Number & Algebra: Strands 3 & 4

#1
A Relations Approach to Algebra: Linear Functions
#2
A Relations Approach to Algebra: Quadratic, Cubic & Exponential Functions
#3
Applications of Sequences & Series
#4
Applications of Sequences & Series

Project Maths Tionscadal Mata
Development Team

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Name: _____

School: _____

Linking Depreciation and Compounding to Prior Knowledge on Exponential Functions using Tables, Graphs and Formulae

2007 JC HL Q1 (b)

A snowman has a mass of 12 kg.

It melts at a rate of 0·2% of its mass per minute.

What will be the mass of the

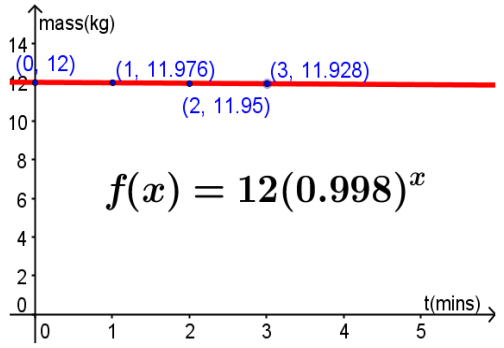
snowman after 3 minutes?

Give your answer correct to 2 decimal places.

Poorly answered.

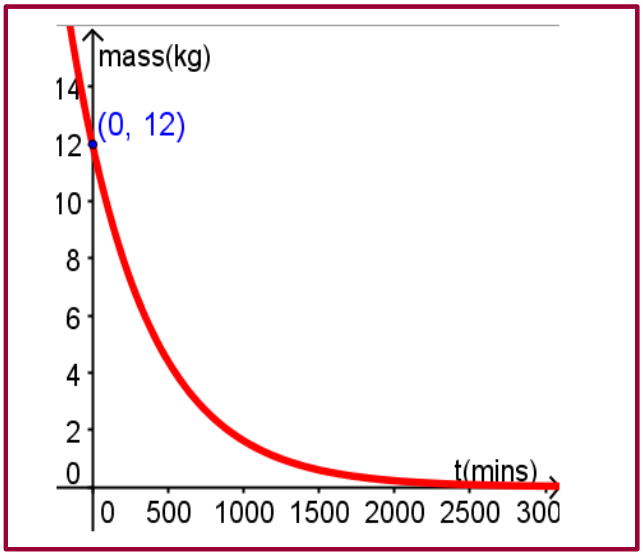
Common errors: **1.** *Ignoring cumulative loss of mass.*

2. *Mistake in % or decimal.*



Looks linear.....???

The bigger picture.....



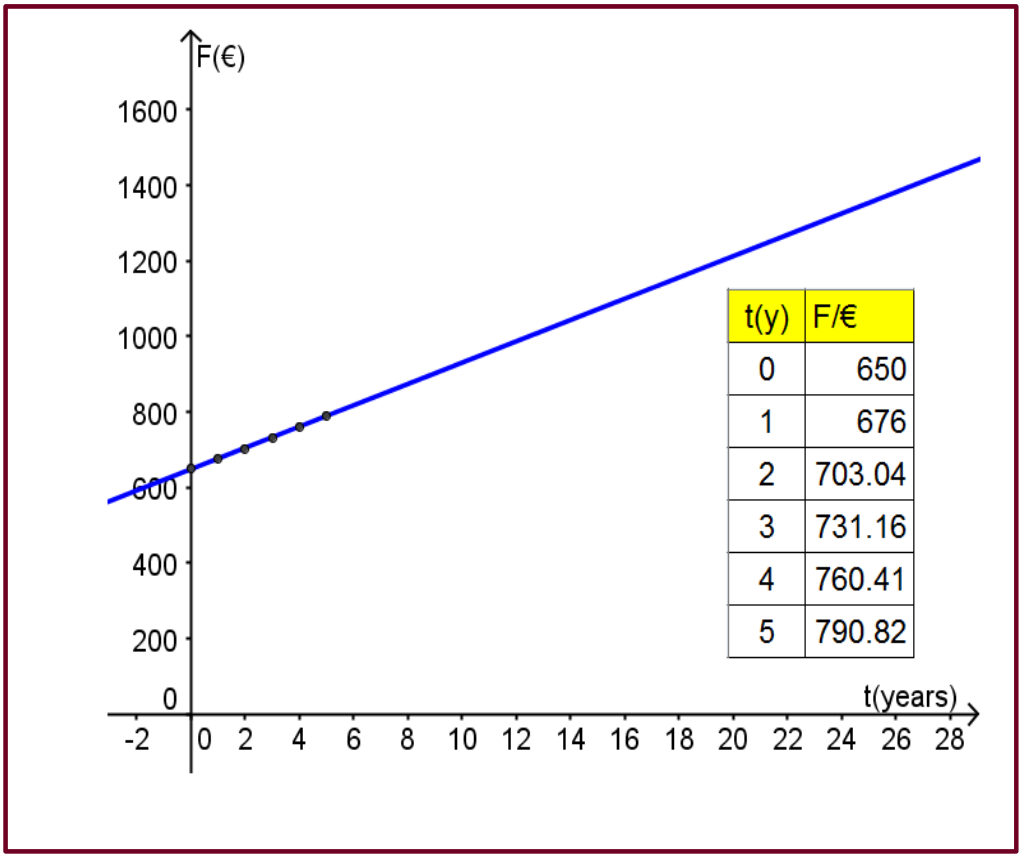
€650 is deposited in a fixed interest rate bank account. The amount in the account at the end of each year is shown in the following table.

End of year	1	2	3	4	5
Final value/€	676	703.04	€731.16	760.41	790.82

(a) Explain whether or not the relationship between final value and time can be modelled by a linear, quadratic or exponential function or by none of these?

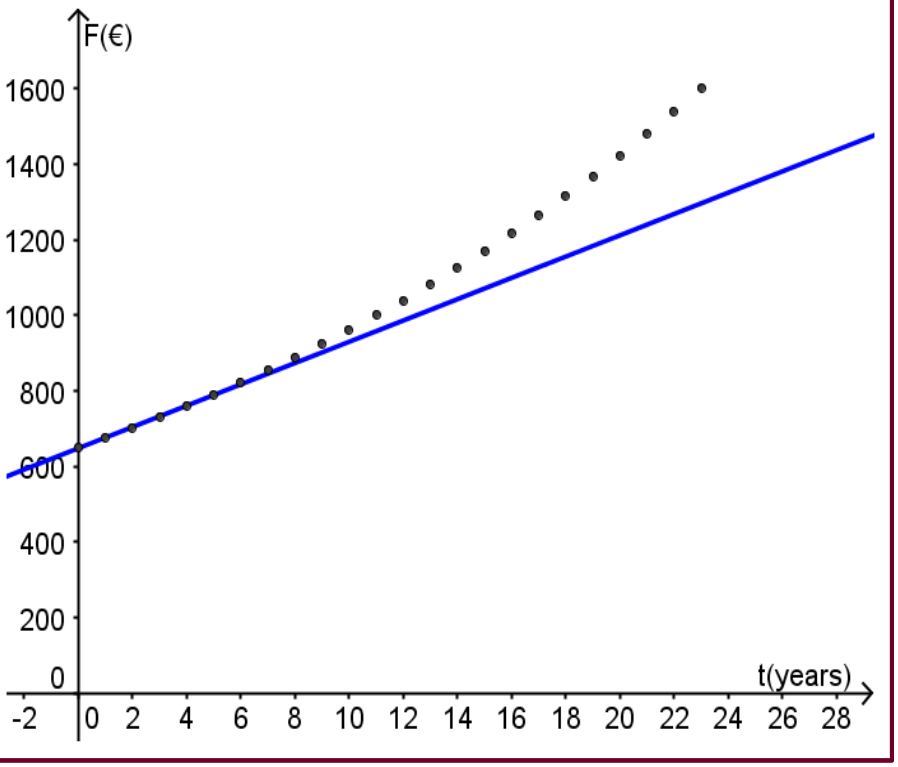
(b) If you plot a graph of final value against time what does the graph look like for this limited range of times?

Looks linear.....



t(y)	F/€
0	650
1	676
2	703.04
3	731.16
4	760.41
5	790.82

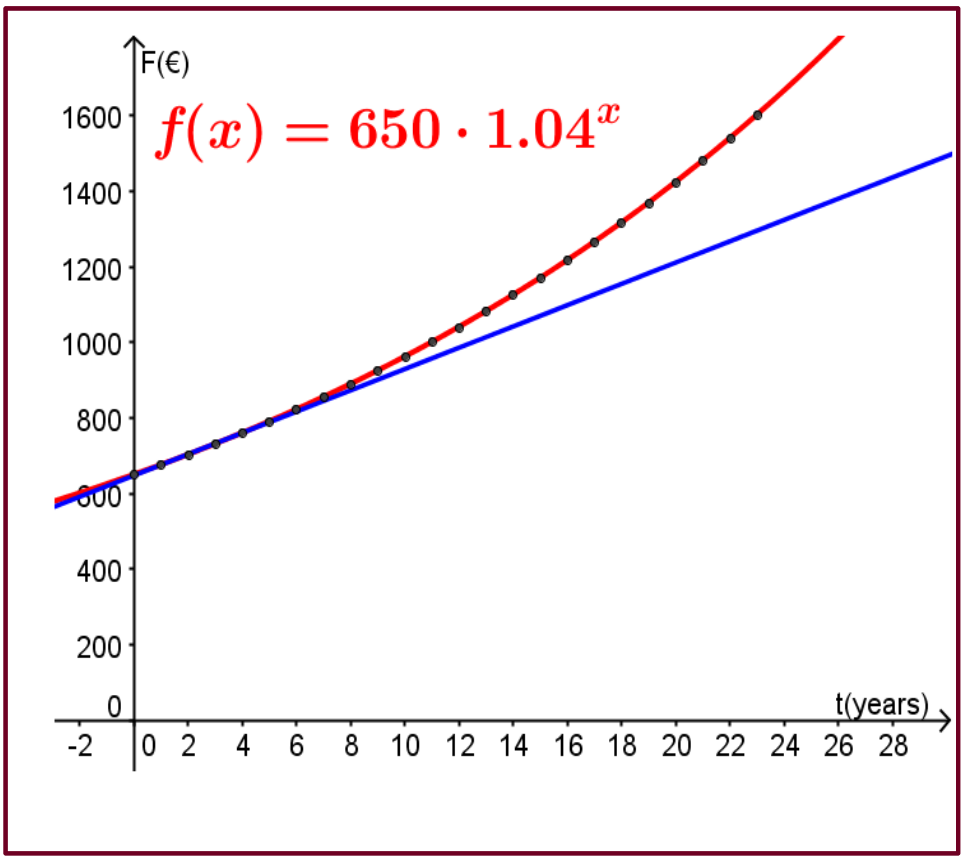
On further investigation.....



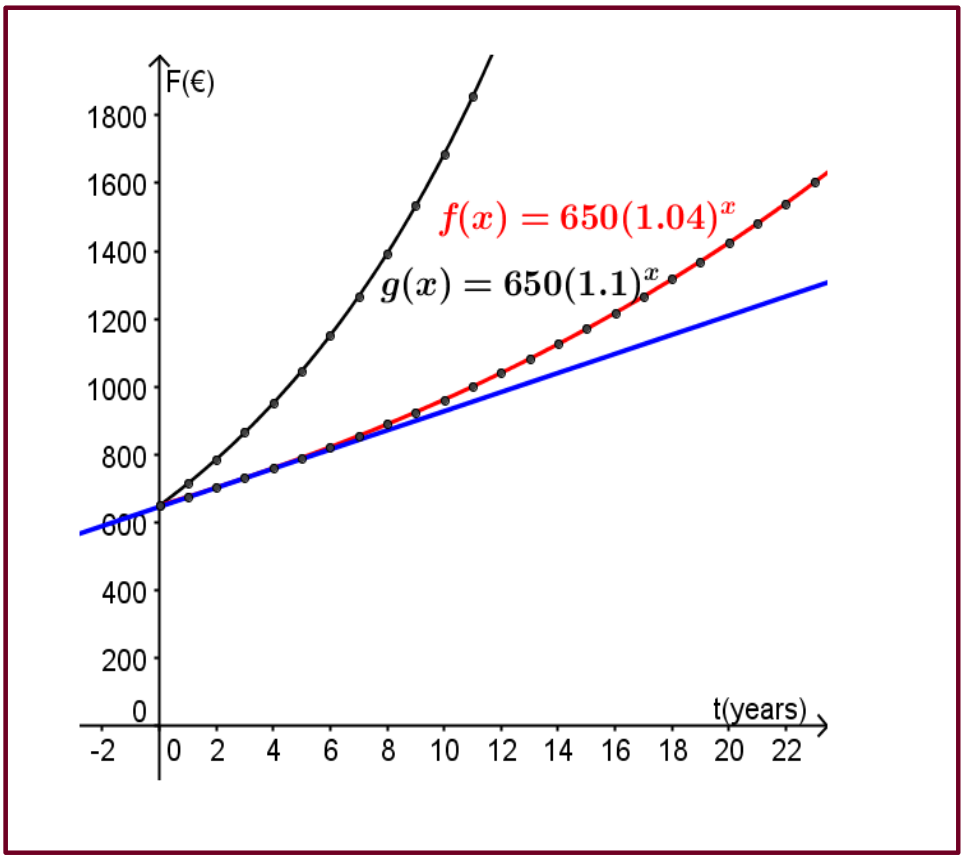
What formula expresses the final value in *t* years given an initial value of €650?

t(y)	F/€
0	650
1	676
2	703.04
3	731.16
4	760.41
5	790.82
6	822.46
7	855.36
8	889.57
9	925.15
10	962.16
11	1000.65
12	1040.67
13	1082.3
14	1125.59
15	1170.61
16	1217.44
17	1266.14
18	1316.78
19	1369.45
20	1424.23
21	1481.2
22	1540.45
23	1602.07

Final value is growing exponentially



Comparing a 10% interest rate to a 4% interest rate



Applying Rules for Indices

Complete the following table, without the use of a calculator.
Leave your answers in index form:

Before multiplying	After multiplying	Before multiplying	After multiplying
1. $a^7 \times a^3$		6. $1.02^3 \times 1.02^3$	
2. $8^3 \times 8^2$		7. $1.14^5 \times 1.14^2$	
3. $8.2^5 \times 8.2^2$		8. $1.06^4 \times 1.06$	
4. $(6.4)^5 (6.4)^2$		9. $(1.08)^5 (1.08)$	
5. $1.3^2 \times 1.3^5$		10. 1.07×1.07^5	

Evaluate Using the Calculator *(prior knowledge for method 2)*

1. 7^4

2. 4.5^4

3. 1.8^5

4. 1.06^6

5. 1.325^5

6. $\left(\frac{3}{2}\right)^7$

7. $\left(\frac{1}{2}\right)^4$

8. $10(3)^4$

9. $100(6)^3$

10. $1000(2.5)^3$

11. $300(1.03)^6$

12. $2000(1.025)^5$

13. $250(1.16)^4$

14. $400(1.08)^4$

Final Value Using Compounding (two methods)

Method 1	
Value of the gift (P)	€5000
Interest for the 1st year (I_1) (4% of €5 000)	€200
F_1 =Final value (end of year 1)	€5200
Interest for the 2nd year (I_2)	€208
F_2 = Final Value (end of year 2)	€5408
Interest for the 3rd year (I_3)	€216.32
F_3 =Final Value (end of year 3)	€5624.32

$0.04 = i$

$1.04 = 1 + i$

Write an expression for the final value F in terms of P, i and t.

Method 2

Value at end of year 1 €5000 × **1.04** = €5200	=	Value at end of year 1 €5000 × **1.04**
Value at end of year 2 €5200 × **1.04** = €5408	=	Value at end of year 2 €5000 × **1.04^2** Check with a calculator
Value at end of year 3 €5408 × **1.04** = €5624.32	=	Value at end of year 2 €5000 × **1.04^3** Check with a calculator

What % of P is F_1?	104%	Express this as a number and use it to calculate F_1	1.04
What % of F_1 is F_2?	104%	Express this as a number and use it to calculate F_2	1.04
What % of F_2 is F_3 ?	104%	Express this as a number and use it to calculate F_3	1.04

Finding Roots

Revision using the Calculator

$2 \times 2 \times 2 \times 2 \times 2 = 32$, therefore the 5th root of 32 is 2.

Verify this using the root key $\left[\sqrt[b]{a}\right]$

Evalaute the following:

(a) $625^{\frac{1}{4}}$

(b) $\sqrt[12]{1.043}$

(c) $1.043^{\frac{1}{365}}$

(d) $\sqrt[10]{\frac{5000}{3750}}$

Finding Number of Years (or other time periods) LCHL

Fiona has put €5000 into a savings account at 7% AER.
She needs €10 000 in order to build an extension to her house.
How many years will it take for Fiona to reach her target of €10 000 ?
Give your answer correct to one decimal place.

P	i	$(1+i)$	t	F
€5000	0.07	1.07		€10000

N.B. Prior knowledge of logarithms required

$10000 = 5000(1.07)^t$

$2 = (1.07)^t$

$\log_{1.07} 2 = t$

$t = 10.24$ years

Reducing Balance

Jillian and Noel are each going to buy a games console. It costs €500 and they are getting a loan from the credit union . Jillian says "I am making a lot of money at the moment so I can afford to pay €100 per month." Noel says that he can only afford €80 per month. The credit union is charging them a monthly interest rate of 1% to be paid at the end of each month.
Find their outstanding balances at the end of each month.

(a) A loan is taken out

(b) After 1 month interest is added on

(c) The person then makes his/her monthly repayment.
This process is then repeated until the loan is fully paid off.

Reducing Balance

Compare the total interest paid by Jillian and Noel.

Total interest J = €15.55
Total interest N = €18.98

Compare the time taken by Jillian and Noel to pay off the loan.

t_J = 6 months
t_N = 7 months

Jillian

Initial loan	€500.00
Interest 1	€5.00
Total	€505.00
Payment 1	€100.00
Balance 1	€405.00
Interest 2	4.05
Total	€409.05
Payment 2	€100.00
Balance 2	€309.05
Interest 3	3.0905
Total	€312.14
Payment 3	€100.00
Balance 3	€212.14
Interest 4	2.121405
Total	€214.26
Payment 4	€100.00
Balance 4	€114.26
Interest 5	1.14261905
Total	€115.40
Payment 5	€100.00
Balance 5	€15.40
Interest 6	0.154045241
Total	€15.56
Payment 6	€15.56
Balance 6	€0.00

Noel

Initial loan	€500.00
Interest 1	€5.00
Total	€505.00
Payment 1	€80.00
Balance 1	€425.00
Interest 2	4.25
Total	€429.25
Payment 2	€80.00
Balance 2	€349.25
Interest 3	3.4925
Total	€352.74
Payment 3	€80.00
Balance 3	€272.74
Interest 4	2.727425
Total	€275.47
Payment 4	€80.00
Balance 4	€195.47
Interest 5	1.95469925
Total	€197.42
Payment 5	€80.00
Balance 5	€117.42
Interest 6	1.174246243
Total	€118.60
Payment 6	€80.00
Balance 6	€38.60
Interest 7	0.385988705
Total	€38.98
Payment 7	€38.98
Balance 7	€0.00

Depreciation

A company buys a new lorry for €50 000. After 4 years it needs to sell the lorry. The value of the lorry reduces by 15% each year. What is the value of the lorry after 4 years?

Depreciation

Method 1	
Original Value of Lorry P	**€50000**
Depreciation in year 1 =€50000(0.15)	**€7500**
F_1 =Final value (end of year 1)	**€42500**
Depreciation in year 2	**€6375**
F_2 = Final Value (end of year 2)	**€36125**
Depreciation in year 3	**€5418.75**
F_3 =Final Value (end of year 3)	**€30706.25**

Method 2

Value at end of year 1 €50000 × **0.85** = €42500	=	Value at end of year 1 €50000 × **0.85**
Value at end of year 2 €42500 × **0.85** = €36125	=	Value at end of year 2 €50000 × **0.85^2** **Check with a calculator**
Value at end of year 3 €536125 × **0.85** = €30706.25	=	Value at end of year 3 €50000 × **0.85^3** **Check with a calculator**

$0.04 = i$

$1.04 = 1 + i$

Write an expression for the final value F in terms of P, i and t.

What % of P is F_1?	85%	Express this as a number and use it to calculate F_1	0.85
What % of F_1 is F_2?	85%	Express this as a number and use it to calculate F_2	0.85
What % of F_2 is F_3 ?	85%	Express this as a number and use it to calculate F_3	0.85

AER, EAR, CAR
&
Interest Rates other than Annual

Savings and Investments

AER (annual equivalent/effective rate) tells you what interest you will earn annually, which depends on how often interest is added.

- ✓ Used for savings and investments
- ✓ It may or may not include charges;
- ✓ Allows investors to make comparisons between savings accounts which pay interest at different intervals
- ✓ Takes into consideration the effect of compounding interest

The financial regulator's office considers the terms AER/EAR and CAR all to be equivalent. The term CAR is approved for use in relation to tracker bonds – for other investment products the regulator considers the acronym AER or EAR should be used.

AER

Leaving Certificate 2010 Sample Paper 1 Foundation Level Q2

A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate **(AER)** of 6%.
Find the value of the investment when it matures in eight years' time.

$$F = P(1+0.06)^8 = €7969.24$$

Leaving Certificate 2010 Sample Paper 1 Ordinary Level Q2

(a) A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate **(AER)** of 6%. Find the value of the investment when it matures in eight years' time.
(b) A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.

$$6000 = 5000(1+i)^8 \Rightarrow (1+i) = 1.2^{\frac{1}{8}} = 1.02305 \Rightarrow i = 0.02305$$

$$\text{AER} = 2.305\%$$

Unwrap the Terminology

Advertisement Christmas 2010 with AER

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2011

15 month fixed term savings account

Justify that that 5.25% fixed for 15 months is the same as 4.18% AER