Algebra

"For effective learning, algebraic thinking must be nurtured in parallel with arithmetic understanding"
Lynn Arthur Steen

Pre-Algebra →	**Understanding Variables →**	**Algebra →**	**Extension**
Number Theory Solid understanding of Number Theory from Strand 3 Useful Methodology: Array Models, T&L on Integers, Fractions & Ratio **1**		**Algebra skills seen as "generalised arithmetic"**. Make an explicit association between symbols and numbers. Use array models and algebra tiles (drawings) to help misconceptions. **3**	"Most of the major principles of algebra and geometry emerge as generalisations of patterns in number and shape"
Patterns **2** **Fostering 'Algebraic Thinking' through exposure to patterns, relationships, generalising and problem solving.** Develop pattern-based thinking - recognise, construct and extend patterns (T&L on Patterns) - use tables to represent a pattern (patterns with unifix cubes) - use patterns to represent real-world situations - develop language to describe patterns precisely, both orally and in writing, as a prelude to using symbols. - use patterns to solve problems (Locker Problem) Deliberate focus on relationships involving two variables - develop an understanding of how one quantity changes as a result of the change in another quantity: $y = mx + c$ - Methodologies: Money Box Problem/ Sunflowers Problem - Students use tables and graphs to represent a relationship - Students introduced to linear relationships, constant rate of change, variables, increasing/decreasing change, slope = rise/run Generalising using symbols - Simplification: Letters employed to reduce the language used to describe patterns. (Doesn't matter what letter/symbol is used) - Students generalise the pattern, using symbols, and make their first formula. The Power of Pattern-Based Thinking: Problem Solving - Patterns and relationships are used to model maths and real-world situations, particularly for solving problems. - Symbols are used to generalise the rule of a pattern observed in a situation. Then that rule can be used to solve the problem. *By doing Patterns first:* *Algebra is seen as the language we use to describe patterns and relationships for the ultimate goal of problem solving.* *Students also get a very good introduction to a variable as a changing quantity.*	**2** "Algebra provides finite ways of managing the infinite." **Variables can be used in 4 different ways:** - A formula like $A = l \times b$ (infinite amount of possibilities) - A Law/Identity like the Commutative Law, $x + y = y + x$ (for all cases) - A Relationship/Rule like $\{(x, y) \mid y = 2x + 3,\ x \epsilon R\}$ (infinite amount of points that fit a rule) - An unknown like $2x = 6$ (one number from an infinite set of possibilities) All of the above can be explored using patterns. **Problem Solving:** Using a variable as an unknown can be introduced and explored through problem solving. Example: For how many days did John need to save in order to accumulate €45 for a new computer game?	**3** **Money Box Problem extended:** We can show adding like terms as part of a real-world problem solving question. For example: 2 family members combining their savings to buy a computer console costing €249 **Skills for Solving Equations:** After Money Box / Sunflowers Problem is used to explain an unknown in context of a real-world problem, extend this to teach the skills for solving equations. Methodology: T&L on Equations, stabilisers **Solving Word Problems using Algebra**: Show that algebra allows choice and flexibility in solving problems. Let students discover that algebra is often the most efficient way to solve a problem, especially word problems. ***Overview of the learning outcome for teaching algebra:*** *The relationship based approach to learning algebra should culminate in students having a deep understanding of algebra which allows easy movement between story, table, graph and equation.* *Learners should also have an appreciation that the power of algebra lies in its capacity to describe relationships for the purpose of problem solving.*	- **Factorising** - Construct some **Perimeter and Area Formulae** using patterns and variables - Discover **theorems** through patterns - Extend rise over run triangle into the formula for slope, then the distance between 2 points. **Co-ordinate Geometry** understood as the marriage of geometry and algebra. - Discover **quadratic, cubic and exponential relationships** through patterns - Look at patterns in **Statistics** - Discover **Trigonometric Ratios** through patterns - Investigate patterns of change in **Periodic and Trigonometric functions** - Rates of change observed in patterns can be extended to change at an instantaneous point in **Calculus.** - Extend patterns and symbols into **Sequences and Series**
Functions Introduce the terms inputs, outputs, a mapping, domain and range. Money Box Problem N → N, Sunflowers Question N → R	Play "Guess the Rule" game.		- Formalise **Functions**