicle depreciating in value by a constant amount each year (Straight Line depreciation) (This is not the reducing balance method.)



What do you notice about these pictures?

Are these pictures in proportion?

How could you tell if they are in proportion?

If you double the length you must double the width

If you multiply the length by *n* you must multiply the width also by *n*

Enlargements are proportional relationships

Proportional and non Proportional Situations

$$f : x \to 2x$$
$$f : x \to 3x$$
$$f : x \to nx$$

Notice when x = 0, y = 0 so these functions always go through (0, 0)

Proportional and non Proportional Situations

$$f(x) = x + 4$$
$$f(1) = 5$$
$$f(2) = 6$$

This function is non proportional

Proportional always linear and of the form y=mx

Learned so Far

- Tables, Graphs, Words and Formula (Multi Representational Approach)
- Linear Patterns
- Rate of change $= \dfrac{\text{Rise}}{\text{Run}} =$ Slope
- Equation of the line is $y = mx + c$, where m is the rate of change (slope) and c is where the line cuts the y–axis (y intercept)
- Variables and Constants
- Independent and dependent variables
- Proportional and non proportional situations

Original Gym A Problem: Membership €50 and €10 per visit

	Total cost
0	50
1	60
2	70
3	80
4	90

Term	Pattern	Formula
1	50	a
2	50 +10	$a + d$
3	50 +10 + 10	$a + d + d$
4	50 +10 + 10+10	$a + d + d + d$
5	50 +10 + 10+10 +10	$a + d + d + d + d$

Sequence = {50, 60, 70, 80, 90, ….} (a = first term) (d = common difference)

Can you predict what the 10th term will be in terms a and d?	$a + 9d$
Can you predict what the nth term of the sequence will be in terms of a, d and n?	$T_n = a\ (n - 1)d$

Formula and Tables booklet page 22

n is the position of the term in the sequence, not the number of visits to the gym

Arithmetic Sequences (Linear Patterns)

Sequences that have a first term, *a* and you add a common difference, *d* to the previous term to get the next term are known as arithmetic sequences.

1, 2, 3, 4, 5,…	Yes
1, 2, 4, 8, 16,…	No

Given the nth term of a arithmetic sequence is $T_n = 2n+3$.
What is the $(n+1)^{th}$ term?

$$T_{n+1} = 2(n+1)+3$$
$$T_{n+1} = 2n+5$$

Given we are dealing with an arithmetic sequence in terms of *a*, *d* and *n* what should $T_{n+1} - T_n$ equal?
Answer: *d*

Was this true for this sequence?

$T_{n+1} - T_n$	$T_2 - T_1$
$= 2n+5-[2n+3]$	$T_1 = 5$ and $T_2 = 7$
$= 2$	$= 7-5 = 2$
	$= d$

To prove $T_{n+1} - T_n = d$ for all arithmetic sequences

$$T_{n+1} + T_n$$
$$= a+(n+1-1)d-[a+(n-1)d]$$
$$= a+nd-a+nd+d$$
$$= d$$

1.7 Pg. 5

Proof of Formula

The sum of the first *n* terms of an arithmetic series

(Students will not be required to prove this formula.)

$S_n = T_1 + T_2 + T_3 + T_4 + \cdots + T_{n-1} + T_n$

$S_n = a + a + d + a + 2d + a + 3d + \cdots + a + (n-2)d + a + (n-1)d$ **[1]**

Note S_n can also be written as: $T_n + T_{n-1} + \cdots + T_4 + T_3 + T_2 + T_1$

Writing Sn in reverse:

$S_n = a + (n-1)d + a + (n-2)d + \cdots + a + 3d + a + 2d + a + d + a$ **[2]**

Adding **[1]** and **[2]**

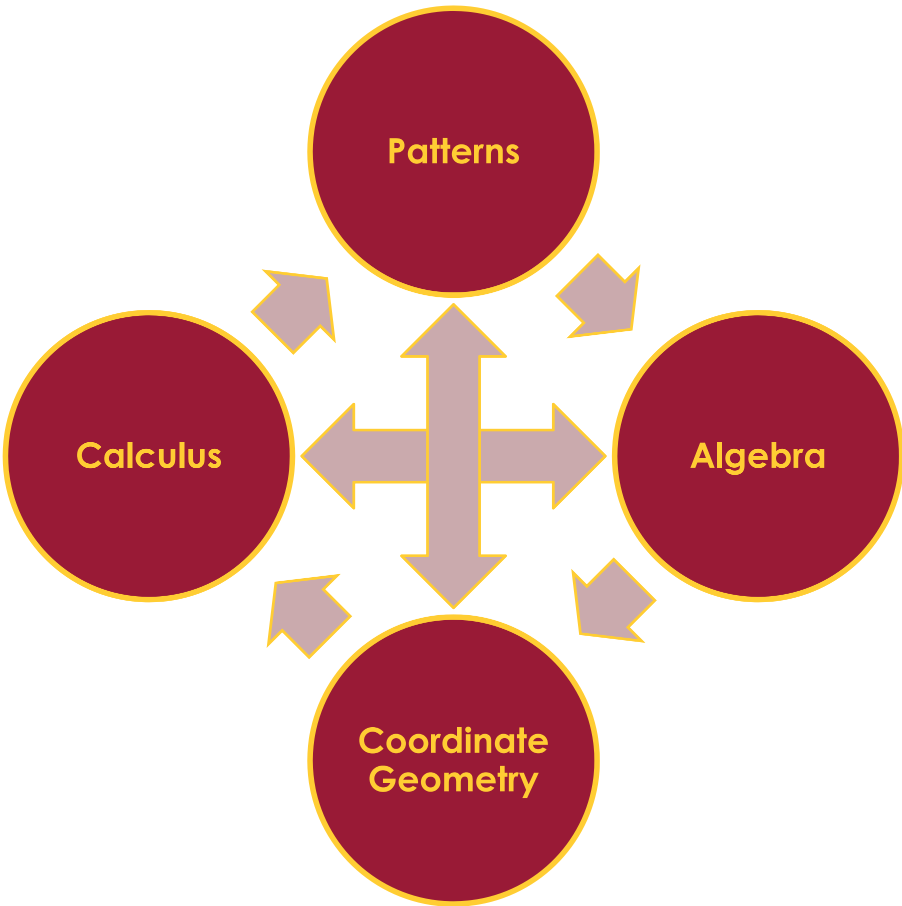
$S_n = a + a + d + a + 2d + a + 3d + \cdots + a + (n-2)d + a + (n-1)d$

$S_n = a + (n-1)d + a + (n-2)d + \cdots + a + 3d + a + 2d + a + d + a$

$2S_n = \{2a + (n-1)d\} + \{2a + (n-1)d\} + \{2a + (n-1)d\} + \cdots + \{2a + (n-1)d\} + \{2a + (n-1)d\}$

$2S_n = n\{2a + (n-1)d\}$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$ Formula as per formula and tables booklet



Introduction to Quadratic Graphs

PRIOR KNOWLEDGE: PROPERTIES OF LINEAR GRAPHS

WHAT ARE THE BASIC PROPERTIES OF A LINEAR GRAPH?

Let's Begin…

- There is a mythical creature called a "Walkasaurs"
- The table provided shows how "Walkasaurs" height changes with time

Time (years)	*Height (metres)*
0	1
1	2
2	4
3	7
4	
5	
6	
7	

Points to Ponder...

- Do you notice a pattern in the rate of growth of the walkasaurus?
- Is the change in height each year the same? (a constant number)
- Can you complete the table for the remaining years?
- If I draw this graph will it be a straight line(linear) ?
- Why or why not?

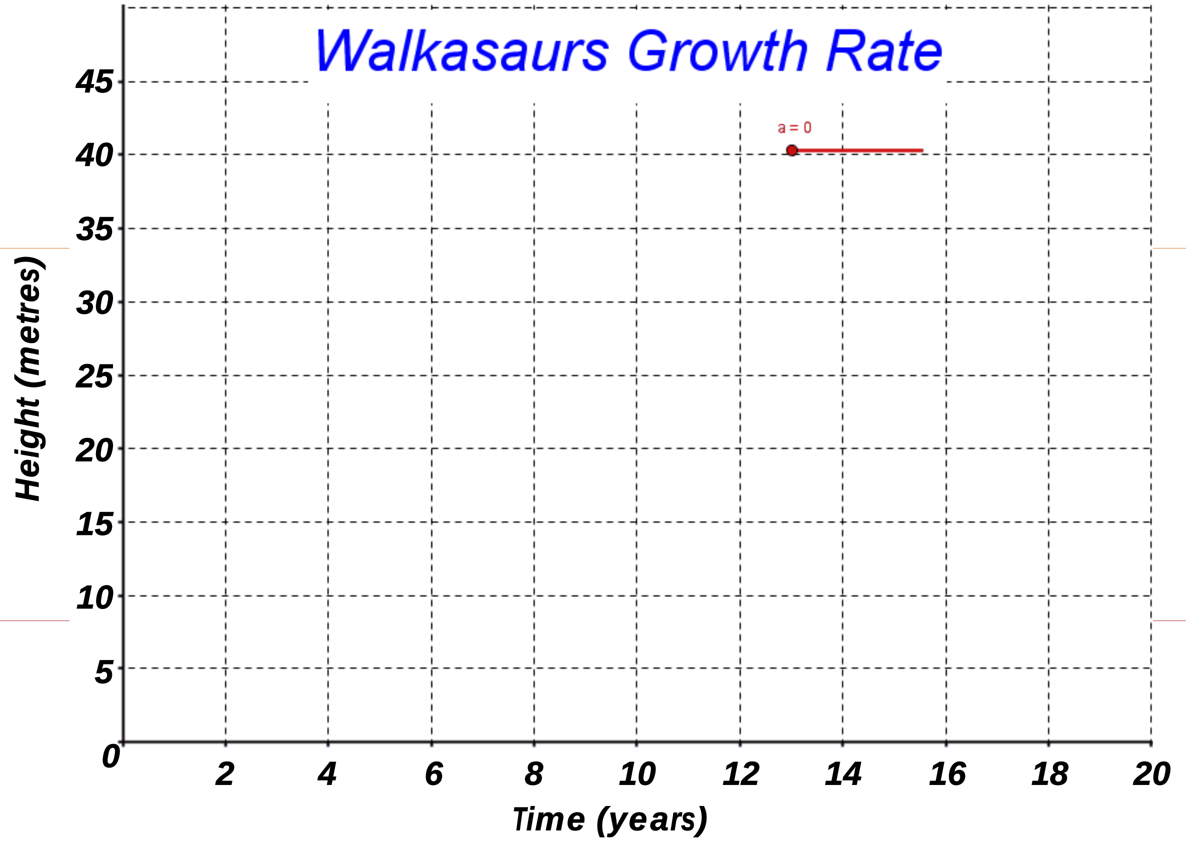
Time (years)	Height (metres)
0	1
1	2
2	4
3	7
4	
5	
6	
7	

Drawing the graph

Complete the table and plot the graph.

Time (years)	Height (metres)
0	1
1	2
2	4
3	7
4	
5	
6	
7	

The Graph



Finding the Pattern

Time (years)	Height (metres)	1st change	2nd change
0	1		
1	2	1	
2	4	2	1
3	7	3	1
4	11	4	1

The first change is not a constant number, as is the case in a linear graph, however the 2nd change is a constant, this is one of the properties of a quadratic graph.

Aeroplane Lift Off

For a given wing area the lift of an aeroplane is proportional to the square of its speed. The table below shows the lift of a Boeing 747 jet airline at various speeds.

Speed (km/h)	180	240	300	360	420	480	540	600
Lift (net upward force) (Newtons)	11340	45360	102060	181440	283500	408240	555660	725760

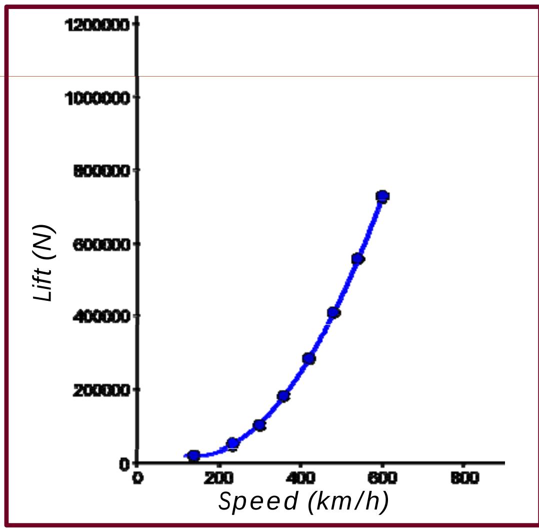
(a) Is the pattern of lifts quadratic? Give a reason for your answer.
(b) Sketch the graph to show how the lift increases with speed.

A Boeing 747 weighs 46000 Newtons at takeoff .
(c) Estimate how fast the plane must travel to get enough lift to take flight.
(d) Explain why bigger planes need longer runways.

Speed (km/h)	180	240	300	360	420	480	540	600
Lift (net upward force) (Newtons)	11340	45360	102060	181440	283500	408240	555660	725760
1st Change	34020	56700	79380	102060	124740	147420	170100	
2nd Change		22680	22680	22680	22680	22680	22680	

Because the **second differences** are constant, the pattern is quadratic.

Geogebra File Link



Angry Birds!!

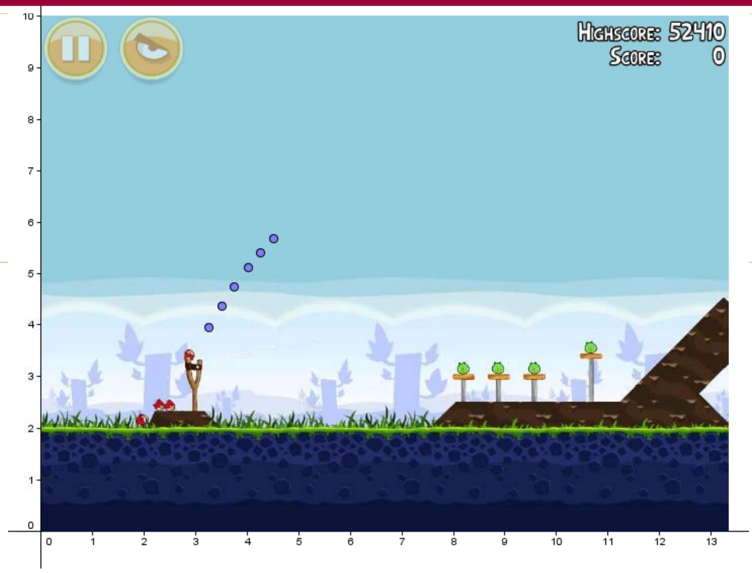
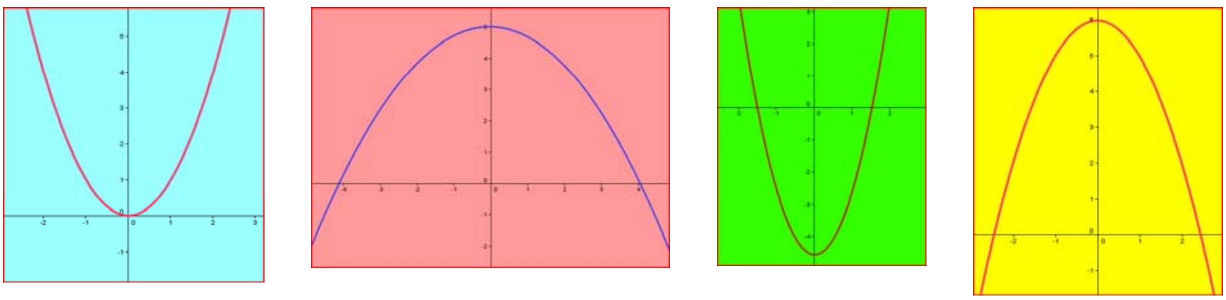


Table of Values:

Height	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5
Distance	1.5	2.375	3.1	3.675	4.1	4.375	4.5	4.475	4.3	3.975	3.5	2.875	2.1	1.175

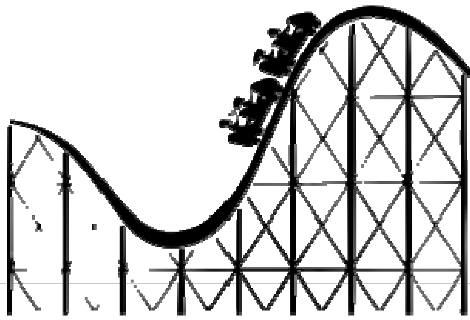
Create a List of the Properties of Quadratic Graphs



1. They are curved.
2. The 1st change is not constant, but the 2nd change is constant
3. They can occupy all 4 quadrants of the plane

Introduction to Cubic Graphs

Cubic Graphs

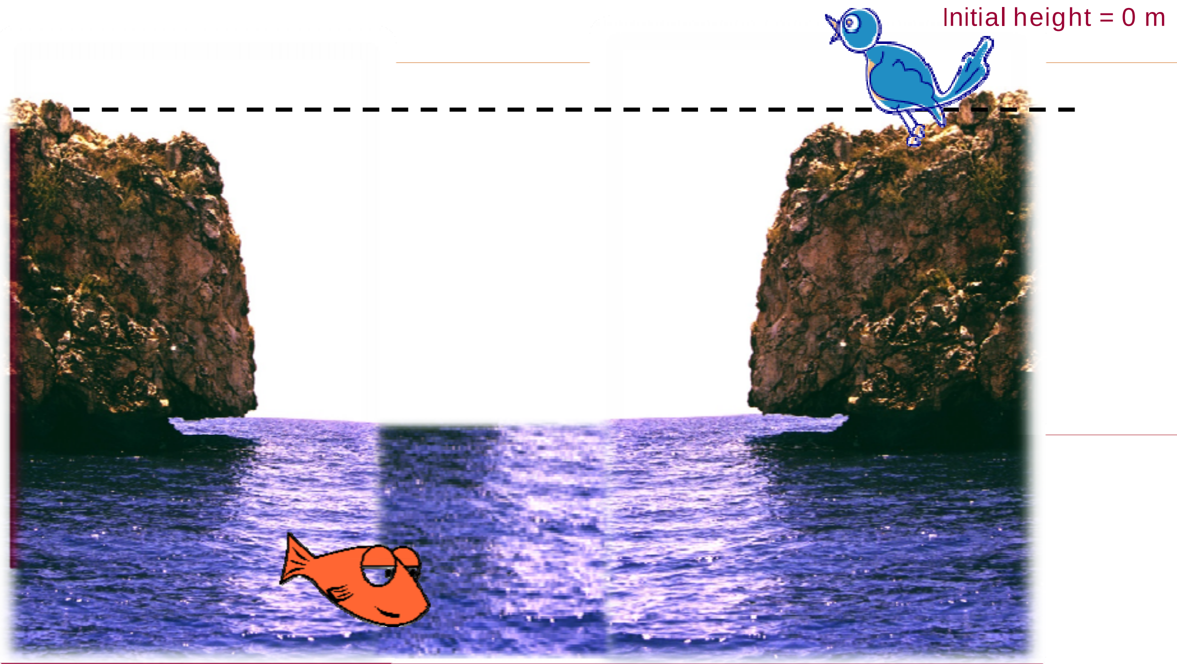


As previously discussed, not every thing can be described by a straight line, nor can everything be described by a “ ∪ ” or “∩” shaped curve.

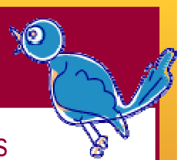
Lets take a look at the shape of a roller coaster.

It looks like 2 quadratics stuck together. But does it have the properties of a quadratic, i.e. The second differences will be constant?

Bird Journey



Looking at the Data

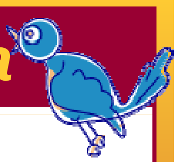


The distance the bird travelled and its change in height relative to its starting position is given in the table below:

Distance Travelled (m)	2	3	4	5	6	7	8
Change in height (m)	12	10	0	– 12	– 20	– 18	0

If we were to graph this data, what shape would the graph be?

Looking at the Change in the Data



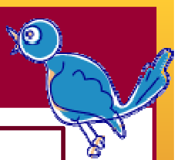
Distance Travelled (m)	2	3	4	5	6	7	8
Change in height (m)	12	10	0	– 12	– 20	– 18	0
1st Change	– 2	– 10	– 12	– 8	2	18	
2nd Change	– 8	– 2	4	10	16		
3rd Change	6	6	6	6			

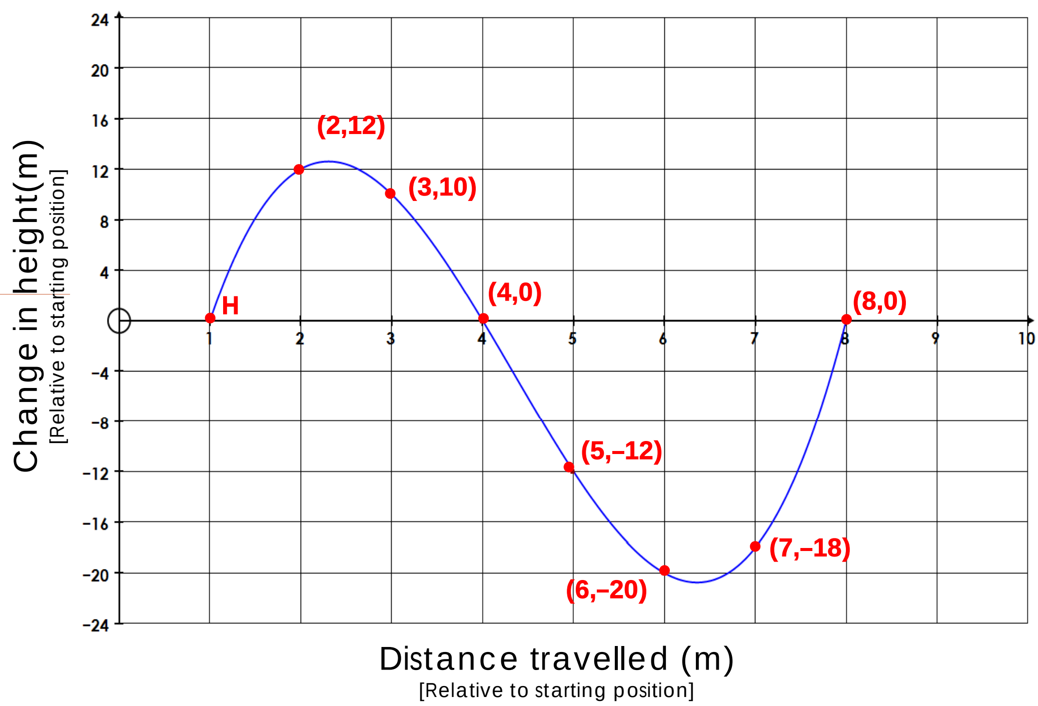
First change not a constant, so graph will not be **LINEAR**

Second change not a constant, so graph will not be **QUADRATIC**

Third change is a constant, this means the graph is a **CUBIC**

Graph of Bird's Journey





Introducing Exponential Functions

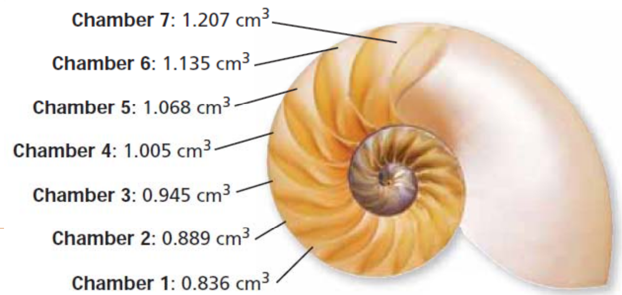
RECOGNIZE AND DESCRIBE AN EXPONENTIAL PATTERN.
USE AN EXPONENTIAL PATTERN TO PREDICT A FUTURE EVENT.
COMPARE EXPONENTIAL AND LOGISTIC GROWTH.

Recognising an Exponential Pattern

- A sequence of numbers has an exponential pattern when each successive number increases (or decreases) by the same percent.
- Here are some examples of exponential patterns:
  - Growth of a bacteria culture
  - Growth of a mouse population during a mouse plague
  - Decrease in the atmospheric pressure with increasing height
  - Decrease in the amount of a drug in your bloodstream

Recognising an Exponential Pattern

Describe the pattern for the volumes of consecutive chambers in the shell of a chambered nautilus.



Source: Larson Texts

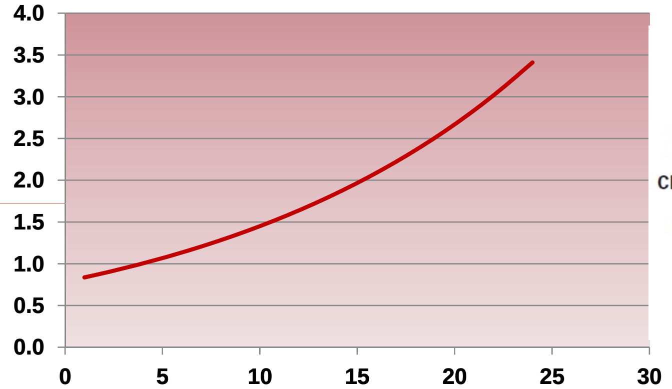
Solution: It helps to organize the data in a table.

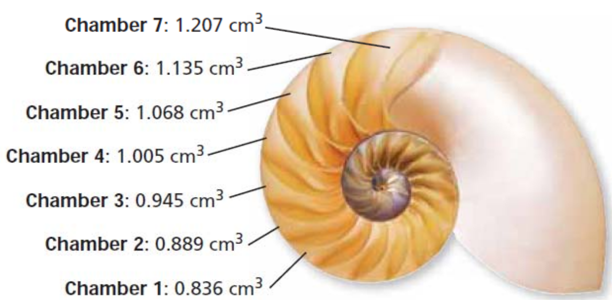
Chamber	1	2	3	4	5	6	7
Volume (cm^3)	0.836	0.889	0.945	1.005	1.068	1.135	1.207

Begin by checking the differences of consecutive volumes.

Recognising an Exponential Pattern

Chamber	1	2	3	4	5	6	7
Volume (cm^3)	0.836	0.889	0.945	1.005	1.068	1.135	1.207





Begin by checking the differences of consecutive volumes to conclude that the pattern is *not linear or Quadratic.*
*Then find **the ratios** of consecutive volumes.*

Checking the Ratios

Chamber	1	2	3	4	5	6	7
Volume (cm^3)	0.836	0.889	0.945	1.005	1.068	1.135	1.207

$$\frac{0.889}{0.836} \approx 1.063 \qquad \frac{0.945}{0.889} \approx 1.063 \qquad \frac{1.005}{0.945} \approx 1.063$$

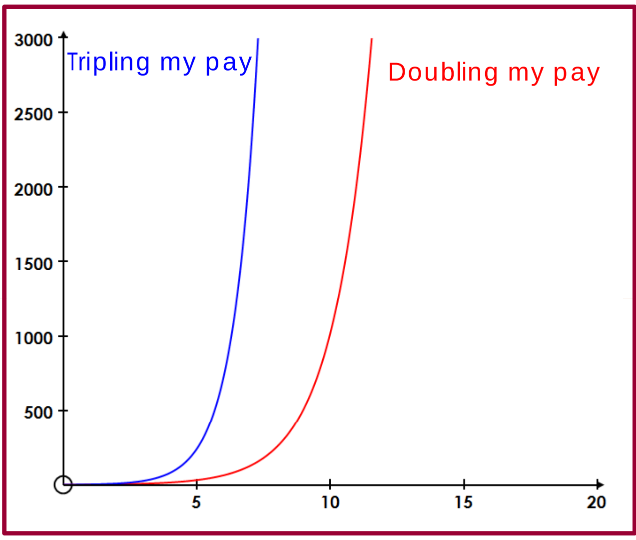
$$\frac{1.068}{1.005} \approx 1.063 \qquad \frac{1.135}{1.068} \approx 1.063 \qquad \frac{1.207}{1.135} \approx 1.063$$

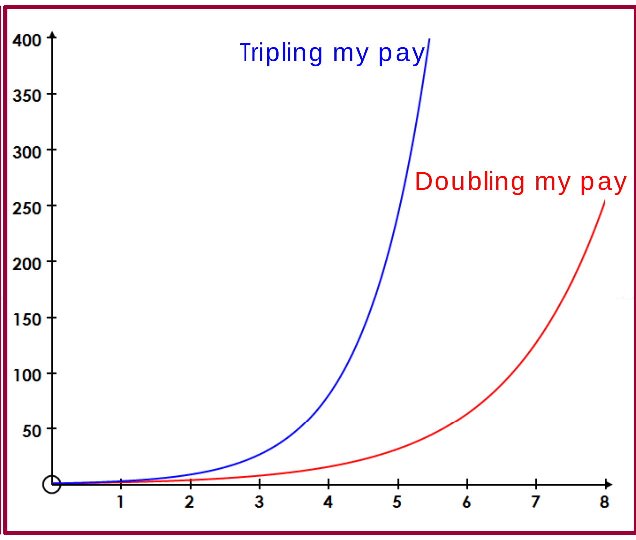
The volume of each chamber is about 6.3% greater than the volume of the previous chamber. So, the pattern is exponential.

Notice the difference between linear and exponential patterns. With **linear patterns**, successive numbers increase or decrease by the **same amount**.

With **exponential patterns**, successive numbers increase or decrease by the **same ratio**.

Both Graphs the Same but the Scales are Different





Exponential Graphs: Equation

$$y = a\,b^{x}$$

Intervals of time

Final Amount

Starting Value

Growth Factor

Table for the First 10 Days

View handout

Time (days)	*Money (Cents)*	*Pattern*
0	2	$2 \times 2^0 = 2^1$
1	4	$2 \times 2^1 = 2^2$
2	8	$2 \times 2^2 = 2^3$
3	16	$2 \times 2^3 = 2^4$
4	32	$2 \times 2^4 = 2^5$
↓	↓	↓
27	268,435,456	$2 \times 2^{27} = 2^{28}$
28	536,870,912	$2 \times 2^{28} = 2^{29}$

$$y = a\,b^{x}$$

Can you identify how the variables in the above formula relate to the values in the table?

Conclusion

- If a graph is Linear, the first change is constant
- If a graph is quadratic, the second change is constant
- If a graph is a cubic, the third change is constant
- If a graph is exponential, successive numbers increase or decrease by the same ratio.

Further Exploration of Patterns

Abstract Quadratic Patterns

	$a+b+c$		$4a+2b+c$		$9a+3b+c$		$16a+4b+c$		$25a+5b+c$	
1st change		$3a+b$		$5a+b$		$7a+b$		$9a+b$		
2nd change			$2a$		$2a$		$2a$			Coefficient of n^2 is $\frac{1}{2}(2a) = a$.

Constant second change, therefore it is quadratic starting with an^2.

Write out the an^2 series $a, 4a, 9a, 16a, \ldots$ and subtract from the original series.

$$\begin{array}{rccccc} & a+b+c, & 4a+2b+c, & 9a+3b+c, & 16a+4b+c, & 25a+5b+c,\ldots \\ - & a & 4a & 9a & 16a & 25a,\ldots \\ \hline & b+c & 2b+c & 3b+c & 4b+c & 5b+c,\ldots \end{array}$$

1st change $\quad b \quad b \quad b \quad b \quad$ This is linear bn

If you now go down form the 1st term which is $(b+c)-b=c$

The General Term is now an^2+bn+c

Generalising Quadratic Sequences

n^2	1,	4,	9,	16,	25,…
1st change	3	5	7	9	
2nd change	2	2	2		

$2n^2$	2,	8,	18,	32,	50,…
1st change	6	10	14	18	
2nd change	4	4	4		

When you get half the 2nd change you get the coefficient of n^2

$4n^2$	4,	16,	36,	64,	100,...
1st change	12	20	28	36	
2nd change	8	8	8		

$0.5n^2$	0.5,	2,	4.5,	8,	12.5,...
1st change	1.5	2.5	3.5	4.5	
2nd change	1	1	1		

More Difficult Quadratic Patterns – Method 1

	6,	12,	20,	30,	42,…
1st change	3	5	7	9	
2nd change	2	2	2		

Therefore it is n^2 with other terms.

Write out the n^2 series 1, 4, 9, 16,... and subtract from the original series.

$$\begin{array}{rrrrrr} & 6, & 12, & 20, & 30, & 42,\ldots \\ - & 1, & 4, & 9, & 16, & 25,\ldots \\ \hline & 5, & 8, & 11, & 14, & 17,\ldots \end{array}$$

1st change 3 3 3 3 This is linear $3n$

If you now go down form the 1st term which is $5 - 3 = 2$

The general term is now $n^2 + 3n + 2$

More Difficult Quadratic Patterns – Method 2

$$6,\ 12,\ 20,\ 30,\ 42,\ldots$$

Look for lowest common factor, here it is 2

$$2\times 3,\ 3\times 4,\ 4\times 5,\ 5\times 6,\ 6\times 7,\ldots$$

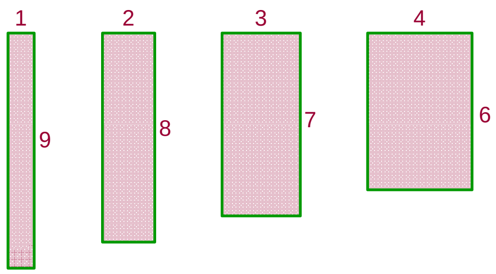
This is an AP $\times$ AP

First AP $\quad 2,\ 3,\ 4,\ 5,\ 6,..\quad T_n = a+(n-1)d = 2+(n-1)1 = n+1$

Second AP $\quad 3,\ 4,\ 5,\ 6,\ 7,\ldots\quad T_n = a+(n-1)d = 3+(n-1)1 = n+2$

$$T_n \times T_n = (n+1)(n+2) = n^2+3n+2$$

Given you have 20 metres of wire, what is the maximum rectangular shaped area that you can enclose?



Area	9	16	21	24
1st change	7	5	3	
2nd change		–2	–2	

Therefore it is $-n^2$ with other terms.

Write out the $-n^2$ series $-1,\ -4,\ -9,\ -16,\ldots$ and subtract from the original series.

$$\begin{array}{rrrrr} & 9, & 16, & 21, & 24,\ldots \\ - & -1, & -4, & -9, & -16,\ldots \\ \hline & 10, & 20, & 30, & 40, \end{array}$$

This is linear $10n$

The general term for the area is $-n^2+10n$

Geometric

$T_1 = 2$
$T_2 = 4$
$T_3 = 8$
$T_4 = 16$
$\vdots$
$T_n = ?$

This is a Geometric Sequence with $T_1 = 2 = a$

Each term is multiplied by 2 to get the next term: $r(\text{Common Ratio}) = 2$

$$\boxed{\frac{T_n}{T_{n-1}} = r}$$

Finding the T_n of a Geometric Sequence

$T_1 = a$
$T_2 = ar$
$T_3 = (ar)r = ar^2$
$T_4 = (ar^2)r = ar^3$
$T_5 = (ar^3)r = ar^4$
$\vdots$
$T_n = ar^{n-1}$ Exponential

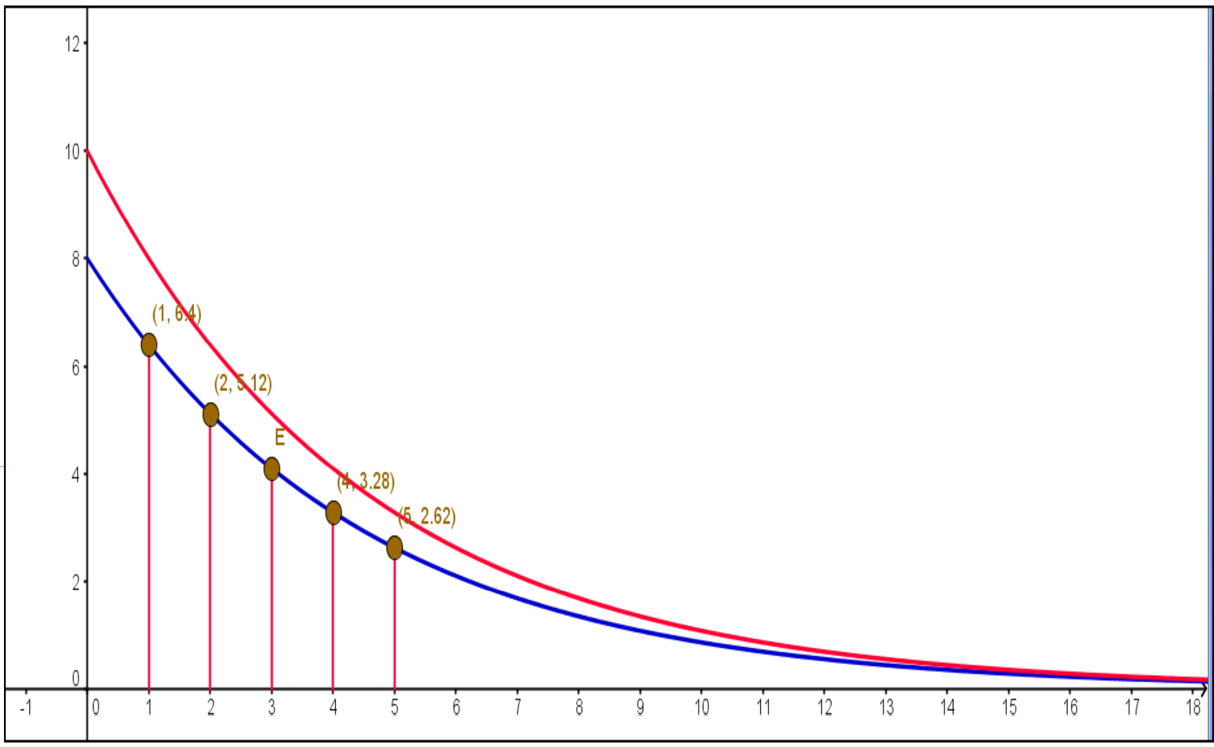
A ball is dropped from a height of 8 m. The ball bounces to 80% of its previous height with each bounce. How high (*to the nearest cm*) does the ball bounce on the fifth bounce.

	1st	2nd	3rd	4th	5th
8,	6.4,	5.12,	4.96,	3.2768,	2.62144

2.62144m = 262 cm

Is this a Linear, Quadratic or Cubic pattern?

Let's look at the graph of this.



Blue: $f(x) = 8(0.8)^x$ or $T_n = 8(0.8)^n$

Write down the function which describes the red graph.
What is the total distance travelled by the ball when it hits the ground for the 5th time?

What if we were asked to find the total distance travelled when the ball hits the ground for the 20th time. Is there any general way of doing it?

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_n = \frac{a(1 - r^n)}{1 - r}$$

Finding the S_n of a Geometric Series

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a + 0 + 0 + 0 + 0 + \cdots - ar^n$$ Subtracting

$$(1 - r)S_n = a - ar^n$$

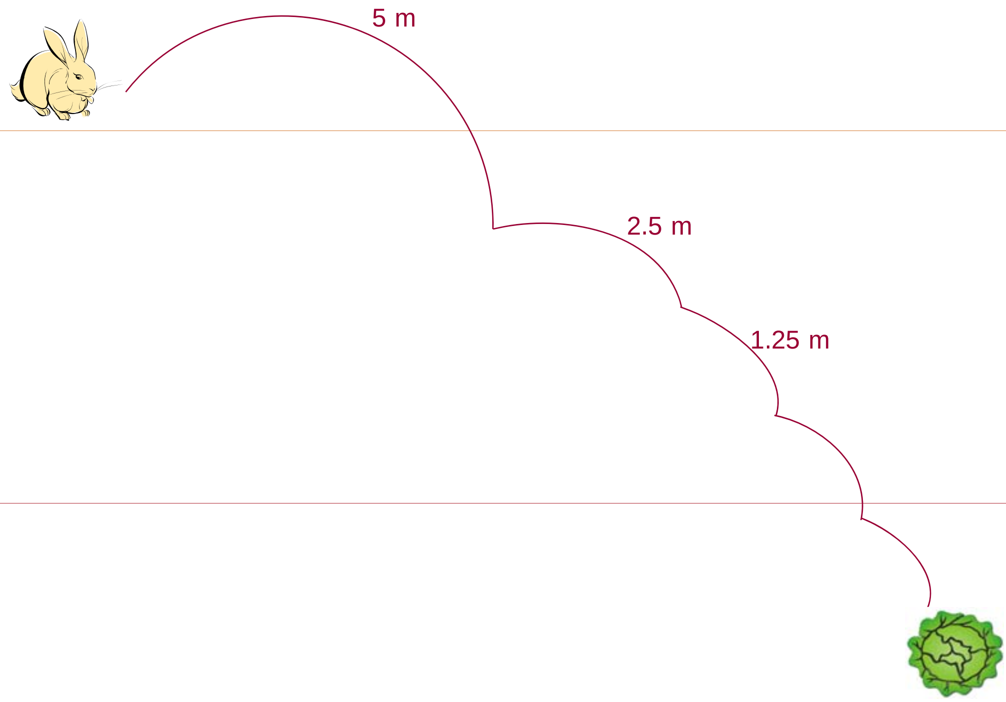
$$S_n = \frac{a - ar^n}{1 - r}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{or} \quad S_n = \frac{a(r^n - 1)}{r - 1}$$

These formulas can also be proved by Induction

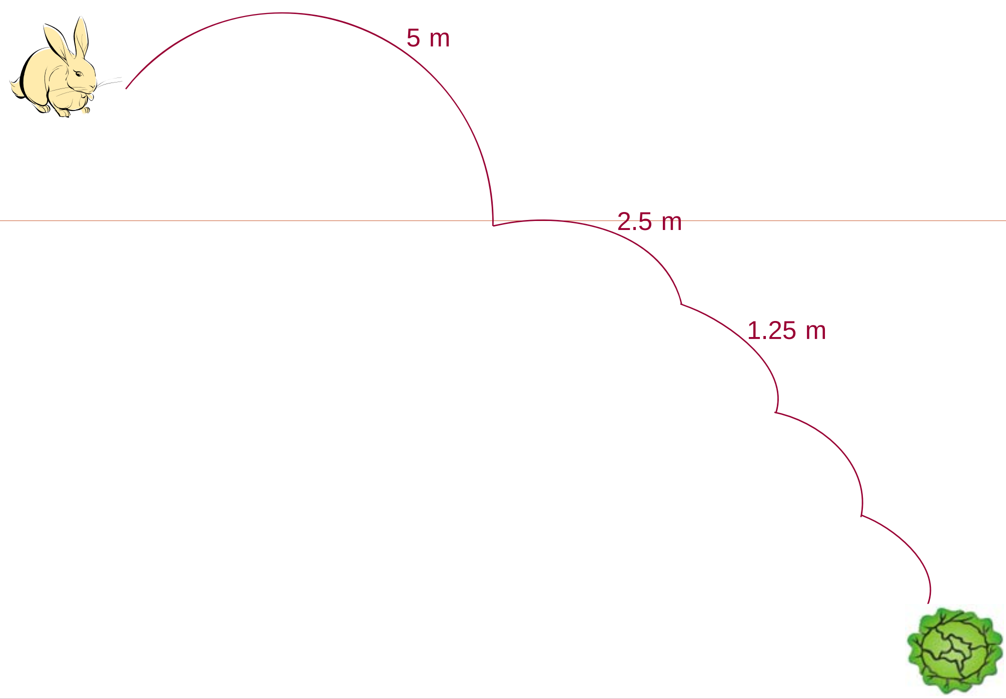
Link to Student's CD

A rabbit is 10 metres away from a some food. It hops 5 metres, then hops 2.5 metres, then 1.25 metres, and so on, hopping half its previous hop each time. What will the length of the 6^{th} hop be?



A rabbit is 10 metres away from some food. It hops 5 metres, then hops 2.5 metres, then 1.25 metres, and so on, hopping half its previous hop each time.

If the rabbit kept hopping forever, what in theory would be the total distance travelled by it?



The Sum to Infinity of a GP

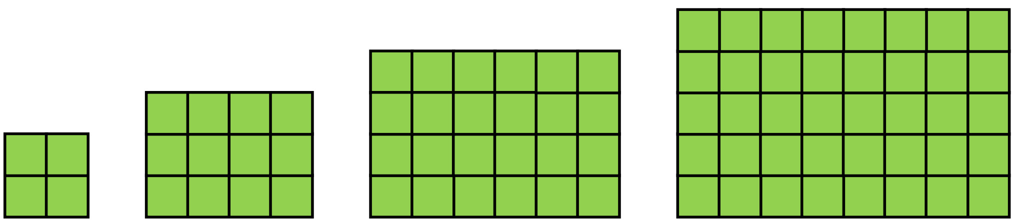
For $|r|<1 \qquad r^{\infty}=0$

$$S_n=\frac{a(1-r^n)}{1-r}=\frac{a}{1-r}-\frac{ar^n}{1-r}$$

$$S_{\infty}=\frac{a}{1-r}-\frac{0}{1-r} \qquad as\ r^{\infty}=0 \ \text{ for } |r|<1$$

$$S_{\infty}=\frac{a}{1-r} \ \text{ for } \ |r|<1$$

Extending the Blocks Question



Find the total number of blocks required to make the first 25 patterns.

$$T_n=2n^2+2n$$

$$S_n=2\sum_{r=1}^{n}r^2+2\sum_{r=1}^{n}r$$

$$S_n=2\left[\frac{n(n+1)(2n+1)}{6}\right]+2\left[\frac{n(n+1)}{2}\right]$$

$$S_n=\left[\frac{n(n+1)(2n+1)}{3}\right]+n(n+1)$$

$$S_{25}=\left[\frac{25(25+1)(2(25)+1)}{3}\right]+25(25+1)$$

$$S_{25}=11{,}700$$

Three Formulae

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2} \qquad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

These formulas can be proved by Induction

Summary of GP formulae

$$T_n \text{ of a GP} = ar^{n-1}$$

$$S_n \text{ of an GP} = \frac{a(r^n - 1)}{r - 1} \text{ for } r > 1 \quad \text{or} \quad \frac{a(1 - r^n)}{1 - r} \text{ for } r < 1$$

$$T_n = S_n - S_{n-1}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

Express $1.\dot{2}$ in the form of $\frac{a}{b}$ where a and $b \in \mathbb{N}$

Method 1

$1.\dot{2} = 1.222222222\ldots$

$= 1 + 0\cdot 2 + 0\cdot 02 + 0\cdot 002 + 0\cdot 0002 + 0\cdot 00002 + \cdots$

$= 1 + \left[\frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + \cdots\right]$

This is an infinite GP with $a = \frac{2}{10}$ and $r = \frac{1}{10}$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{\frac{2}{10}}{1-\frac{1}{10}} = \frac{2}{9}$

$\therefore 1.\dot{2} = 1 + \frac{2}{9} = \frac{11}{9}$

Method 2

Let $x = 1.222222222\ldots$

$10x = 12.22222222\ldots$

$x = 1.222222222\ldots$

$9x = 11$

$\therefore x = \frac{11}{9}$

Express $1.\dot{4}\dot{3}$ in the form of $\frac{a}{b}$ where a and $b \in \mathbb{N}$

Method 1

$1.\dot{4}\dot{3} = 1.43434343\ldots$

$= 1 + 0\cdot 43 + 0\cdot 0043 + 0\cdot 000043 + \cdots$

$= 1 + \left[\frac{43}{100} + \frac{43}{10000} + \frac{43}{1000000} + \cdots\right]$

This is an infinite GP with $a = \frac{43}{100}$ and $r = \frac{1}{100}$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{\frac{43}{100}}{1-\frac{1}{100}} = \frac{43}{99}$

$\therefore 1.\dot{2} = 1 + \frac{43}{99} = \frac{142}{9}$

Method 2

Let $x = 1.43434343\ldots$

$100x = 143.43434343\ldots$

$x = 1.4343434343\ldots$

$99x = 142$

$\therefore x = \frac{142}{99}$

Other Types of Series

$$\boxed{\frac{1}{AP \times AP}}$$

Show that $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$

Fill in the various values in the square brackets and find the sum to n terms of the series whose T_n is $\dfrac{1}{n(n+1)}$

Find the sum of the first 20 terms of the series

$$T_1 = \frac{1}{[\ ]} - \frac{1}{[\ ]}$$

$$T_2 = \frac{1}{[\ ]} - \frac{1}{[\ ]}$$

$$T_3 = \frac{1}{[\ ]} - \frac{1}{[\ ]}$$

$$\vdots \quad \vdots \quad \vdots$$

$$T_{n-1} = \frac{1}{[\ ]} - \frac{1}{[\ ]}$$

$$T_n = \frac{1}{[\ ]} - \frac{1}{[\ ]}$$

$$S_n =$$

Solution

$$T_1 = \frac{1}{1} - \cancel{\frac{1}{2}}$$

$$T_2 = \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$$

$$T_3 = \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}$$

$$\vdots \quad \cancel{\vdots} \quad \cancel{\vdots}$$

$$T_{n-1} = \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}$$

$$T_n = \cancel{\frac{1}{n}} - \frac{1}{n+1}$$

$$S_n = 1 - \frac{1}{n+1}$$

$$S_{20} = 1 - \frac{1}{20+1}$$

$$S_{20} = \frac{20}{21}$$

Arithmethico Geometric AP x GP

$$1 + 2r + 3r^2 + \cdots + nr^{(n-1)}$$

Find the T_n of the following sequence $2, 8, 24, 64, 160\ldots$

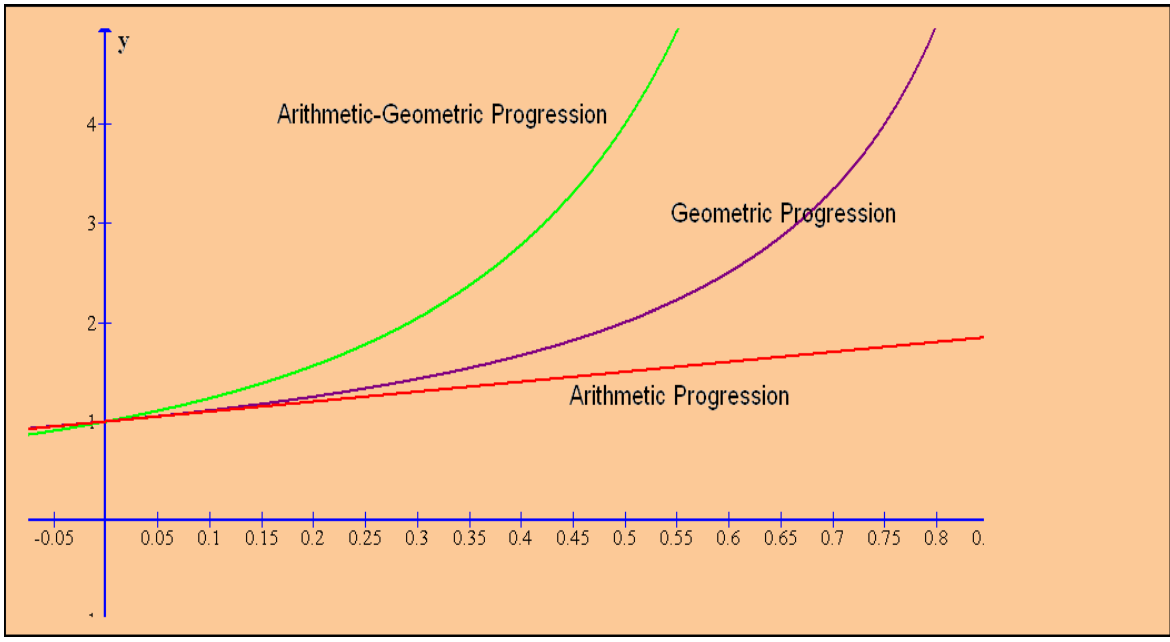
Find the T_n of the following sequence $1\times 2, 2\times 4, 3\times 8, 4\times 16, 5\times 32\ldots$

Each term in this sequence is an AP $\times$ GP

T_n of AP $= n$

T_n of GP $= 2(2)^{n-1} = 2^n$

Combined, $T_n = n(2^n)$

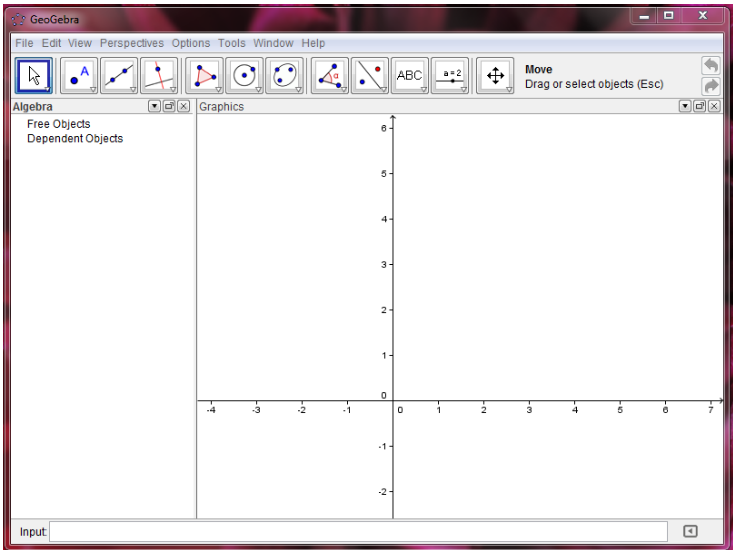


GeoGebra Three Graphs

Function Inspector

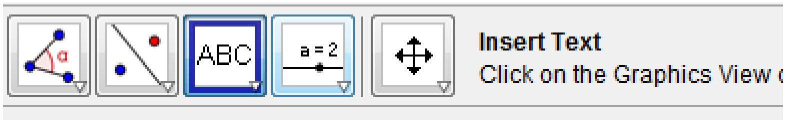
Function Inspector

1. Open a new GeoGebra file.



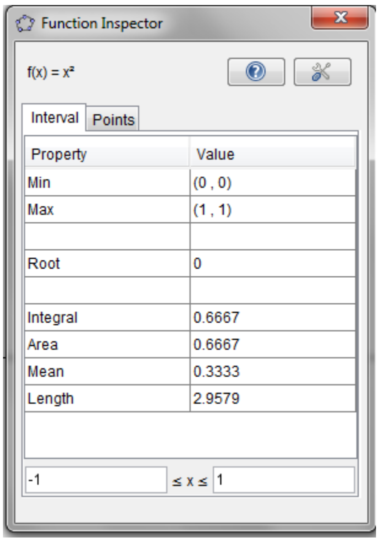
2. Type in some quadratic function into the input. Use f(x) = x^2

3. Click on the small arrow at the side of the ABC icon.

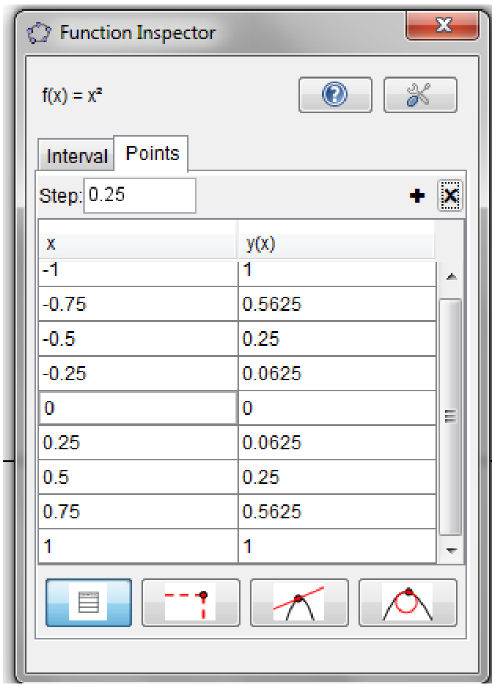


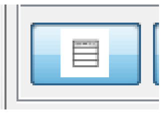
4. Select the Function Inspector icon and then click on the graph of the function.

5. The following should appear.

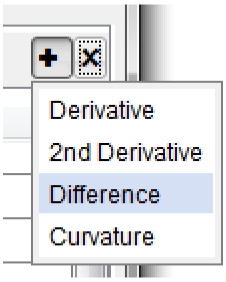


6. Click on the points tab and the following will appear.



7. Make sure that the first icon at the bottom  is selected. Note you can also change the Steps, it is now set at 0.25.

8. Click on the + in the top right hand corner and the following will appear.



9. Click on difference and the first change will be shown.

10. If you click again on the + you will get the second difference. This second change (difference) is a constant as we would expect for quadratic functions.