Content Course 5

Inferencial Statistics & Functions

Name: _____

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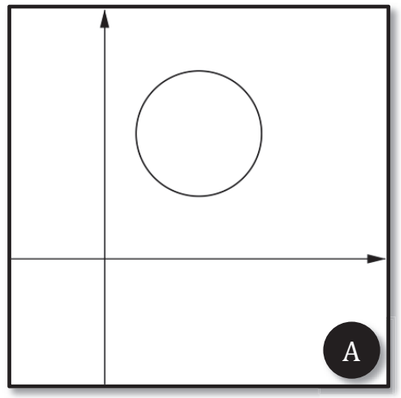
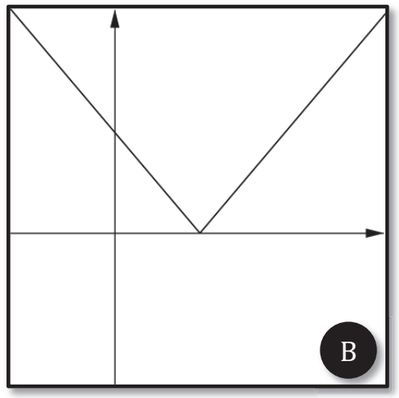
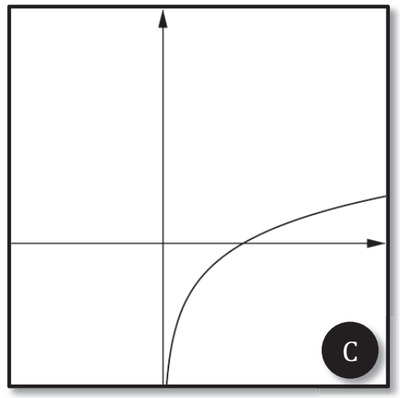
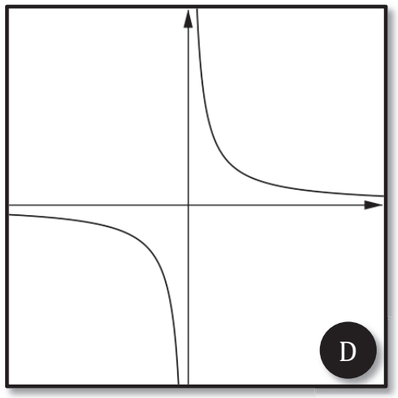
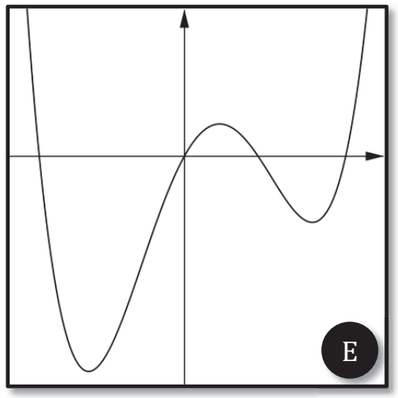
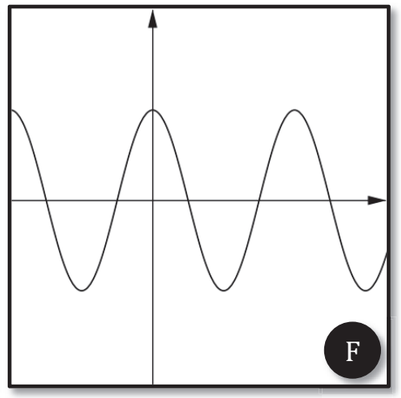


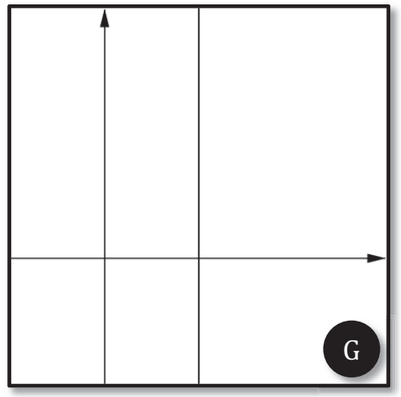


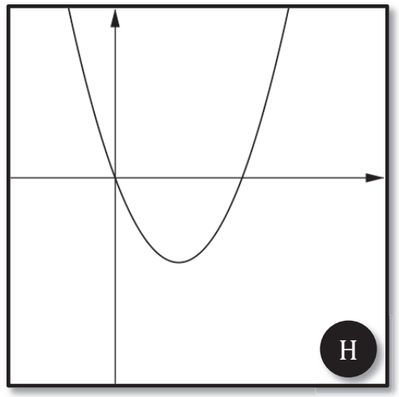


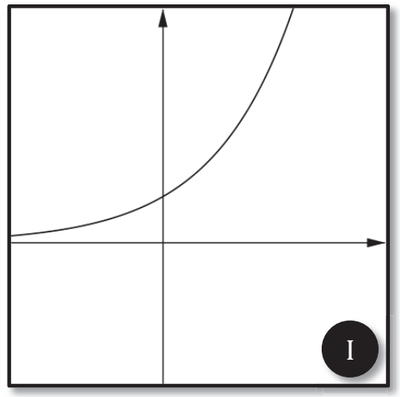


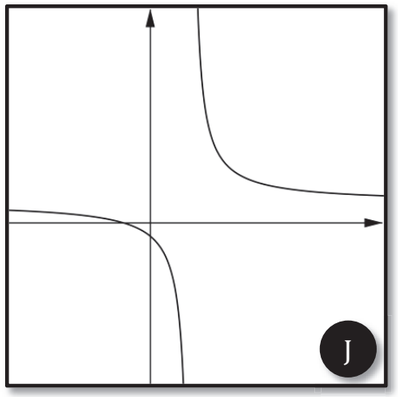


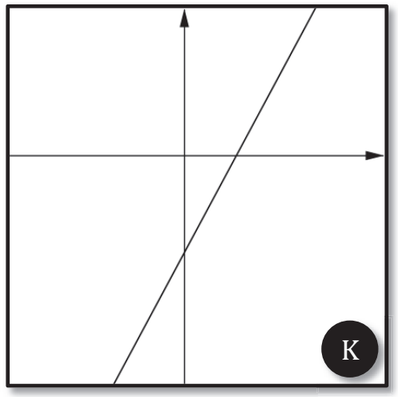


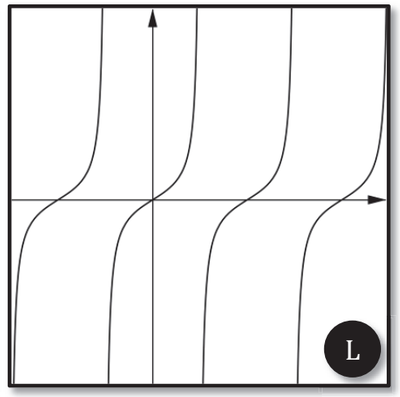


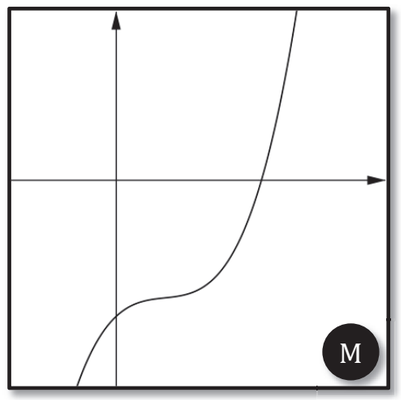


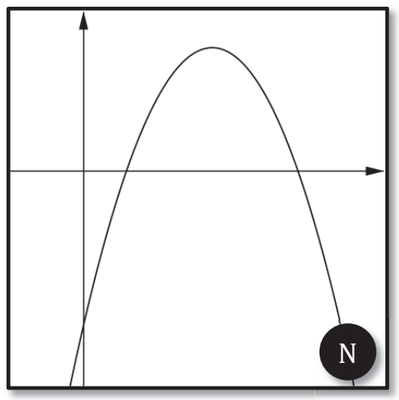


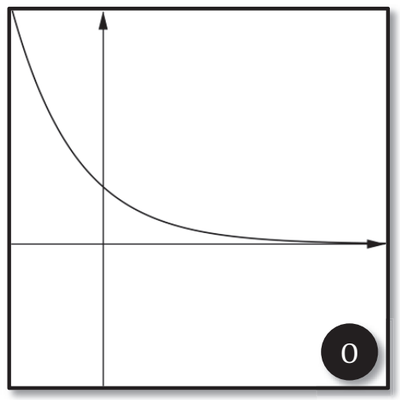


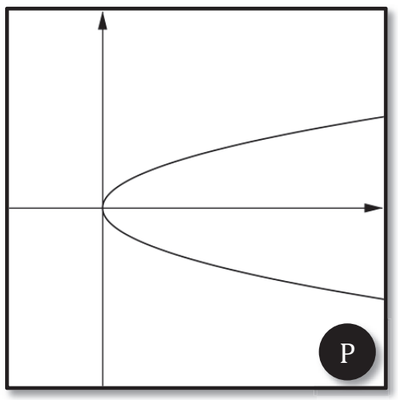


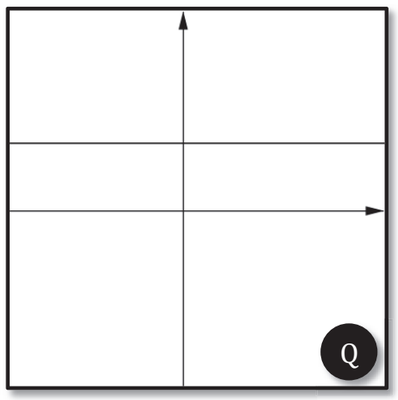


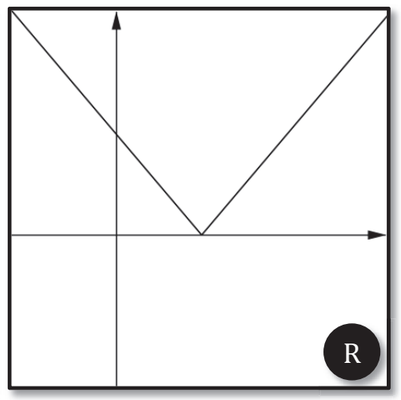


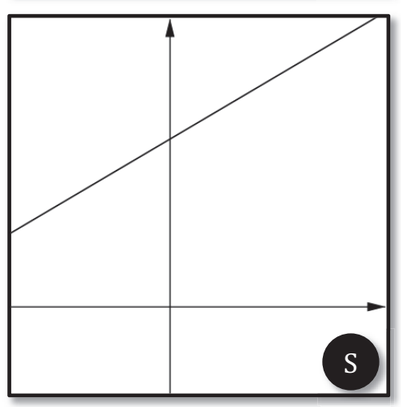


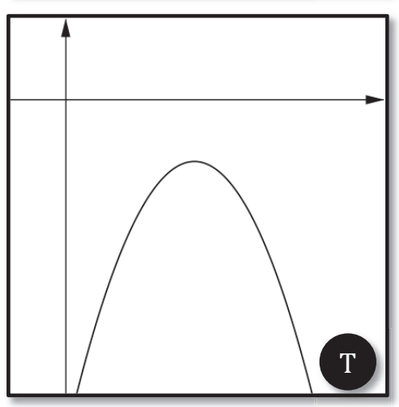


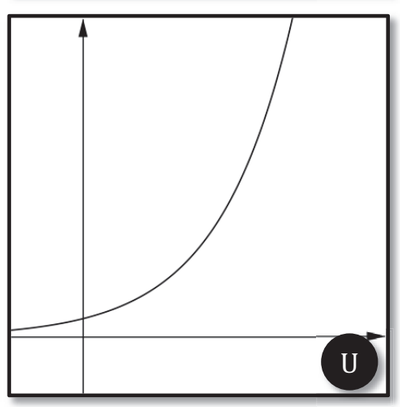


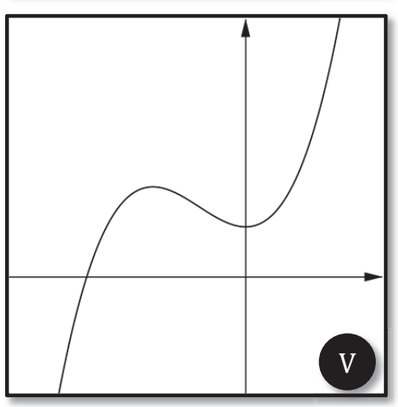


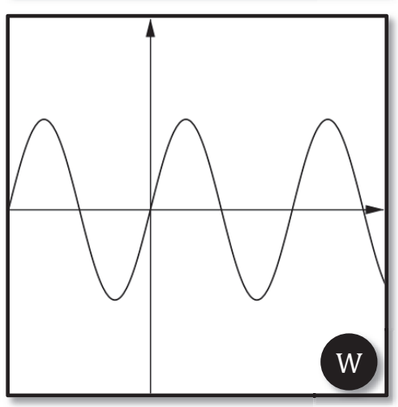


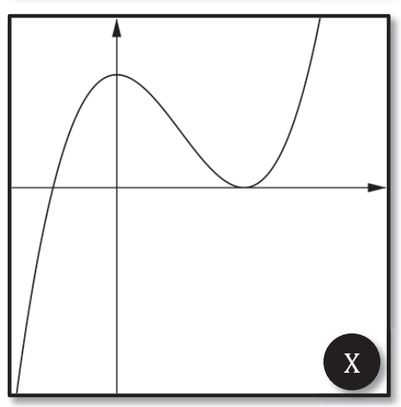












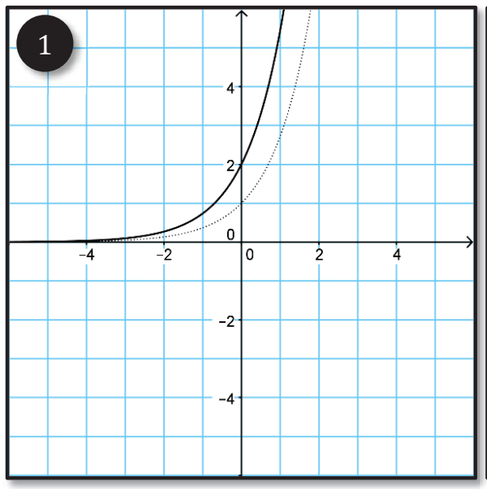
The dotted function in each of the graphs below is f. The graph g is a transformation of f.

	$f(x)$	$g(x)$
1.	$f(x) = e^x$	
2.	$f(x) = x^3 - 3x^2 + 5$	
3.	$y = \ln(x)$	
4.	$f(x) = x^2 - 4x$	
5.	$f(x) = x^3 - 2x^2 - 8x$	

From the graph see can you figure out an equation for each of the $g(x)$ functions.

	$f(x)$	$g(x)$
6.	$f(x) = e^x$	
7.	$f(x) = x^3 - 2x^2 - 8x$	
8.	$f(x) = (x-3)^2 + 4$	
9.	$f(x) = x^3 - 6x^2 + 9x - 5$	
10.	$f(x) = 2 + 4x - x^2$	

1



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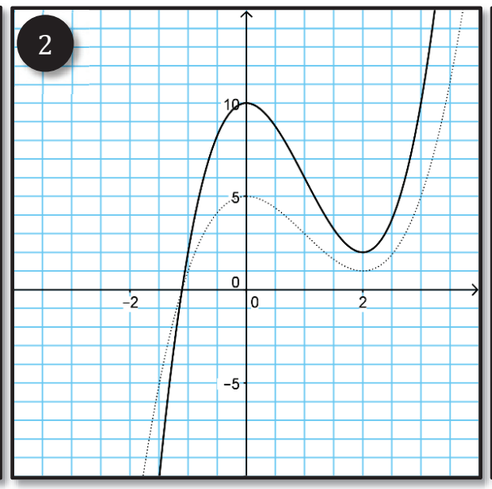

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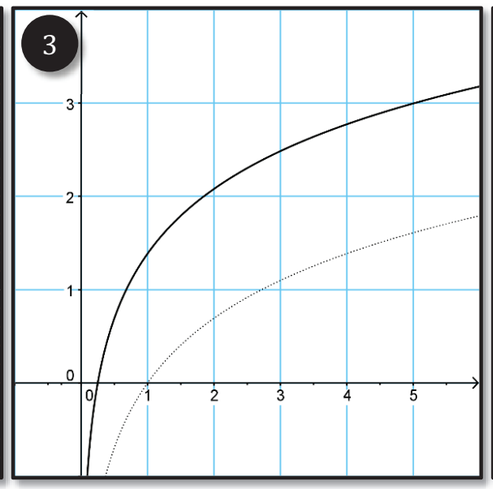

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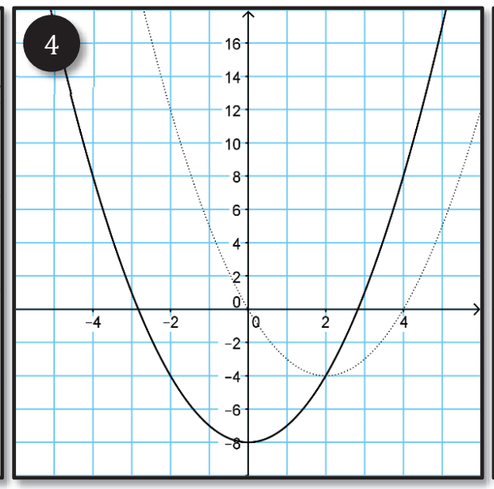

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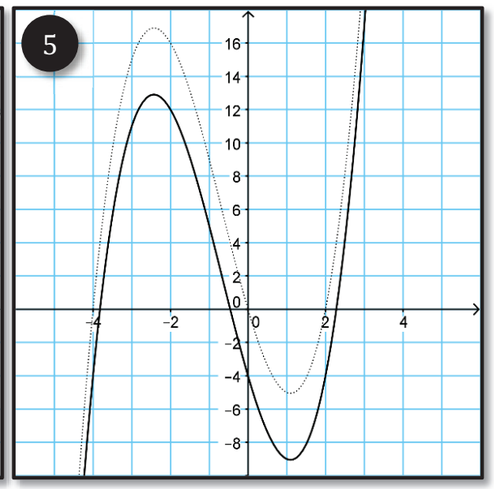

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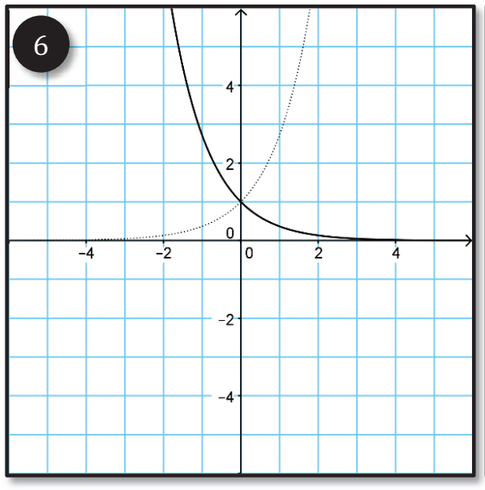

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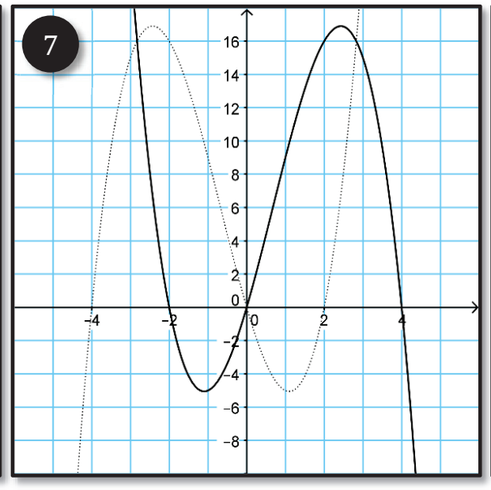

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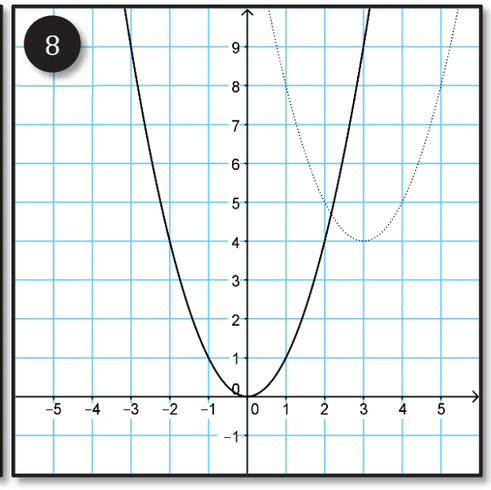

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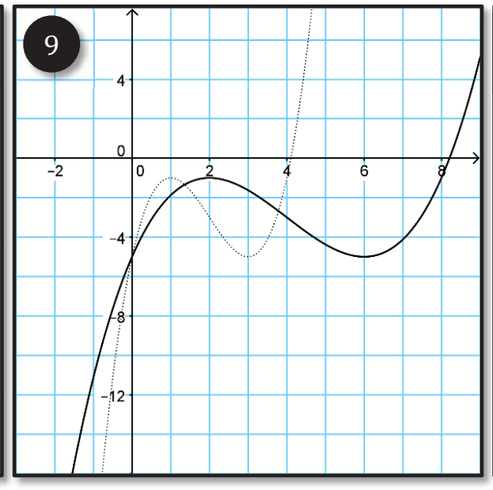

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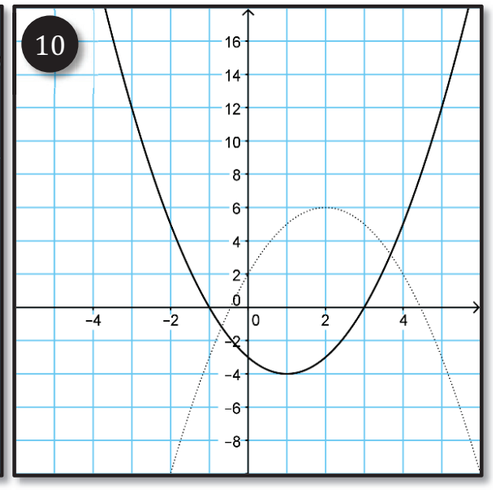

4

x	$y = f(x) - 2$
-2	
-1	
0	
1	
2	

8

x	$y = g(x - 2)$
-2	
0	
2	
4	
6	

3

x	$y = f(x) + 1$
-2	
-1	
0	
1	
2	

7

x	$y = g(x + 1)$
-5	
-3	
-1	
1	
3	

12

x	$y = -g(x)$
-4	
-2	
0	
2	
4	

2

x	$y = \frac{1}{2}f(x)$
-2	
-1	
0	
1	
2	

6

x	$y = g\left(\frac{1}{2}x\right)$
-8	
-4	
0	
4	
8	

11

x	$y = g(-x)$
-4	
-2	
0	
2	
4	

1

x	$y = 2f(x)$
-2	
-1	
0	
1	
2	

5

x	$y = g(2x)$
-2	
-1	
0	
1	
2	

10

x	$y = -f(x)$
-2	
-1	
0	
1	
2	

9

x	$y = f(x)$
-2	
-1	
0	
1	
2	

x	$y = g(x)$
-4	
-2	
0	
2	
4	

x	$y = f(-x)$
-2	
-1	
0	
1	
2	

EXERCISE SET A

Q1 The table below shows the prices charged per room of 40 B & B's houses in Galway.

Race - Week B&B prices per room (€)									
56	75	60	70	80	70	50	90	80	75
75	50	75	50	70	60	65	60	50	70
84	70	70	60	60	70	70	70	40	60
70	80	60	65	55	50	70	80	50	55

(i) Calculate, correct to one decimal place, the mean and standard deviation of the data.
(ii) Show that the empirical rule holds true for 1 standard deviation around the mean.
(iii) Show that the empirical rule holds true for 2 standard deviations around the mean.

Q2. A sweet company claims that 10% of the M & M's it produces are green.
Students found that in a large sample of 500 M & M's 60 were green.

(i) Calculate the margin of error.
(ii) State weather 60 greens from 500 is an unusually high proportion of green M & M's if the claim from the company is assumed to be true.

Q3. Go Fast Airlines provides internal flights in Ireland, short haul flights to Europe and long haul flights to America and Asia. Each month the company carries out a survey among 1000 passengers. The company repeatedly advertises that 70% of their customers are satisfied with their overall service. 664 of the sample stated they were satisfied with the overall service. Would you say that the company were correct in saying that 70% of their customers were satisfied? State the null hypotheses and state your conclusions clearly.

Q4. RTÉ claim that 60% of all viewers watch the Late Late Show every Friday night. An independent survey was carried out on 400 randomly selected viewers to see if the claim were true. The result of the survey was that 180 were watching the Late Late Show.

(i) Calculate the margin of error.
(ii) State the Null and Alternative Hypothesis.
(iii) Would you accept or reject the Null Hypothesis according to this survey? Give a reason for your conclusion.

EXERCISE SET B

Q1. Use the z-tables to find the following probabilities:

(i) $P(Z \leq 1.31)$
(ii) $P(Z \geq 1.32)$
(iii) $P(Z \leq -0.74)$
(iv) $P(-1.32 \leq Z \leq 1.29$

Q2. The amounts due on a mobile phone bill in Ireland are normally distributed with a mean of €53 and a standard deviation of €15. If a monthly phone bill is chosen at random, find the probability that the amount due is between €47 and €74.

Q3. The mean percentage achieved by a student in a statistic exam is 60%. The standard deviation of the exam marks is 10%.

(i) What is the probability that a randomly selected student scores above 80%?
(ii) What is the probability that a randomly selected student scores below 45%?
(iii) What is the probability that a randomly selected student scores between 50% and 75%?
(iv) Suppose you were sitting the exam and you are offered a prize for getting a mark which is greater than 90% of all the other students sitting the exam.
What percentage would you need to get in the exam to win the prize?

Q4. The Sunday Independent reports that the government's approval rating is at 65%. The paper states that the poll is based on a random sample of 972 voters and that the margin of error is 3%.
Show that the pollsters used a 95% level of confidence.

Q5. It is known that 30% of a certain kind of apple seed will germinate. In an experiment 85 out of 300 seeds germinated. Is this considered a significantly poor germination at a 5% level of significance?

Q6. A survey was carried out to find the weekly rental costs of holiday apartments in a certain country. A random sample of 400 apartments was taken. The mean of the sample was €320 and the standard deviation was €50.

Form a 95% confidence interval for the mean weekly rental costs of holiday apartments in that country.

EXERCISE SET C

Q1. A neurologist is testing the effect of a drug on response time by injecting 36 rats with a unit dose of a new drug.

The neurologist measures the response time of each rat to a stimulus.
The neurologist knows that the mean response time for rats not injected is 0.75 seconds.
The mean of the 36 injected rats' response time is 0.6 seconds with a standard deviation of 0.2 seconds.
Can you conclude that the drug has an effect on response time?

Q2. In an examination taken by a large number of students the mean mark was 51.5 and the standard deviation was 8.5. In a random sample of 49 students in a particular town, it was found that among the students in this town the mean mark was 50.

At the 5% level of significance, investigate if there is evidence to conclude that the students of this town did as well as students in general.

Q3. The weights of new-born babies in Ireland is known to have a mean of 3.42 kg and a standard deviation of 0.9 kg. Assuming that the weights are normally distributed, a random sample of 500 babies whose mothers smoked heavily during pregnancy is taken. If the mean weight of this sample is 3.28 kg, can we conclude at the 5% significance that heavy smoking of mothers during pregnancy has an effect on the weight if their babies at birth?

Q4. The mean hourly wage in an EU country is €10. A sample of 35 individuals in the capital city of the country has mean hourly wage of €10.83 with a standard deviation of €3.35 per hour.

(i) Construct a 95% confidence interval for the mean hourly wage in the capital city. Interpret this interval.

(ii) Is their evidence to suggest that hourly wages for workers in the capital city are different from the national hourly wage?
Test the hypothesis using a 5% level of significance.
Clearly state the null and alternative hypotheses and your conclusion.
Give a p-value for this hypothesis test and interpret this p-value.

EXERCISE SET D

Q1 Let A and B be two nonempty sets where $A = \{1\ ,2, 3,4\}$ and $B = \{a, b, c\}$.
Consider each of the following relations:
$T = \{(1, a), (2, b), (2, c), (3, c), (4, b)\}$
$U = \{(1, a), (2, b), (4, b)\}$
$V = \{(1, a), (2, b), (3, c), (4, b)\}$
Which of these relations (T, U and V) qualify as functions?

Q2 **(i)** Although the relation V in **Q1** is a function, it is not a one-to-one (or injective) function. Why?
(ii) V is an onto (surjective) function. Why?
(iii) Does the function f, defined by the relation V, have an inverse

Q3 For each of the relations $\{Q, R, S, T, U, V\}$ below, determine whether the relation is a function. If the relation is a function, determine whether the function is injective and/or surjective.

(i) $A = \{1, 2, 3\}, \quad B = \{a, b, c, d\}$
$Q = \{(1, a), (2, d), (3, b)\}$

(ii) $A = \{1, 2, 3\}, \quad B = \{a, b, c\}$
$R = \{(1, a), (2, b), (3, c)\}$

(iii) $A = \{1, 2, 3\}, \quad B = \{a, b, c\}$
$S = \{(1, a), (2, b), (3, b)\}$

(iv) $A = \{1, 2, 3\}, \quad B = \{a, b, c, d\}$
$T = \{(1, a), (2, b), (2, c), (3, d)\}$

(v) $A = \{1, 2, 3\}, \quad B = \{a, b\}$
$U = \{(1, a), (2, b), (3, b)\}$

(vi) $A = \{1, 2, 3\}, \quad B = \{a, b\}$
$V = \{(1, a), (2, b)\}$

Q4 **(i)** Which of the relations in **Q3** is a bijection?
(ii) For the relation that is a bijection, write down the elements of the inverse function.

Q5. The function $\boldsymbol{f}$ is defined by: $\boldsymbol{f}: \mathbb{R} \to \mathbb{R}: \boldsymbol{x} \mapsto \boldsymbol{x^2 + 2}$.
(i) Give an example to show that f is not injective.
(ii) Give an example to show that f is not surjective.

Q6. The function $\boldsymbol{f}$ is defined by: $\boldsymbol{f}: \mathbb{R} \to \mathbb{R}: \boldsymbol{x} \mapsto \boldsymbol{x^2 - 6x}$.
(i) Give an example to show that f is not injective.
(ii) Give an example to show that f is not surjective.

Q7. For each of the functions below determine which of the properties hold, injective, surjective, bijective. Briefly explain your reasoning.
(i) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$.
(ii) The function $f: \mathbb{R} \to \mathbb{R}^+$ defined by $f(x) = e^x$.
(iii) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (x + 1)x(x - 1)$.
(iv) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (x^2 - 9)(x^2 - 4)$.

EXERCISE SET E

Q1 **(i)** In each part state the natural domain and the range of the given function:

(a) $f(x) = x^2$ **(b)** $g(x) = \ln x$ **(c)** $h(x) = \sqrt{x}$ **(d)** $k(x) = \frac{1}{x}$

(ii) In each part find the natural domain and the range of the given function:

(a) $f(x) = x^2 - 6x + 13$ **(b)** $g(x) = \ln(x + 2)$

(c) $h(x) = \sqrt{1 - x}$ **(d)** $k(x) = \frac{x+2}{x-3}$

(iii) For each of the functions in part **(ii)** that has an inverse, state the domain and range of the inverse function.

EXERCISE SET F

Q1. Which of the following cubic functions have an inverse?
[Hint: Finding the derivative of the function may help!]

(i) $f(x) = x^3 - 6x^2 + 5x + 7$ **(ii)** $f(x) = -x^3 - 6x^2 - 13x + 4$
(iii) $f(x) = x^3 + 3x^2 + 4x + 3$ **(iv)** $f(x) = -x^3 + 3x^2 - x - 1$

Q2. A is the closed interval $[0, 5]$. That is, $A = \{x | 0 \leq x \leq 5, x \in \mathbb{R}\}$.
The function f is defined on A by:

$$\boldsymbol{f: A \to \mathbb{R}: x \mapsto x^3 - 5x^2 + 3x + 5.}$$

(a) Find the maximum and minimum values of f.
(b) State whether f is injective. Give a reason for your answer. [SEC S2014, Q5 P1]

Q3. Consider $f: x \mapsto 2x + 3$.

(a) On the same axes, graph f and its inverse function f^{-1}.
[Hint: Pick two points and reflect them in the line $y = x$. An inverse function is a reflection of the function in the line $y = x$.

(b) Find $f^{-1}(x)$ using **(i)** coordinate geometry and the slope of $f^{-1}(x)$ from **(a)**
(ii) variable interchange.

(c) Check that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$