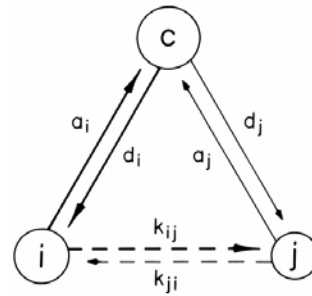


Project Maths Workshop 6



**Exploring Connections & Reasoning
Leading to Proof**

Name: _____

School: _____

Exploring Connections and Reasoning Leading to Proof

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Constructions

Foundation Level

Ordinary Level

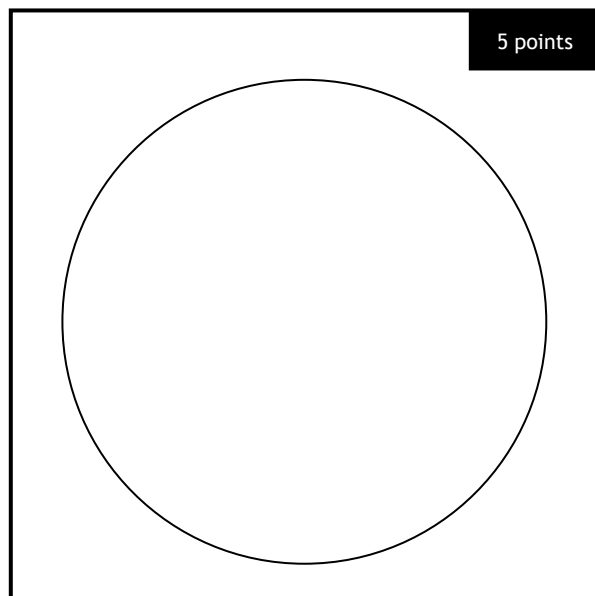
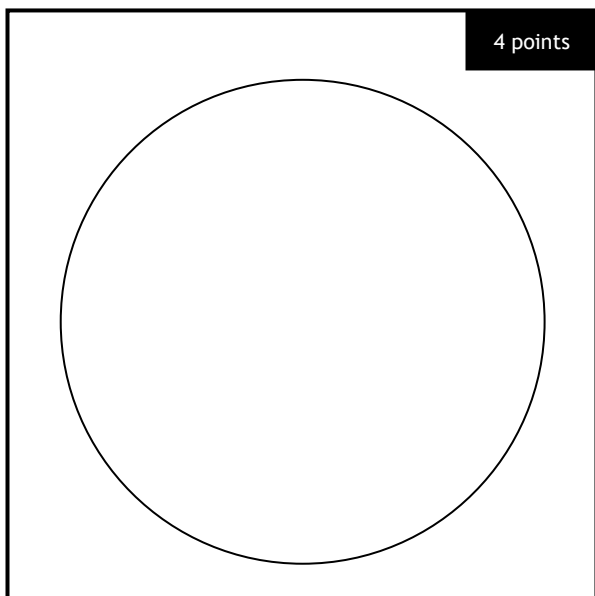
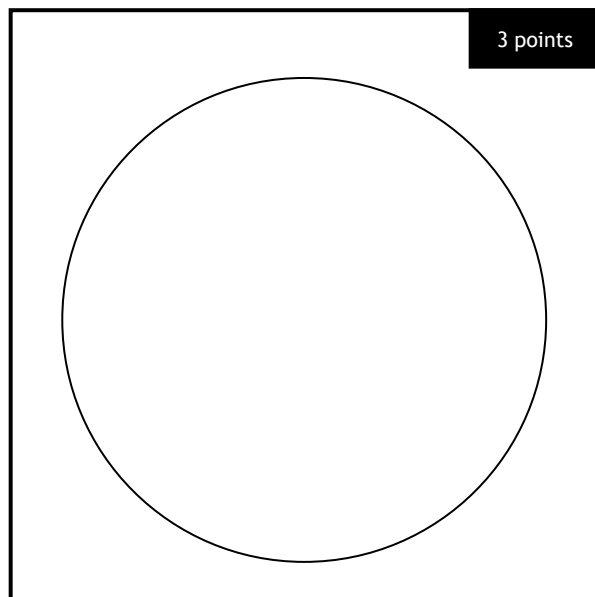
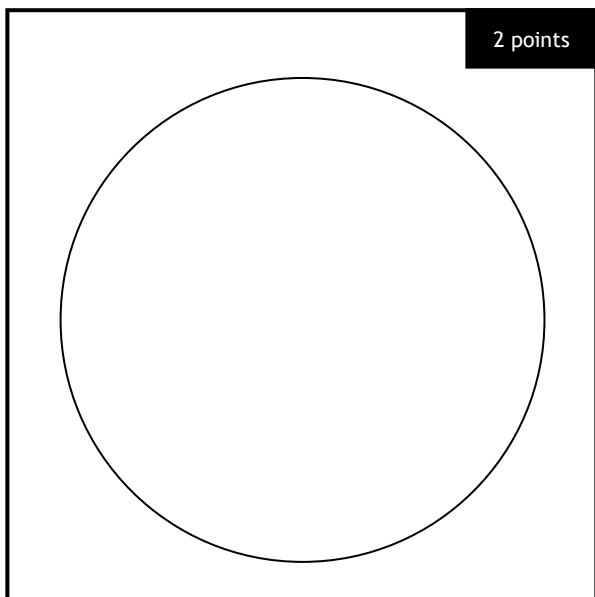
Higher Level

Foundation Level	Ordinary Level	Higher Level
18, 19 & 20	16, 17 & 21	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 & 22
Angle of 60° , without using a protractor or set square.	Circumcentre and circumcircle of a given triangle, using only straight-edge and compass.	Bisector of a given angle, using only compass and straight edge.
Tangent to a given circle at a given point on it.	Incentre and incircle of a given triangle, using only straight-edge and compass	Perpendicular bisector of a segment, using only compass and straight edge
Parallelogram, given the length of the sides and the measure of the angles	Centroid of a triangle.	Line perpendicular to a given line l , passing through a given point not on l .
		Line perpendicular to a given line l , passing through a given point on l .
		Line parallel to given line, through given point.
		Division of a segment into 2, 3 equal segments, without measuring it.
		Division of a segment into any number of equal segments, without measuring it.
		Line segment of given length on a given ray
		Angle of given number of degrees with a given ray as one arm.
		Triangle, given lengths of three sides.
		Triangle, given SAS data.
		Triangle, given ASA data.
		Right-angled triangle, given the length of the hypotenuse and one other side.
		Right-angled triangle, given one side and one of the acute angles (several cases).
		Rectangle, given side lengths
		Orthocentre of a triangle.

**Section A in
2012, 2013 & 2014**

Theorems & Corollaries

Ordinary Level	Higher Level
<p>Investigate Theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 & Corollary 6</p>	<p>Prove Theorems 11, 12 & 13</p>
<p>The angle opposite the greater of two sides is greater than the angle opposite the lesser side.</p>	<p>If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.</p>
<p>(Triangle Inequality). Two sides of a triangle are together greater than the third.</p>	<p>Let $\triangle ABC$ be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio $s : t$, then it also cuts $[AC]$ in the same ratio.</p>
<p>If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.</p>	<p>If two triangles $\triangle ABC$ and $\triangle A'B'C'$ are similar, then their sides are proportional.</p>
<p>Let $\triangle ABC$ be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio $s : t$, then it also cuts $[AC]$ in the same ratio.</p>	
<p>If two triangles $\triangle ABC$ and $\triangle A'B'C'$ are similar, then their sides are proportional,</p>	
<p>For a triangle, base times height does not depend on the choice of base.</p>	
<p>A diagonal of a parallelogram bisects the area.</p>	
<p>The area of a parallelogram is the base by the height.</p>	
<p>(1) Each tangent is perpendicular to the radius that goes to the point of contact. (2) If P lies on the circle s, and a line l is perpendicular to the radius to P, then l is tangent to s.</p>	
<p>(1) The perpendicular from the centre to a chord bisects the chord. (2) The perpendicular bisector of a chord passes through the centre.</p>	



Fill in the following table:

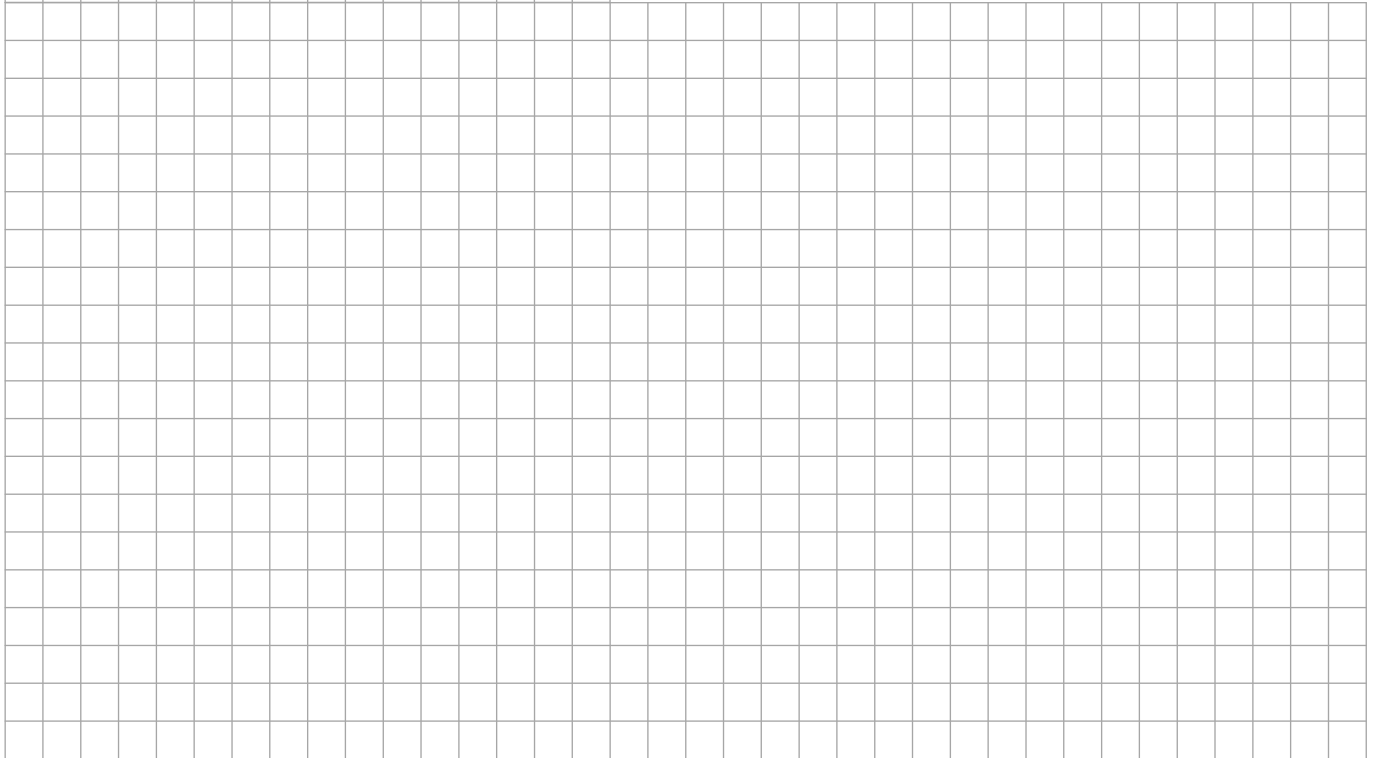
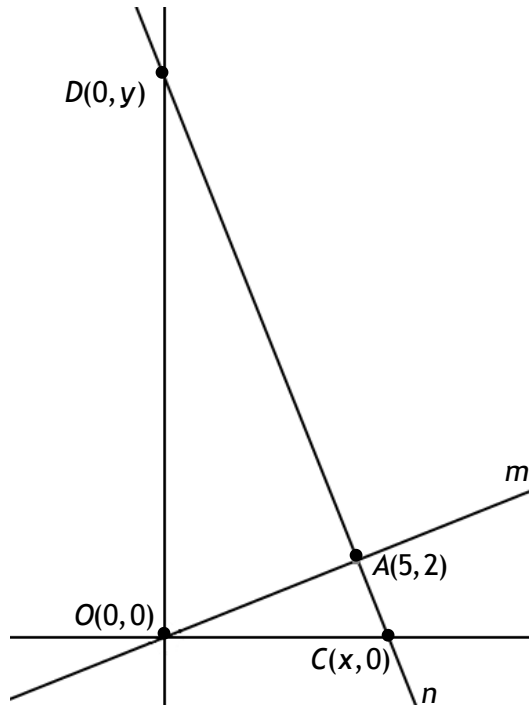
<i>Number of Points</i>	<i>Maximum Number of Regions</i>
2	
3	
4	
5	

WS6.04

Problem to Ponder

Lines m and n are perpendicular.
Line m contains $O(0,0)$.
Lines m and n intersect at $A(5,2)$.

Find x .



This is a Theorem on your course. Tick the box to the title of this theorem.

In an isosceles triangle the angles opposite the equal sides are equal.	
Two sides of a triangle are together greater than the third.	
Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	
If two triangles are similar, then their sides are proportional, in order.	
Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio $m:n$, then it also cuts [AC] in the same ratio.	
The angles in any triangle add to 180° .	

Below are two similar triangles.

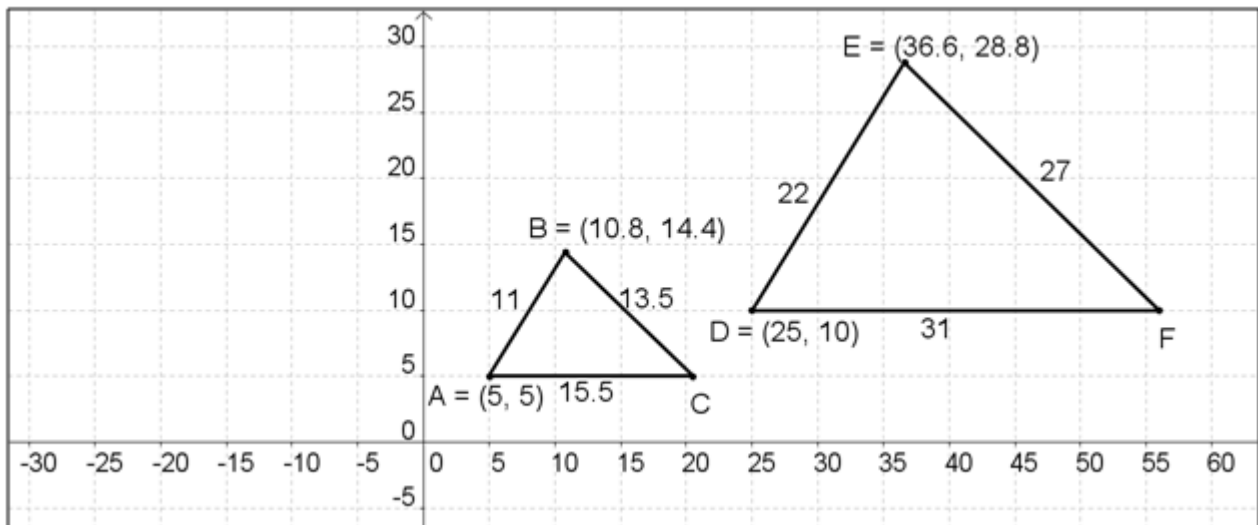
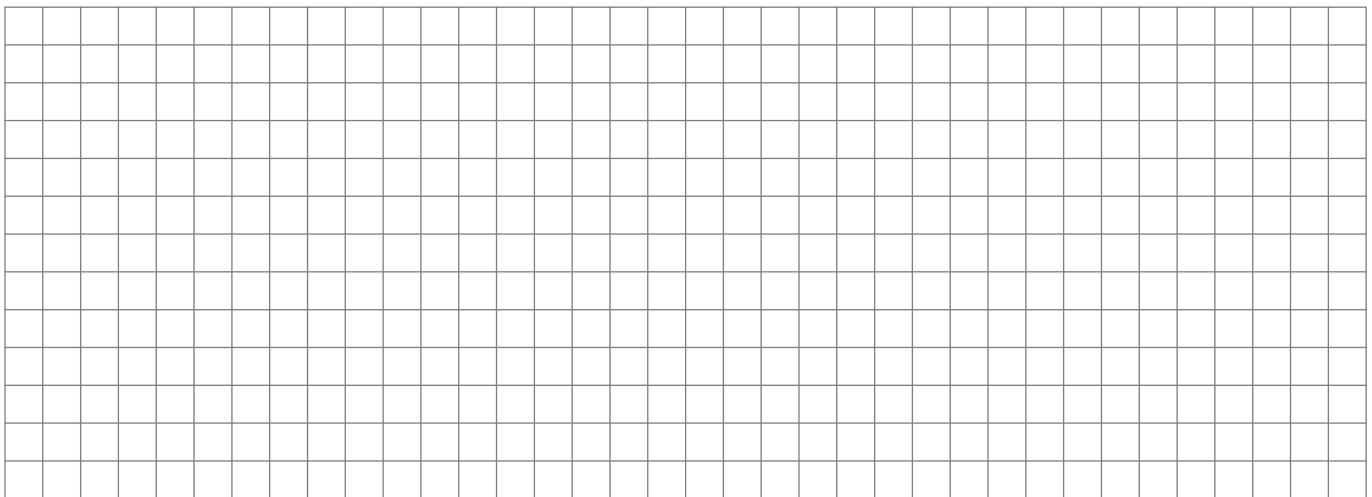


Fig. 6.1

- (i) A and B are the points $(5, 5)$ and $(10.8, 14.4)$ as shown. $|AC|$ is 15.5 cm. Name the co-ordinates of C .

Answer:

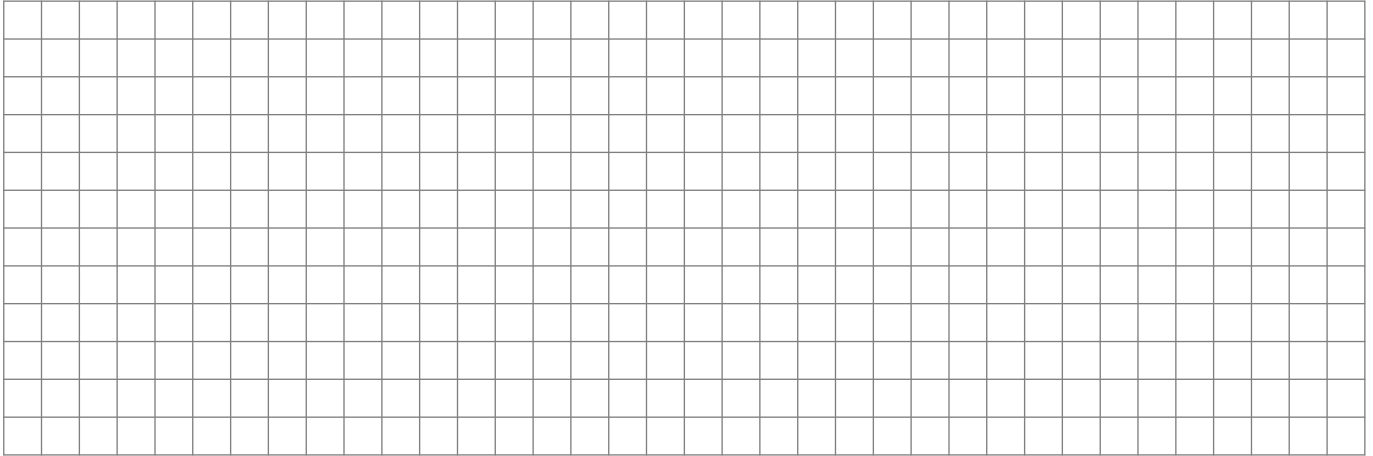
- (ii) Using the length formula, verify that $|AC|$ is 15.5 cm.



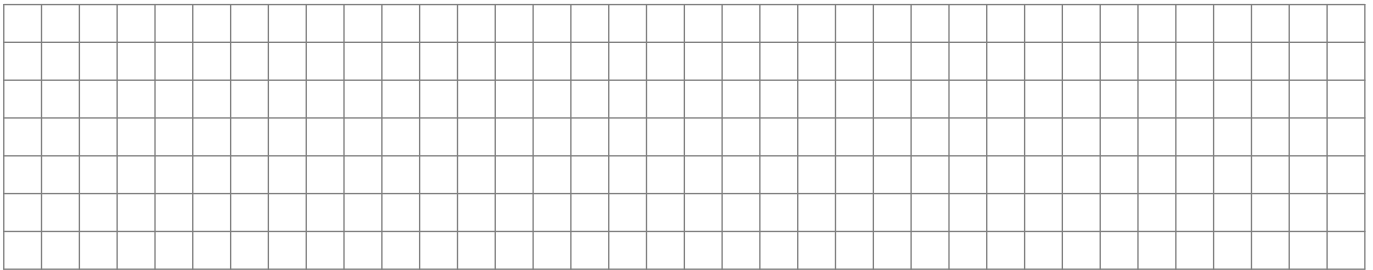
- (iii) D and E are the points $(25, 10)$ and $(36.6, 28.8)$ as shown. $|DF|$ is 31 cm. Name the co-ordinates of F .

Answer:

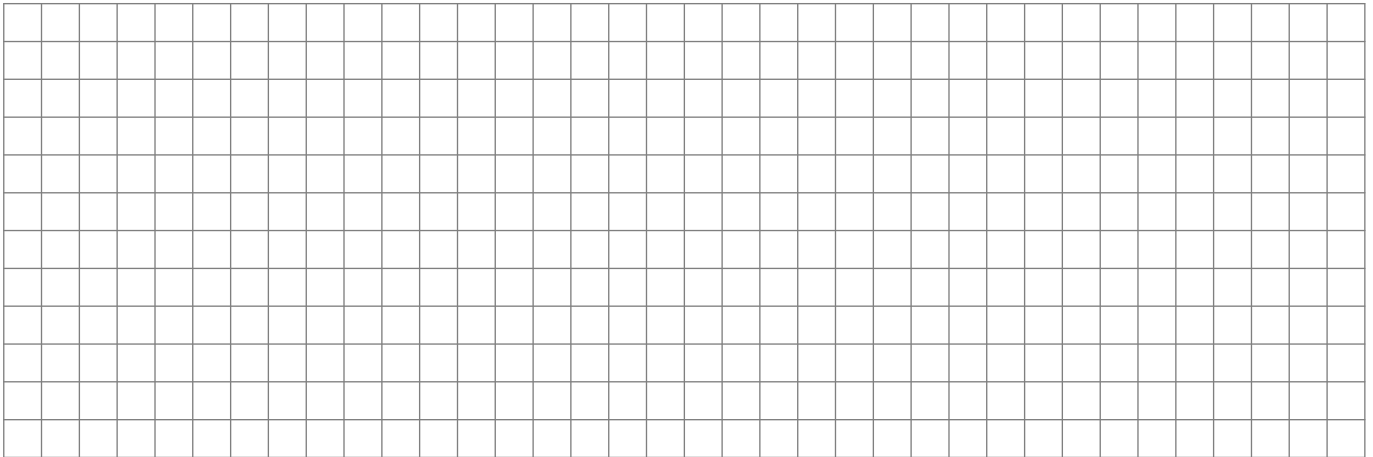
(viii) By using an alternative method to the one above, find the coordinates of the centre of enlargement.



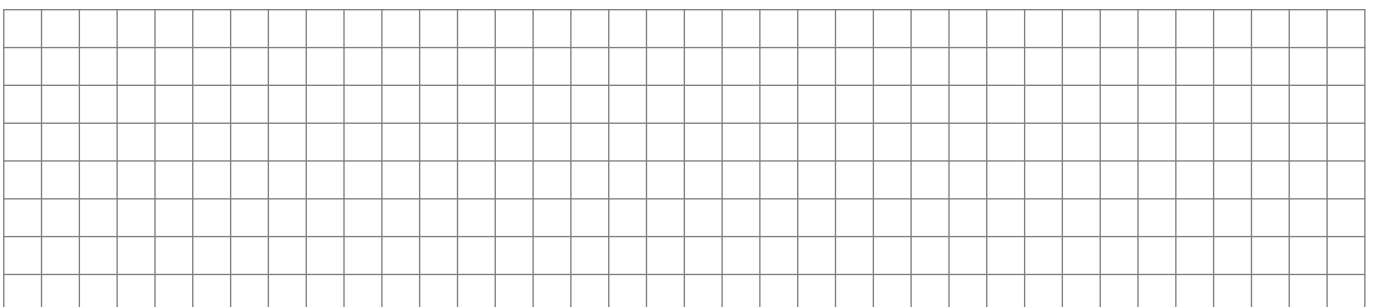
Would it be possible to always use this alternative method for finding the centre of enlargement? Explain.



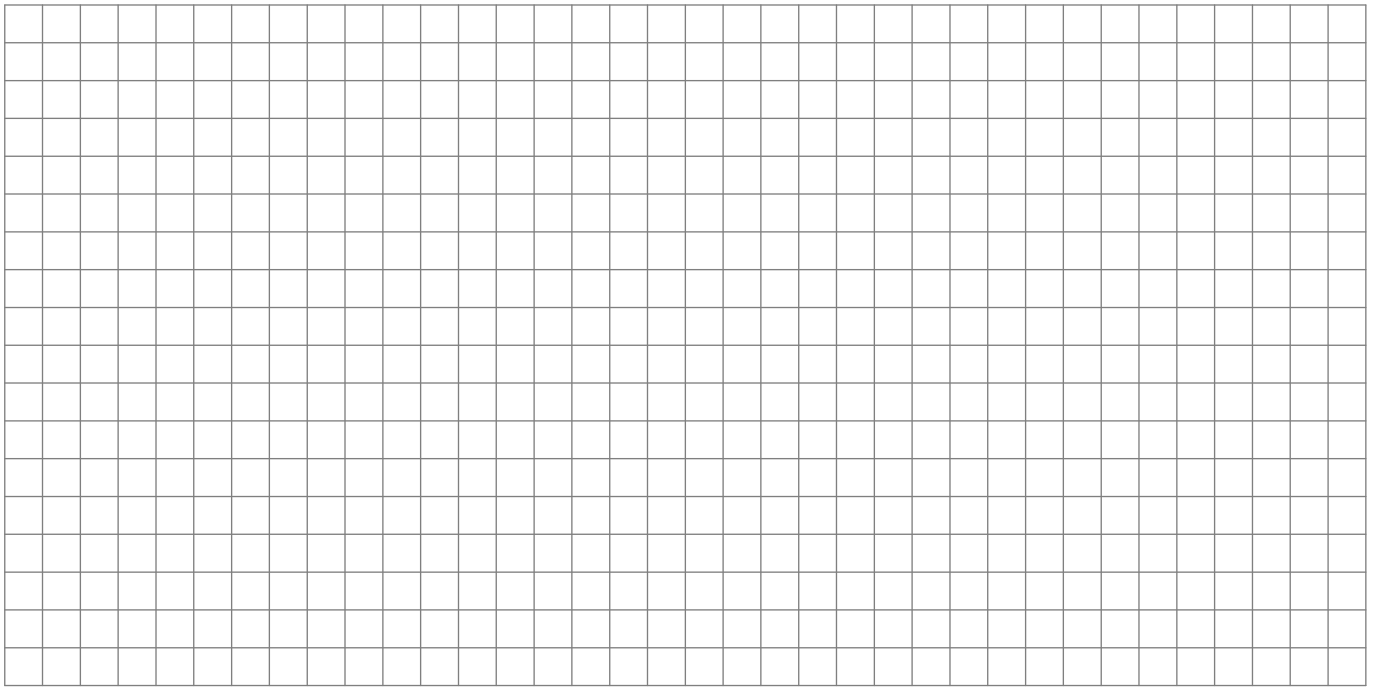
(ix) Find $|\angle ABC|$ using trigonometry.



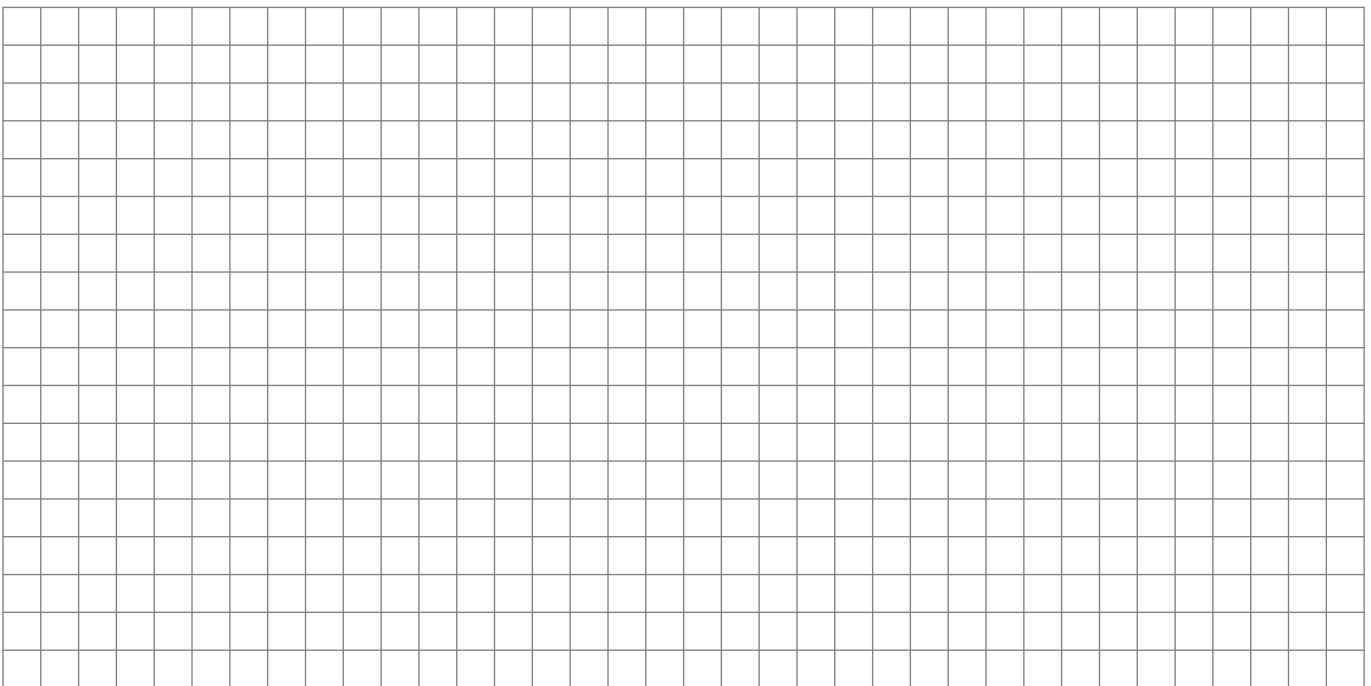
(x) Is there an angle bigger than $|\angle ACB|$ in $\triangle ABC$? Give a reason for your answer. (The use of protractors is not allowed.)



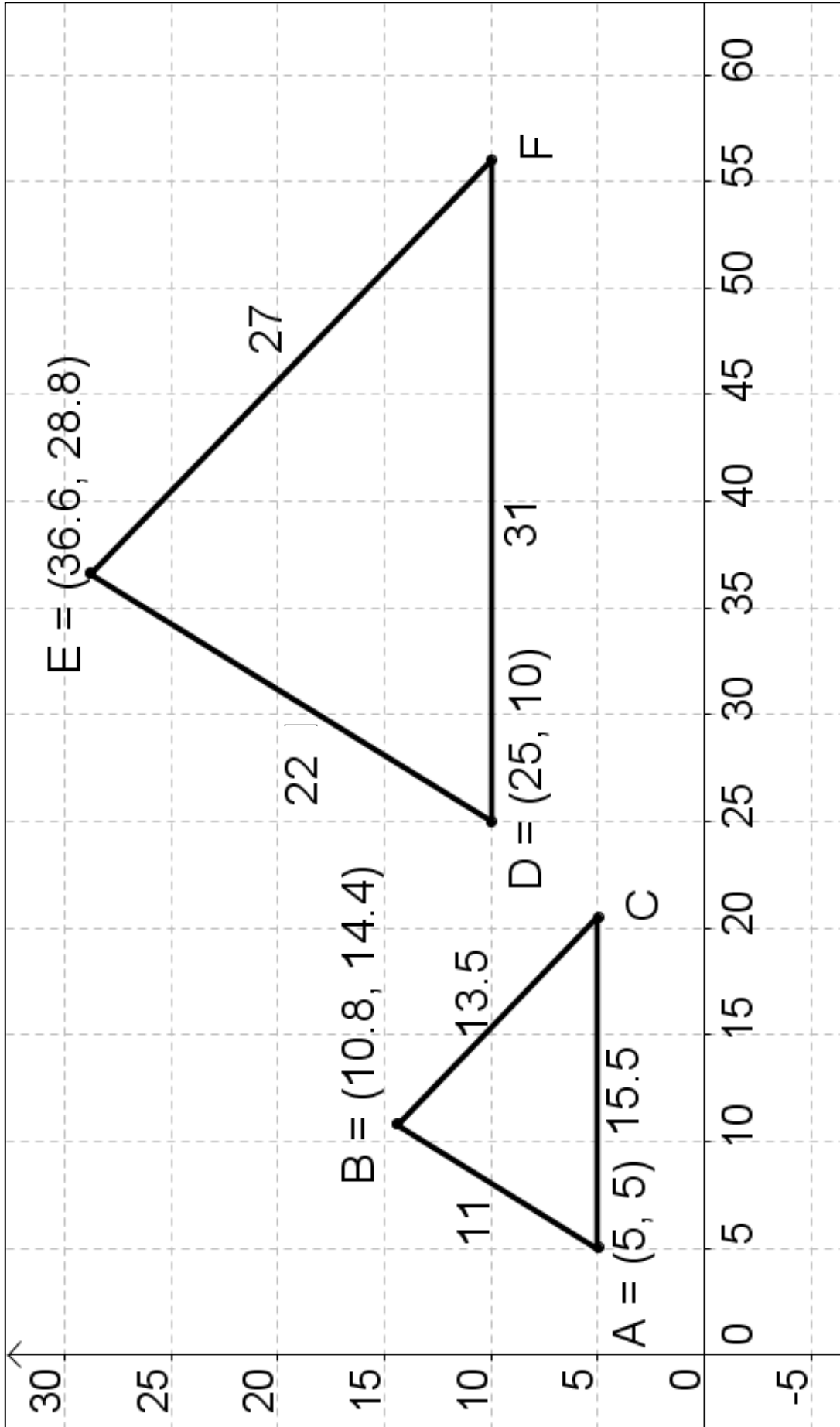
(xiv) Find the area of $\triangle ABC$ by using formula 3.



(xv) Find the area of $\triangle DEF$.



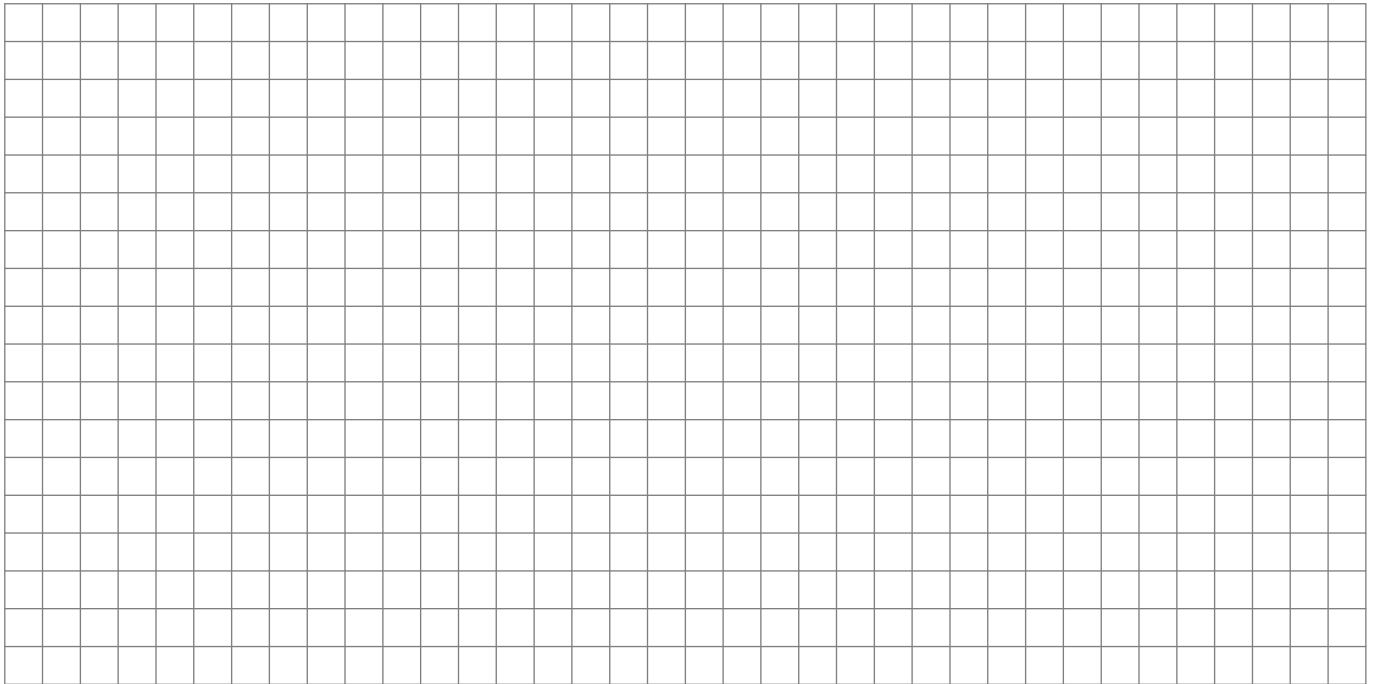
- (xiv) Using the diagram below construct the centroid (S) of $\triangle ABC$ and construct the centroid (T) of $\triangle DEF$.



(xvii) The centroid of a triangle can be calculated using the following formula:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right).$$

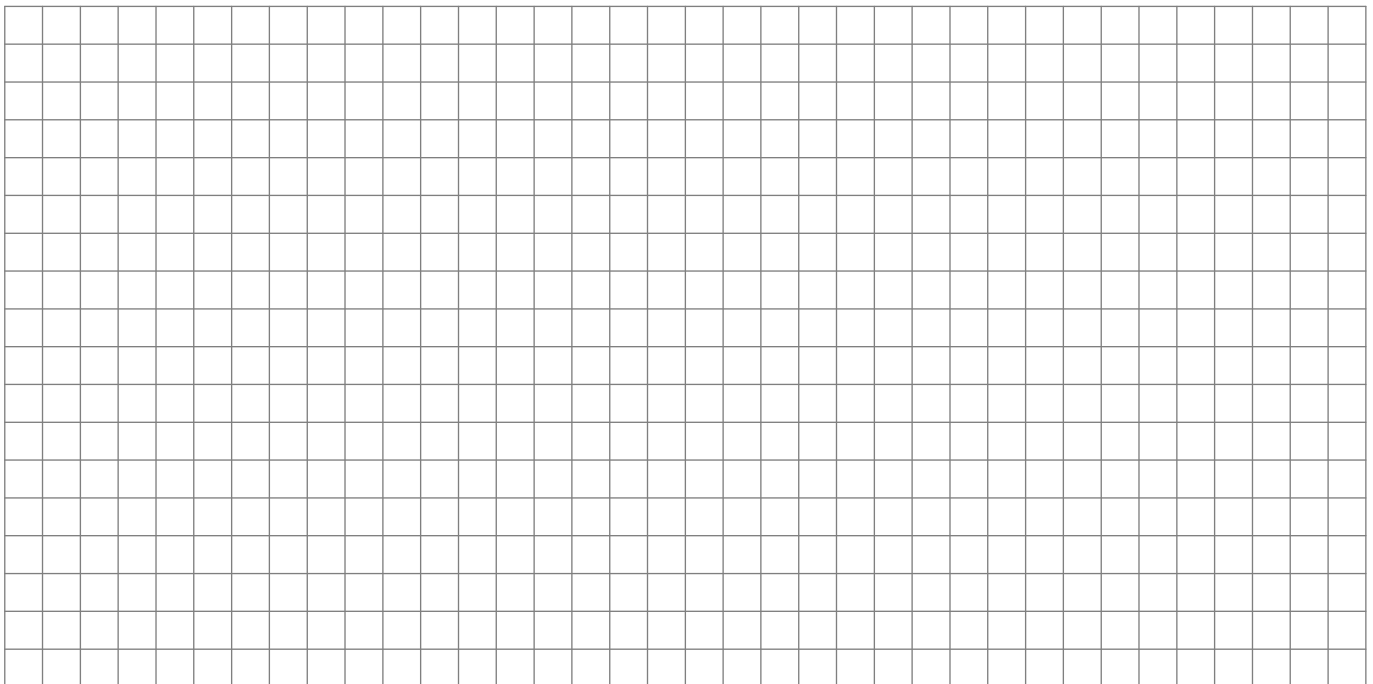
Calculate the centroid for each triangle, correct to 1 d.p.



Centroid of $\triangle ABC$: $S(\quad , \quad)$

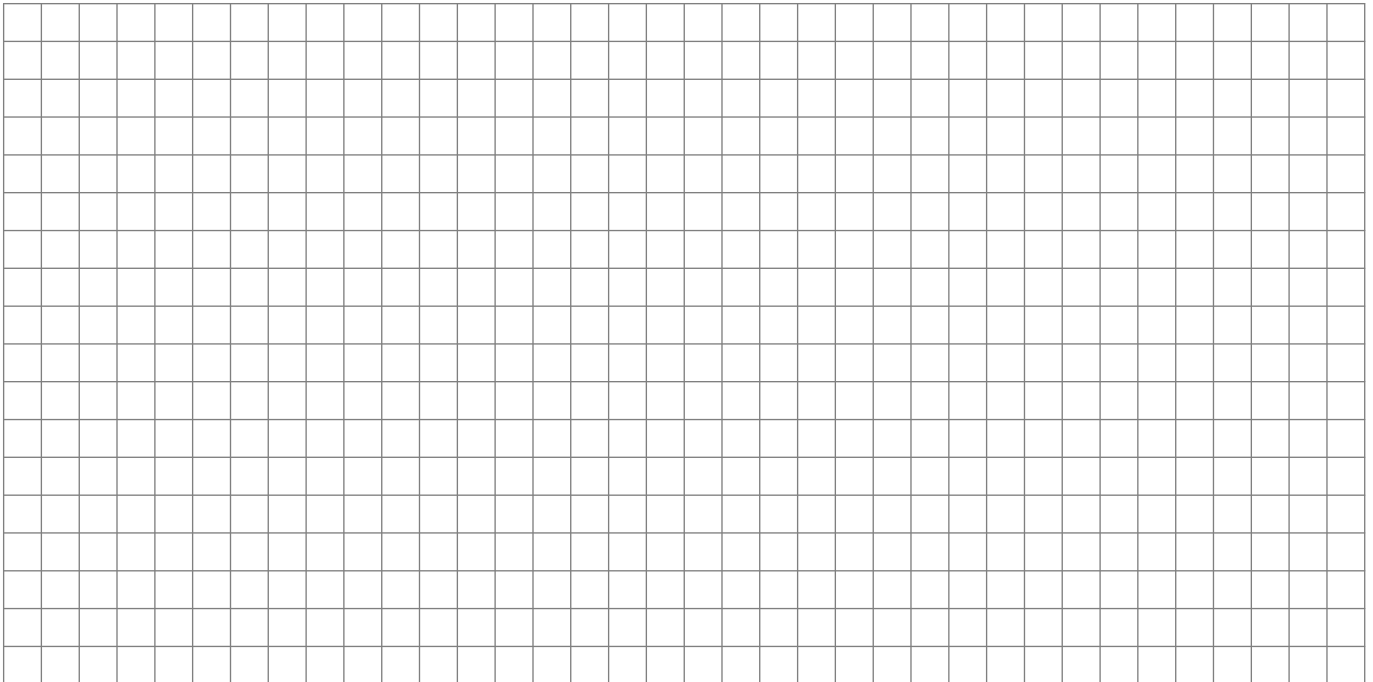
Centroid of $\triangle DEF$: $T(\quad , \quad)$

(xviii) If 2 triangles are similar, then the ratio of 2 corresponding lengths is equal to the scale factor. Show that this statement is true by calculating $|AS|$ and $|DT|$, correct to 1 d.p.

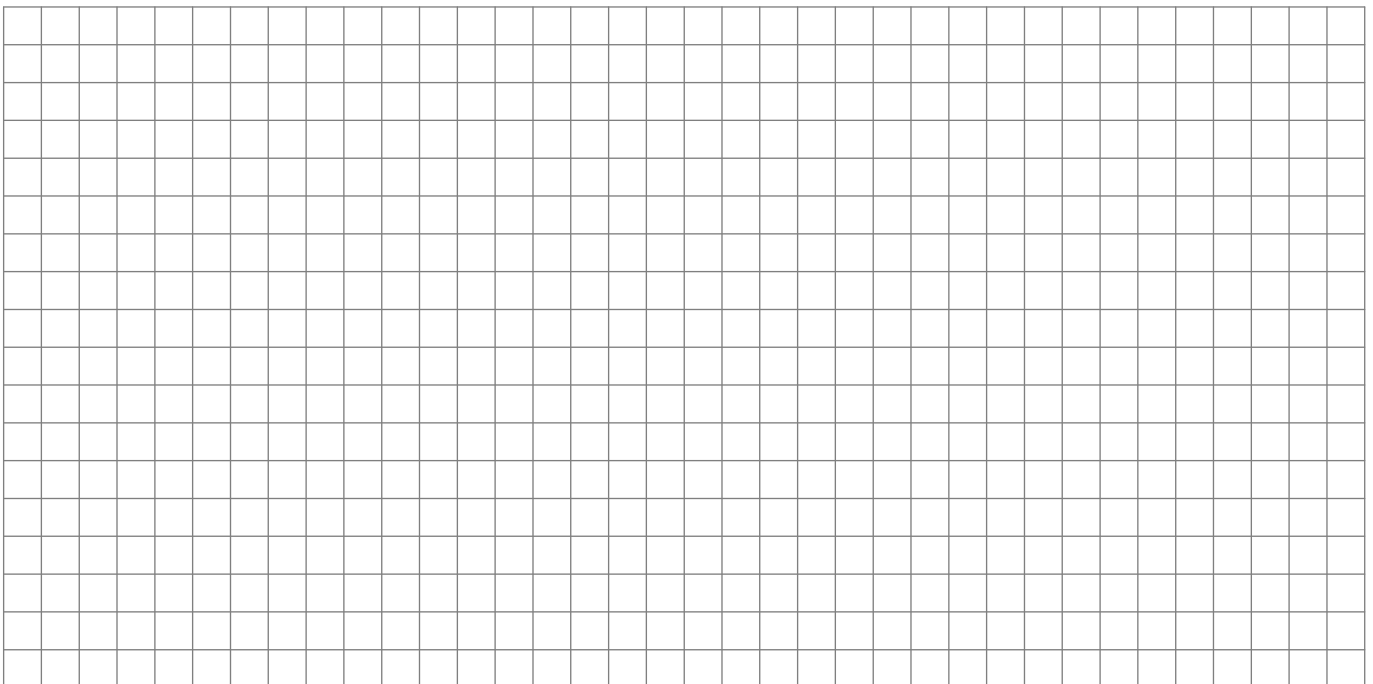


(xx) *LC(HL) Verify, using the above property, that the centroid of a triangle formula is as follows:

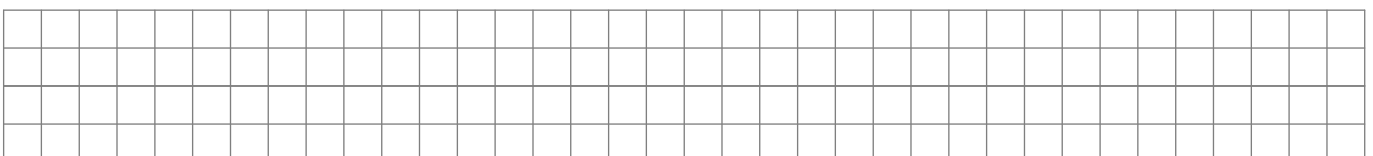
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$



(xxi) *LC (HL) Show that point A (5, 5) divides $[PD]$ internally in the ratio 1:1.



Do you notice any other points dividing a line segment in the ratio 1:1?
If so, name them.



Task 2

In a different remote area of Australia the Royal Flying Doctor Service has an aircraft base located at *P*, another aircraft base located at *Q* and another base located at *S*.

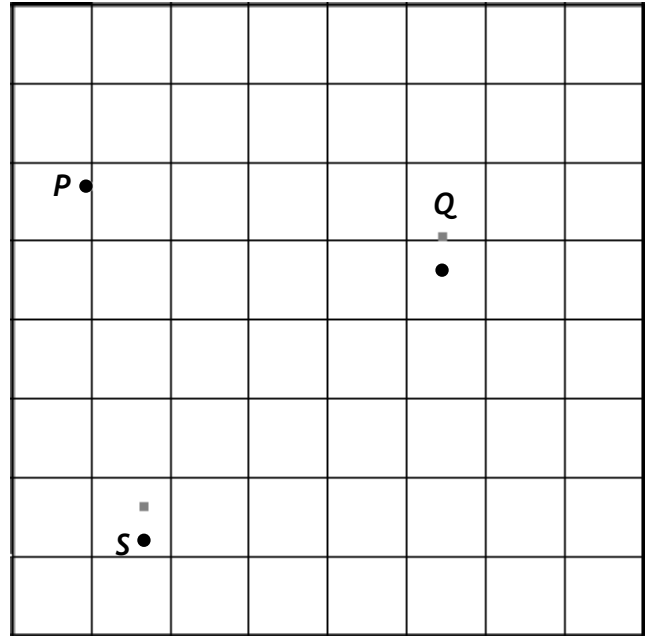
All emergency calls are received at a central call centre and are then transferred to the closest aircraft base.

This map of the area shows the position of the three aircraft bases.

You need to divide the area into three regions so that any emergency is responded to from the nearest aircraft base.

The scale on the map is

“grid square side = 20 km”.



(i) What three lines have you drawn?

(ii) Have these three lines intersected at a common point?

(iii) What do you notice about the distances from the point of intersection to each aircraft base.

(iv) What does the above result indicate to you?

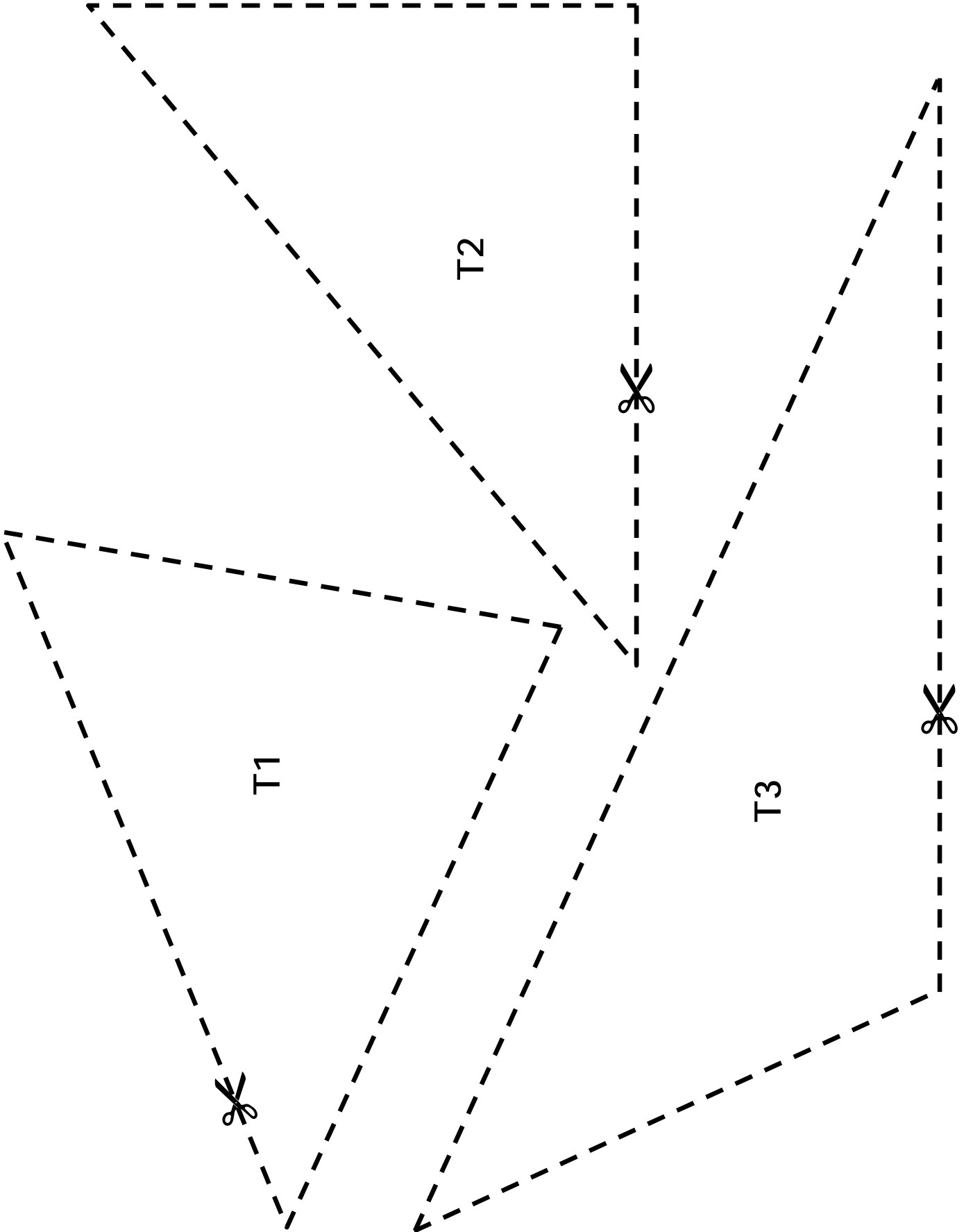
(v) What is the approximate area of each region?

P: *Q*: *S*:

(vi) The number of emergency calls is consistent for all of the area and you have 168 people overall to staff the three bases. How many staff members do you need at each aircraft base.

P: *Q*: *S*:

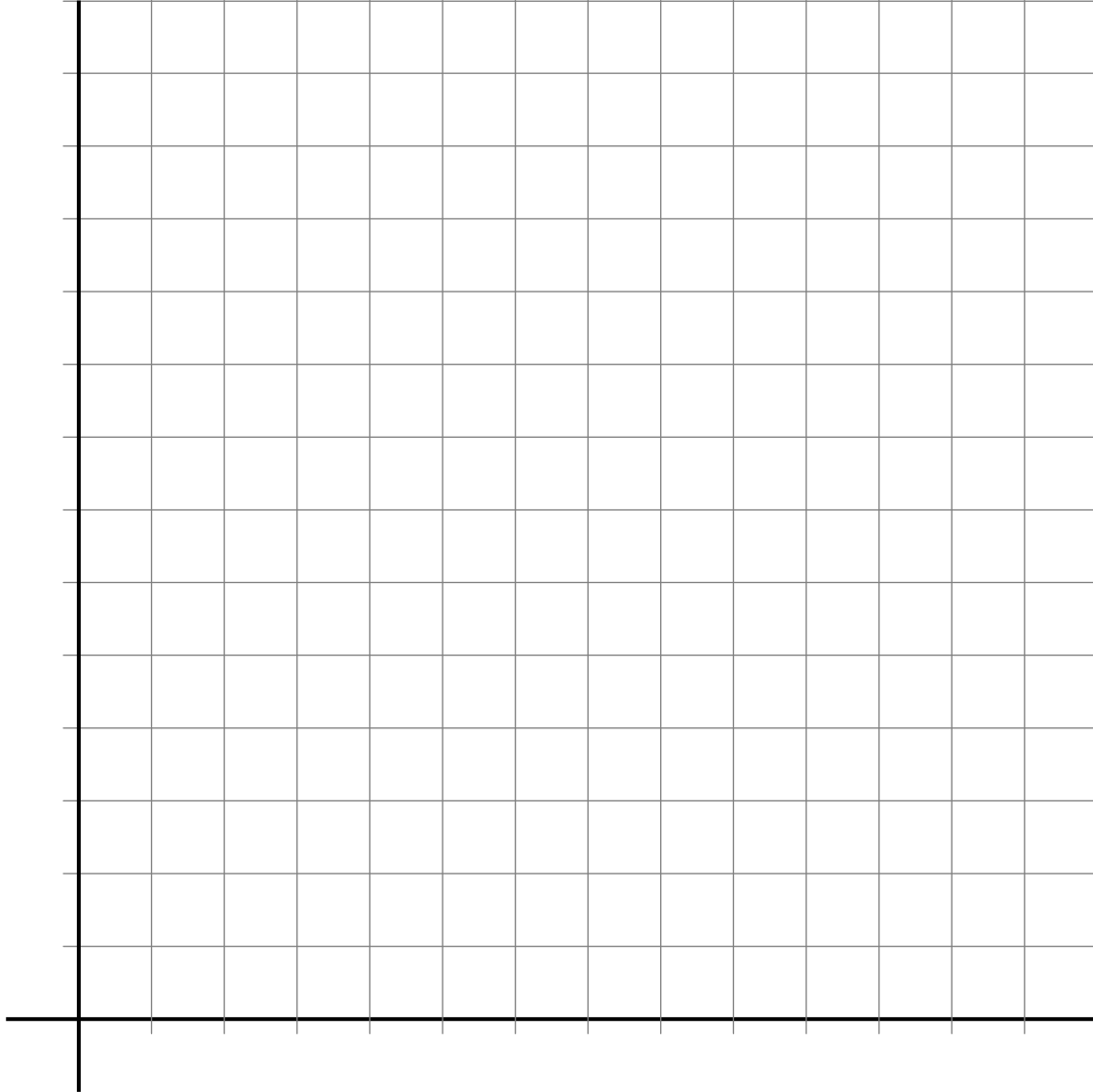
Triangle Cutouts for Task 3



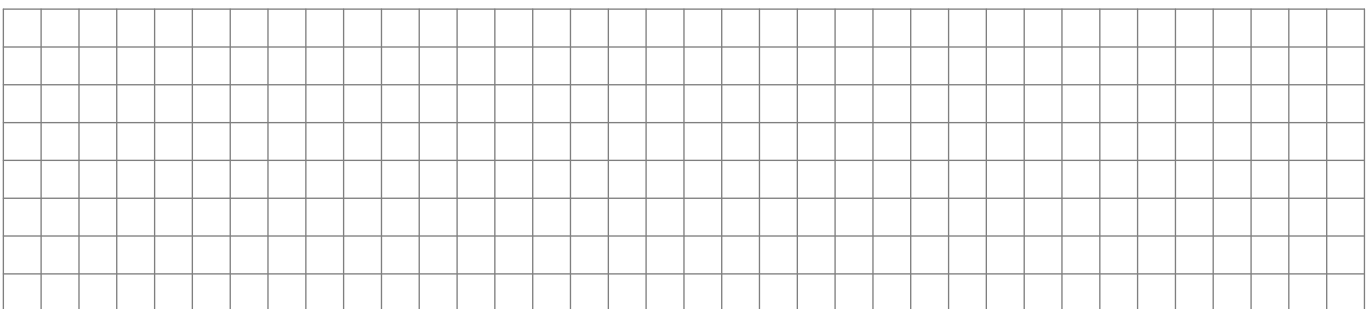
Task 6

In an underdeveloped region an Irish aid organisation intends to build a medical clinic to service three villages located at grid coordinates (0,4), (5,1) and (1,1). Each village is connected to the other by a straight road.

What should the grid coordinates for the clinic be in order to locate it equidistant from all three villages?



Is this the best location for the clinic? Give a reason for your answer.



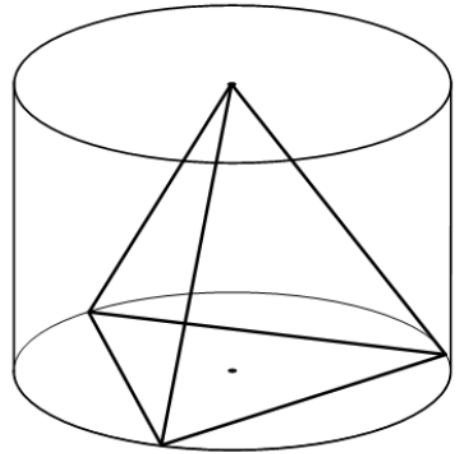
Task 7

A regular tetrahedron has four faces, each of which is an equilateral triangle.

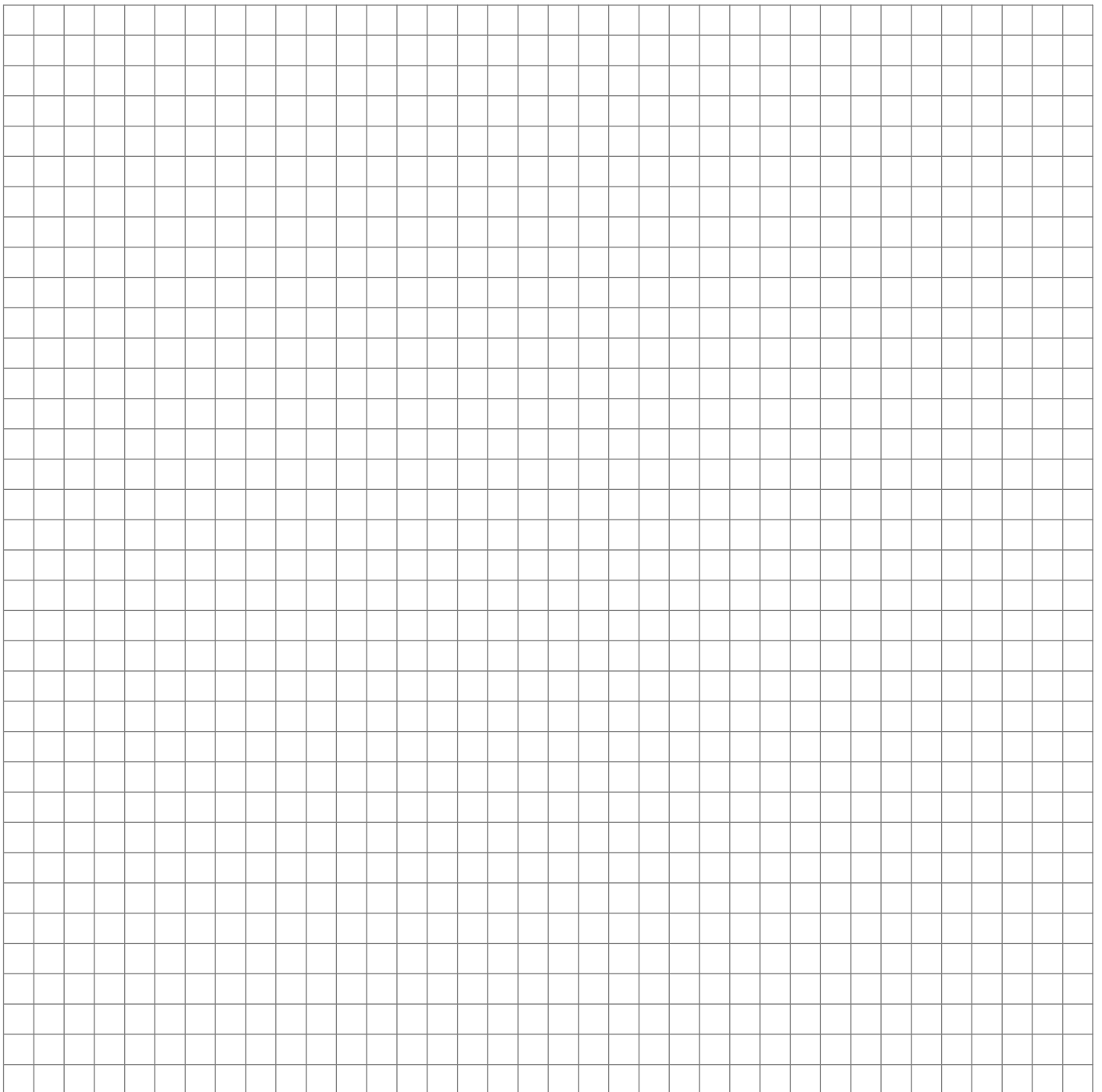
A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron.

The length of one edge of the tetrahedron is $2a$.

The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom and the apex just touching the top.



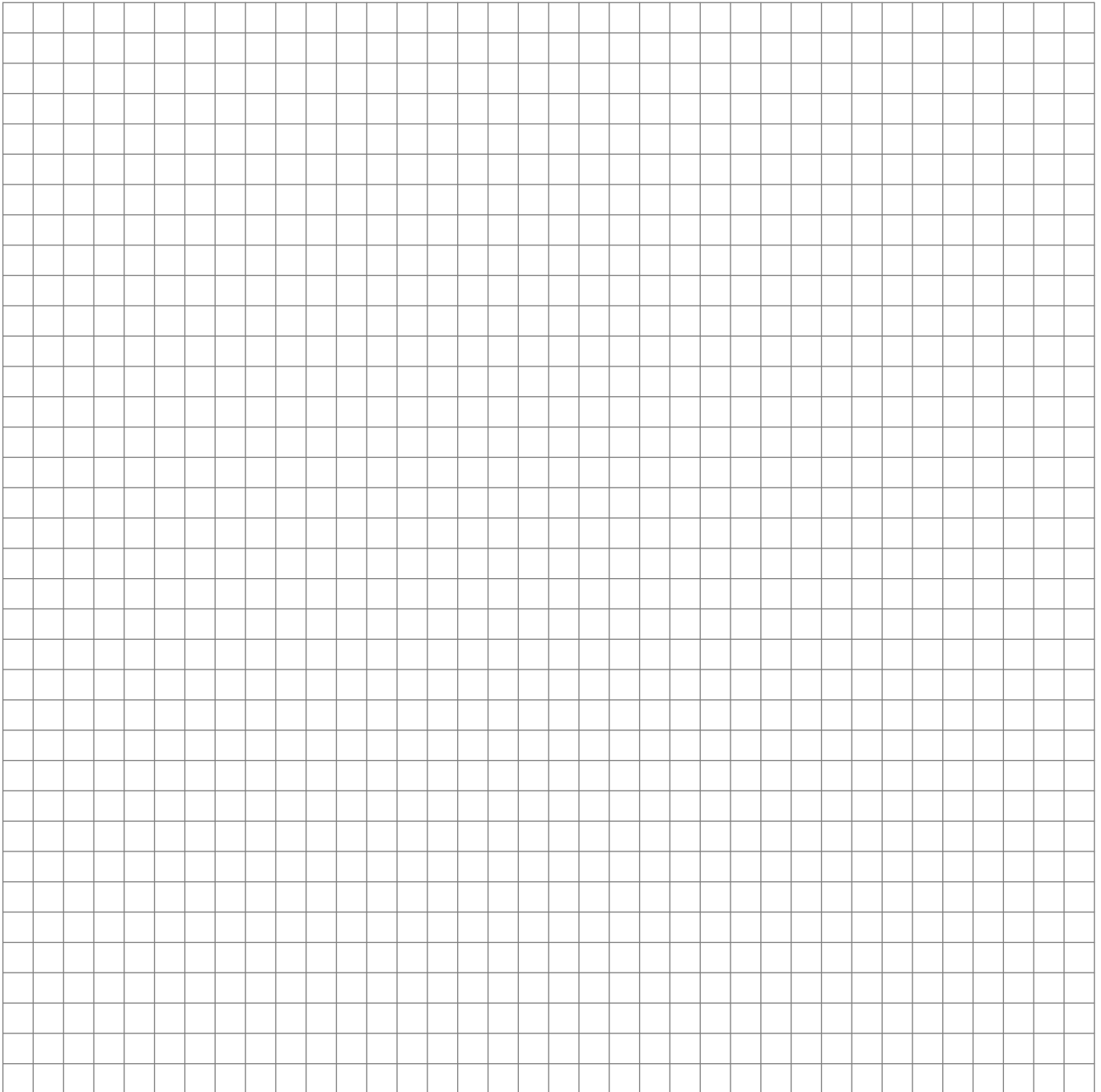
If the volume of the cylindrical container is $\frac{8\sqrt{6}}{9}\pi a^3$ find the height of the puzzle.



Task 8

In a public park the gardener measures the distances between the only three trees using a measuring tape. The distances are 31.83 m, 25.75 m, and 29.27 m. He wishes to fence off a circular play area in which to place swings and have the three trees as part of the perimeter of that play area.

What area of the park will he be fencing off?



Strand 1: Geometry and Trigonometry

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
2.1 Synthetic geometry *	– perform constructions 18, 19, 20 (see <i>Geometry Course for Post-primary School Mathematics</i>)	– perform constructions 16, 17, 21 (see <i>Geometry Course for Post-primary School Mathematics</i>) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom,	– perform constructions 1-15 and 22 (see <i>Geometry Course for Post-primary School Mathematics</i>) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems 11, 12, 13,
2.2 Co-ordinate geometry	– use slopes to show that lines are <ul style="list-style-type: none"> • parallel • perpendicular 	– recognise the fact that the relationships $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$ are linear – solve problems involving slopes of lines – recognise that $(x - h)^2 + (y - k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle centre (h, k) and radius r – solve problems involving a line and a circle with centre (0, 0)	– prove theorems 11, 12, 13, <i>Geometry for Post-primary School Mathematics</i> may the proof of Pythagoras – find the perpendicular distance from a point to a line • the angle between two lines – divide a line segment in a given ratio m:n – recognise that $x^2 + y^2 + 2gx + 2fy + c = 0$ represents the relationship between the x and y co-ordinates of points on a circle centre (-g, -f) and radius r where $r = \sqrt{g^2 + f^2 - c}$ – solve problems involving a line and a circle

Note: This page refers to the syllabus in operation at the time. The syllabus has since changed and no longer offers students a choice in geometry. Please consult the current syllabus!

* In the examination, candidates will have the option of answering a question on the synthetic geometry set out here, or answering a problem-solving question based on the geometrical results from the corresponding syllabus level at Junior Certificate. This option will apply for a three year period only, for candidates sitting the Leaving Certificate examination in 2012, 2013 and 2014. There will be no choice after that stage.

Appendix B – Terms in Logic and Deductive Reasoning (Syllabus pg. 22)

Ordinary Level

Theorem:	A theorem is a statement which has been proved to be true.
Proof:	A proof is a sequence of statements (made up of axioms, assumptions and arguments) leading to the establishment of the truth of one final statement.
Axiom:	An axiom is a statement which is assumed to be true and is used as a basis for developing a system. Example: Axiom 1 - There is exactly one line through any two given points.
Corollary:	A corollary follows after a theorem and is a proposition which must be true because of that theorem. Example: Corollary 6 - This corollary follows Theorem 20 and states: "If two circles share a common tangent line at one point, then the centres and that point are collinear".
Converse:	The converse of a theorem is formed by taking the conclusion as the starting point and having the starting point as the conclusion. Example: The converse of Theorem 2 states "If two angles are equal, then the triangle is isosceles".
Implies:	Implies indicates a logical relationship between two statements, such that if the first is true then the second must be true.

Higher Level

Is equivalent to:	Two things are said to be equivalent if they have the same value but different forms.
If and only if:	Often shortened to "iff". The truth of either one of the connected statements implies the truth of the other. i.e. they both true or both false.
Proof by contradiction:	A proof by contradiction is a proof in which one assumption is made. Then, by using valid arguments a statement is arrived at which is clearly false, so the original assumption must have been false.

Feedback from the SEC on the 2011 Leaving Certificate Examinations for Phase 2 of Project Maths in the Initial Schools

Introduction

This document has been prepared by the Chief Examiner for Leaving Certificate Mathematics 2011, based on the observations of the Examiners, Advising Examiners, and Chief Advising Examiners. Its purpose is to provide feedback to the *Project Maths Development Team* (the support service) and to the NCCA, in order to assist in their ongoing work with teachers.

This document is not intended to be a comprehensive Chief Examiner's report. Rather, it is a distillation of some observations and recommendations that may assist teachers who, in tandem with wanting to provide the best possible learning experiences for their students, will also naturally want to ensure that those students are in a position to demonstrate their competencies to the best effect in the examinations, and thereby gain grades that fully reflect their achievements.

General observations

- At all levels, candidates were better prepared for the material testing Strands 1 and 2 than their counterparts in 2010. This is welcome, and indicates that as teachers and students become more familiar with the syllabus and its requirements, the achievement of the learning outcomes is likely to improve.
- Candidates were less well able to handle the material on Strands 3 and 4 (introduced in Phase 2) than the more established material in Strands 1 and 2 (introduced in phase 1).
- There was a good deal of variation between centres in the quality of response to certain types of questions, such as questions that one might expect to be answered well by people who are used to discussion and exploration. This seems to indicate that some groups of candidates engage in such activities much more often than others.
- While many of the questions on these papers had a significantly greater amount of language than is the case with the examinations on the previous syllabus, it was clear that the vast majority of candidates at all three levels were able to read the questions and understand what was being asked of them, even in cases where they were not able to answer correctly. There was little if any evidence of language issues preventing candidates from engaging with tasks.
- With regard to questions that required text-based responses, candidates seemed better able to describe, evaluate, and draw conclusions than was the case in 2010. For example, many candidates at Ordinary Level were able, given some statistical charts, to draw reasonable inferences about what message the authors of those charts had intended to convey. [P2, Q7(b).] This was the case even though at least one of those charts was of a type that the candidates were unlikely to have seen before. At Higher Level, many candidates were able to explain what it means to say that “correlation does not imply causality”. [P2, Q2(a).] Of those who did not, almost all gave a reasonable explanation of what “correlation” is. Many candidates also showed the ability to give a reasonable evaluation of another person's inferences from data. [P2, Q7(b)(i).]
- At all levels, questions that required understanding of concepts caused considerably more difficulty than those testing the execution of routine skills presented in a familiar way.
- Many candidates had difficulty coping with questions that required the application of their knowledge and skills in a different context from the one in which those skills were developed.

This is not surprising, as it has always presented a difficulty for candidates in mathematics examinations, and is one of the most challenging aspects of mathematics education.

- A number of deficiencies were evident in relation to basic knowledge and understanding of mathematical terms and concepts that should be part of the normal discourse of the mathematics classroom, and without which a proper understanding of the material being studied is impossible. For example, the proportion of Higher-Level candidates who were able to explain what it means to say that $\sqrt{3}$ is not a rational number was much smaller than should be the case, as was the number of ordinary level candidates who were able to explain what an *axiom* is. [HL: P1, Q1(a); OL: P2, Q6A(b).]
- In a number of cases, candidates who were otherwise very competent lost marks on relatively straightforward tasks, whether through carelessness or a failure to gain command of the basics. For example, at Higher Level, some high-scoring candidates were unable to correctly construct a line segment of length $\sqrt{3}$, despite the fact that this task is clearly specified in the syllabus. Some others lost marks by failing to complete some procedural tasks in co-ordinate geometry accurately.
- At all levels, candidates were more likely to attempt all parts of the questions being answered than heretofore, even where they were clearly struggling. In mathematics examinations generally, it is frequently the case that candidates will not make any effort at a question unless they find it familiar and are reasonably sure that they know how to do it. Accordingly, this increased willingness to try is a positive development.
- It was reported that many candidates found the examination long, and, in particular, had insufficient time to spend thinking about the questions of a less familiar type. Nonetheless, many candidates attempted a surplus question on paper 1 (over half of the Higher-Level candidates and over two-thirds of the Ordinary-Level candidates). In general, unless one has plenty of time, attempting a surplus question is not a good examination strategy.

Recommendations to Teachers and Students

- Use the syllabus as the main reference document in preparing for the examination. The examinations will reflect the aim, objectives, and learning outcomes of the syllabus, and will support the development of the key skills of the senior cycle curriculum.
- Remember that the learning outcomes at Ordinary Level are additional to those at Foundation Level, and that those at Higher Level are additional to those at Ordinary and Foundation levels. Accordingly, give due regard to the outcomes listed for the level(s) below the one you are dealing with. Similarly, as the Leaving Certificate syllabus builds on the knowledge and skills developed at Junior Certificate, ensure that you can recall and apply those skills too.
- Try to develop understanding of all mathematical methods employed. Skills will transfer much more readily to unfamiliar scenarios when they are based on understanding. Furthermore, you may be explicitly asked to explain or justify the methods you employ.
- Use the resources provided by the Project Maths Development Team and the NCCA. The examinations are designed on the assumption that candidates have engaged with these activities or ones of a similar type. These materials also help in interpreting the syllabus.
- Engage in activities that draw together skills and understanding from more than one area of the course.
- Be prepared for the unfamiliar. A high level of achievement in mathematics is characterised by the ability to bring insightful knowledge and well-developed skills to bear on new problems. It is not helpful to try to second-guess every conceivable type of problem that might be encountered, in order to learn off the correct method for doing each. It is more productive - both for the achievement of the objectives of the syllabus and for success in the examinations - to develop generic problem-solving skills and to have had plenty of experience in engaging with tasks that vary considerably in their level of familiarity. Teachers should make a concerted effort to expose students to problems that are not like ones they have encountered before, in order to develop their problem-solving skills.
- Ensure that basic skills are not neglected. These too are specified as syllabus outcomes and will be tested directly. Furthermore, problem solving is only possible when the basic tools needed to address the problems are readily available. Fundamental skills in arithmetic, algebra, and geometry need to be continually attended to.
- Ensure you understand the concept of a mathematical proof, that you can use valid reasoning to justify conclusions, and that you can identify and rectify deficiencies in arguments presented by others. Ensure also that you are able to reproduce whatever formal proofs are specified in the syllabus as being directly examinable.
- Be familiar with the terminology and language of the subject. When engaged in discussion and exploration, use the correct terms and seek clarification of any words that are unfamiliar.
- Read questions carefully. Information on examination papers is concise, careful, and deliberate, and it is easy to miss or misread a critical piece of information. Give careful consideration to the question before you begin answering it.

- Use common sense when thinking about questions, and reflect on your answers. If an answer seems unreasonable, this may assist in locating a mistake. Knowledge and skills that have been acquired outside the mathematics classroom are valid and useful.
- Do not be put off or upset if a problem is not working out. Some problems are intended to be challenging. When an examination task is non-routine, then you will be well rewarded for exploring the problem in a reasoned way and applying plausible lines of attack, even if you do not ultimately fully solve the problem.
- Show all your work. Partial credit will be awarded for any substantive work of merit.
- Communicate your thinking as clearly as possible, whether you are solving a mathematical problem or offering a text-based answer.
- Even if you are not asked to draw a diagram, it can often be a very helpful first step. You may gain some credit for the diagram. More importantly, the way forward with the problem very often becomes much clearer when the given information is presented on a diagram.
- Attempt all parts of the questions you are doing. The examiner will always search for merit in what you write. But if you write nothing, you cannot get any marks.
- If you make more than one attempt at a question, make it clear which attempt is your final version. However, you should also ensure that your other attempts remain legible. In most circumstances, you will get credit for your best attempt, even if it has been cancelled in favour of another.
- Ensure that you are thoroughly familiar with your own calculator and capable of using it efficiently and intelligently. Make sure that your calculator conforms to the rules governing the use of calculators in the State examinations, and that it has a sufficient range of features to meet your needs during the examination.

Have you accessed and/or used resources in class from the following websites.....

1. <http://www.projectmaths.ie>:

- "Patterns: A Relations Approach to Algebra" document featured on the homepage which explores linear, quadratic, cubic, inverse and exponential relationships
- Project Maths News updates
- Teaching and Learning plans
- Teacher Handbooks which include a suggested sequence and a suggested number of classes for various lesson ideas
- Student's CD (which now includes all 5 strands) with the Student Activities in class
- Syllabus and Resources Documents which match resources to the syllabus
- Booklets, powerpoints, posters, video links from Workshops 1-5
- Booklets of Sample Papers relevant to LC 2012
- Problem Solving tab?

2. <http://www.censusatschool.ie>

- Census at School to generate a class set of data, source data from students in other countries and access lots of statistical analysis tools and teaching resources

3. <http://www.ncca.ie/projectmaths>

- NCCA Student Resource Materials which are books of questions for JC and LC Strand 1 and Strand 2 and include examples for all levels
- Pre-LC 2010 papers
- Copies of the LC 2012, LC 2013, JC 2013 and JC 2014 syllabuses

4. <http://www.examinations.ie>

- circular S77/2011 from the SEC which came with the 2012 sample papers
- LC 2010 and 2011 and 2012 Sample Papers
- LC 2010 and 2011 Project Maths Papers
- JC 2011 and 2012 Sample Papers
- "Report on the Trial Final", a report that includes model solutions, marking schemes, comments on the answering, exemplars of student work for the 2010 LC Sample Paper 2 with additional notes related to the assessment of candidate work

Have you also.....

- encouraged your students to access the booklets of exam-style questions for Strands 1-4 in the student zone of the NCCA website [<http://www.ncca.ie/projectmaths>]
- encouraged your students to access the Students' tab [<http://www.projectmaths.ie>]
- attended the modular courses for ICT for (i) Strands 1 and 2 or (ii) Strands 3, 4 and 5 or accessed the resources for same on the website or attended the modular courses for content for (i) Strand 1 or (ii) Strand 2 or accessed the resources for same on the website [<http://www.projectmaths.ie>]
- shared more resources with the rest of the teachers in your maths team in recent months
- facilitated the use of concrete resources (e.g. unifix cubes, dice, geometry set, geostrips, clinometers etc.) more often in recent months
- considered the implementation of a common approach in your mathematics department for the teaching of certain topics e.g. algebraic skills, fractions