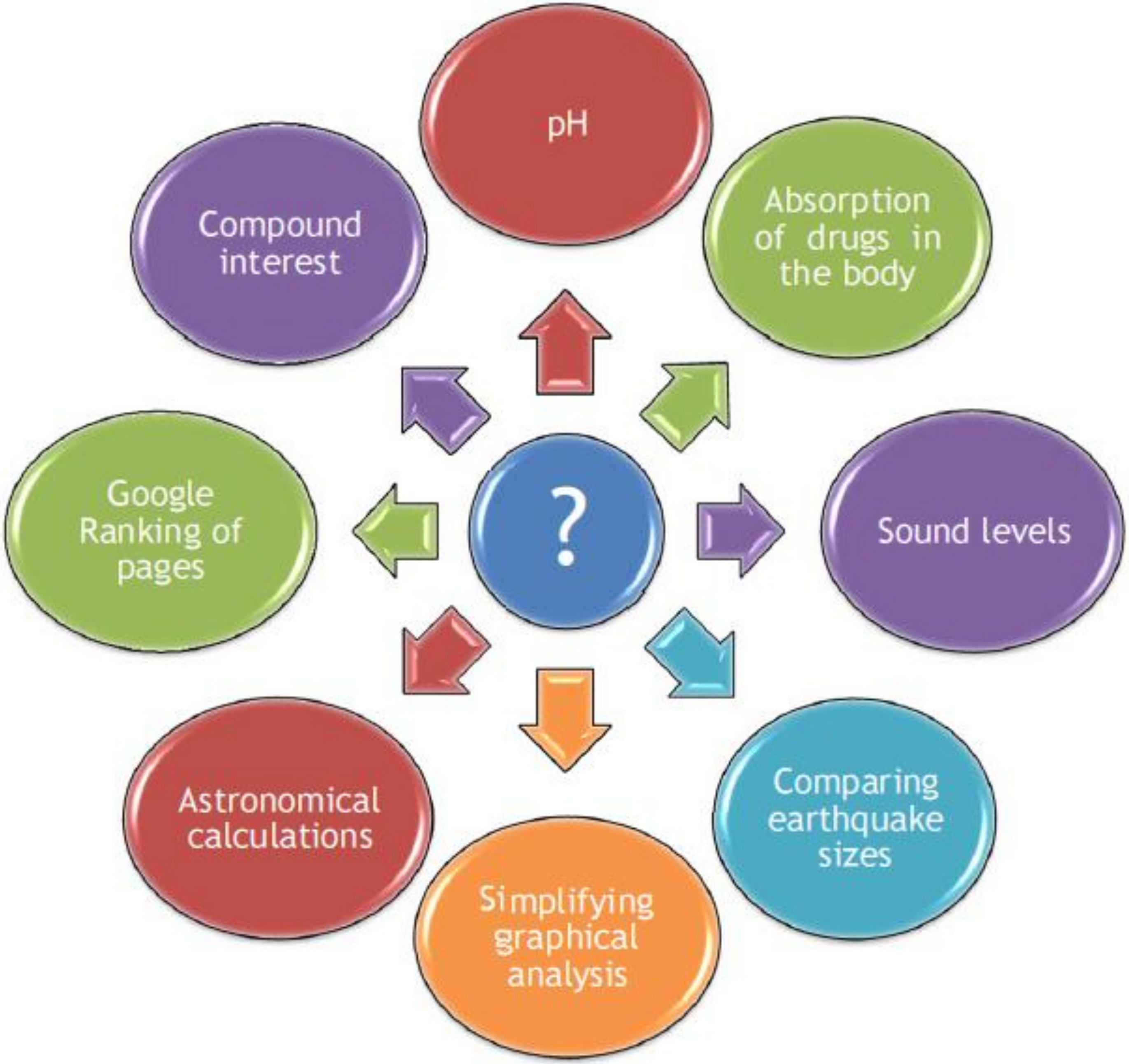

pH
Absorption of drugs in the body
Sound levels
Comparing earthquake sizes
Simplifying graphical analysis
Astronomical calculations
Google Ranking of pages
Compound interest
?

Money matters

"How long will it take for a sum of money to double, if invested at 20% compound interest rate, compounded annually?"

We know the output (effect).

We want to find the input(cause).

The unknown is in the exponent/power/index.

Historical Context
(16th and early 17th centuries)

- Enormous expansion in scientific knowledge, geography, Physics and Astronomy

- Scientists spending too much time doing tedious numerical calculations.

- An invention to free scientists from this burden was required

- John Napier (1550 – 1617) Scottish mathematician took up the challenge.

Prior knowledge

Indices, powers, exponents

$$a^p \times a^q = a^{p+q}$$

$$a^p \div a^q = a^{p-q}$$

$$\left(a^p\right)^q = a^{pq}$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

A.P. G.P.

What types of sequences are shown here?

A simple relation exists between the terms of the G.P. and the corresponding indices or exponents of the common ratio of the G.P.

This relation is the key idea behind Napier's invention.

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

A.P. G.P.

Calculate the following:

(a) 32×128

(b) $4096 \div 512$

(c) 8^4

Use the table and your knowledge of indices.

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

A.P. G.P.

+ ×

Calculate:

(i) 32 x 128

Multiplication reduced to addition!

Check out other examples.

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

A.P. G.P.

Calculate:

(ii) $4096 \div 512$

Division reduced to subtraction!

Check out other examples.

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

×4 ^4

A.P. G.P.

Calculate:

(iii) 8^4

$$8^4 = \left(2^3\right)^4 = \left(2\right)^{3\times 4} = \left(2\right)^{12}$$

Exponentiation reduced to multiplication!

Check out other examples.

Gaps in the table

(fixed number)^{power} = positive number

"If we could write **any** positive number as a power of **some** given fixed number, (later called the base), then multiplication and division of numbers would be reduced to addition and subtraction of their exponents."

He spent 20 years of his life making up tables of powers of a base for any positive number!

LAPLACE

- What power do I put on 2 to give me 256?
- What power do I put on 2 to give me 1024?

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

$$2^x = y \Leftrightarrow \log_2(y) = x$$

Exercise in booklet: switching between exponential and log forms

Power of 2

$\log_2$

$$\log_2(256) = 8$$

$$\log_2(1024) = 10$$

$$\log_2(1) = 0$$

$$\log_2(2) = 1$$

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

The inputs for $y = 2^x$ are "$\log_2$".

Logs reduce a big range of numbers to a more manageable range.

Increase of 1 in the $\log_2$ scale means a ……………. in the original scale.

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

For what values of x is $\log_2(x) < 0$?

Log_2 (numbers between 0 and 1)

x	2^x
0	1
-1	1/2
-2	1/4
-3	1/8
-4	1/16
-5	1/32
-6	1/64
-7	1/128
-8	1/256
-9	1/512
-10	1/1024

x	$\log_2 x$
1	0
1/2	
1/4	
1/8	
1/16	
1/32	
1/64	
1/128	
1/256	
1/512	
1/1024	

Compare $\log_2 x$ and $\log_2 \frac{1}{x}$

$$\log_2 \frac{1}{x} = -\log_2 x$$

Logs expand small variation

x	2^x
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024

x	3^x
0	1
1	3
2	9
3	27
4	81
5	243
6	729
7	2,187
8	6,561
9	19,683
10	59,049

x	5^x
0	1
1	5
2	25
3	125
4	625
5	3,125
6	15,625
7	78,125
8	390,625
9	1,953,125
10	9,765,625

x	6^x
0	1
1	6
2	36
3	216
4	1,296
5	7,776
6	46,656
7	279,936
8	1,679,616
9	10,077,696
10	60,466,176

x	10^x
0	1
1	10
2	100
3	1,000
4	10,000
5	100,000
6	1,000,000
7	10,000,000
8	100,000,000
9	1,000,000,000
10	10,000,000,000

2^x	$\log_2(2^x)$
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9
1,024	10

3^x	$\log_3(3^x)$
1	0
3	1
9	2
27	3
81	4
243	5
729	6
2,187	7
6,561	8
19,683	9
59,049	10

5^x	$\log_5(5^x)$
1	0
5	1
25	2
125	3
625	4
3,125	5
15,625	6
78,125	7
390,625	8
1,953,125	9
9,765,625	10

6^x	$\log_6(6^x)$
1	0
6	1
36	2
216	3
1,296	4
7,776	5
46,656	6
279,936	7
1,679,616	8
10,077,696	9
60,466,176	10

Common logs (Log)

10^x	$\log_{10}(10^x)$
1	0
10	1
100	2
1,000	3
10,000	4
100,000	5
1,000,000	6
10,000,000	7
100,000,000	8
1,000,000,000	9
10,000,000,000	10

Logs put numbers on a human friendly scale. Millions , billions and trillions are really big but written as powers of 10 they become tame! Just plain old 6 and 9 and 12!

The bigger the base the smaller the log of the number to that base.

Bases are always positive!

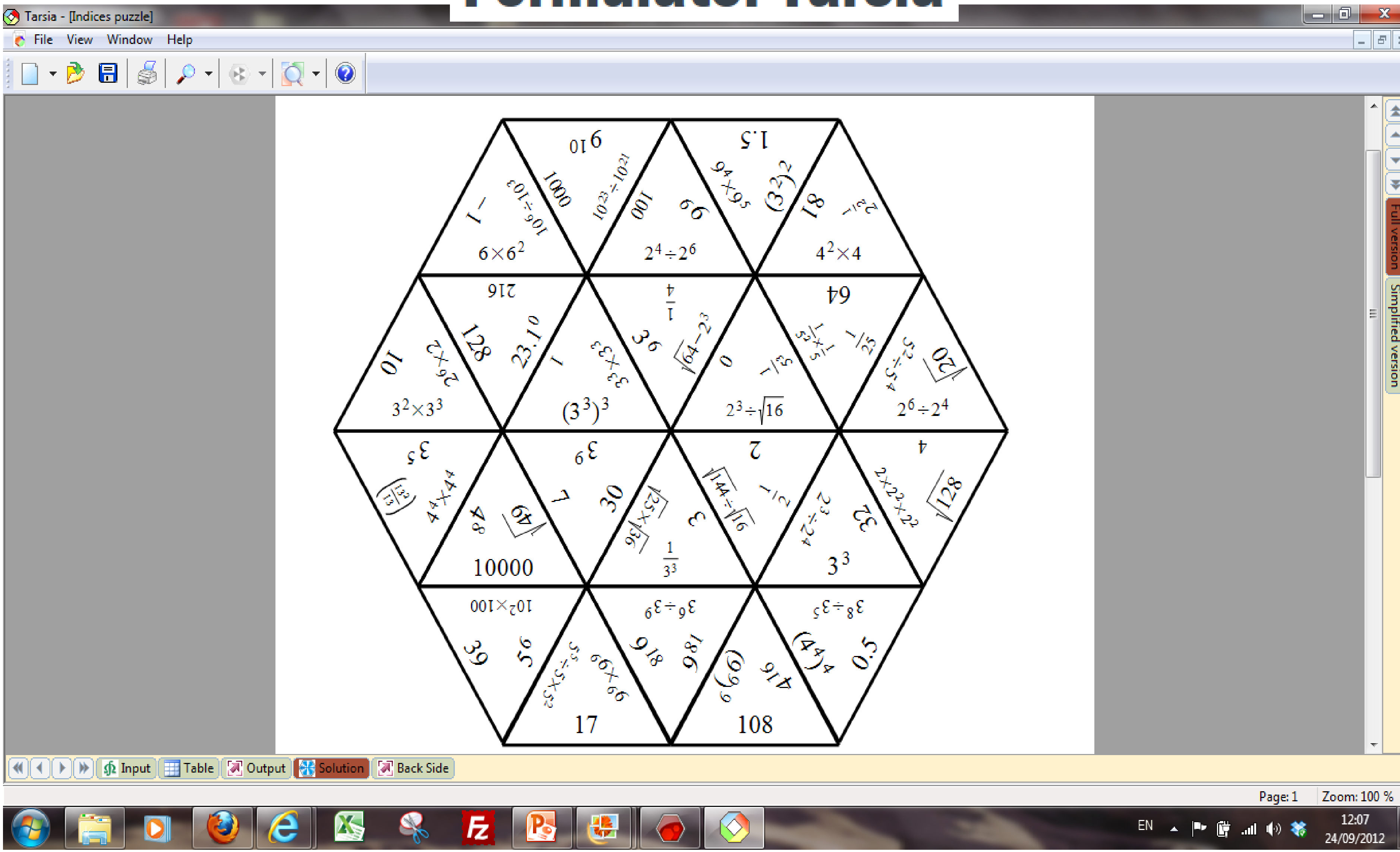
Base *e* and natural logs (ln)

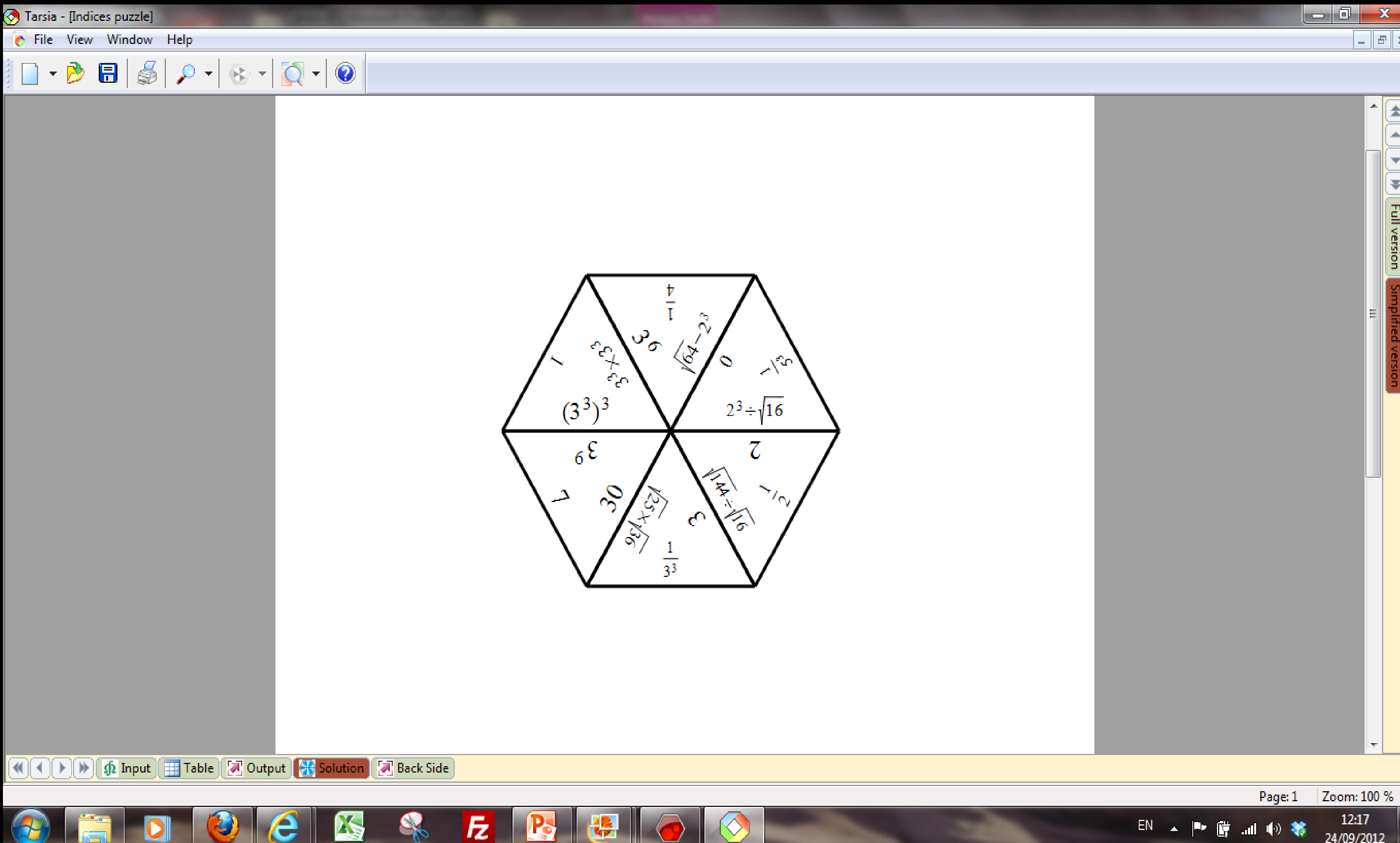
x	e^x
0	1
1	e^1
2	e^2
3	e^3
4	e^4
5	e^5
6	e^6
7	e^7
8	e^8
9	e^9
10	e^{10}

e^x	$\log_e(x) = \ln(x)$
1	0
e^1	1
e^2	2
e^3	3
e^4	4
e^5	5
e^6	6
e^7	7
e^8	8
e^9	9
e^{10}	10

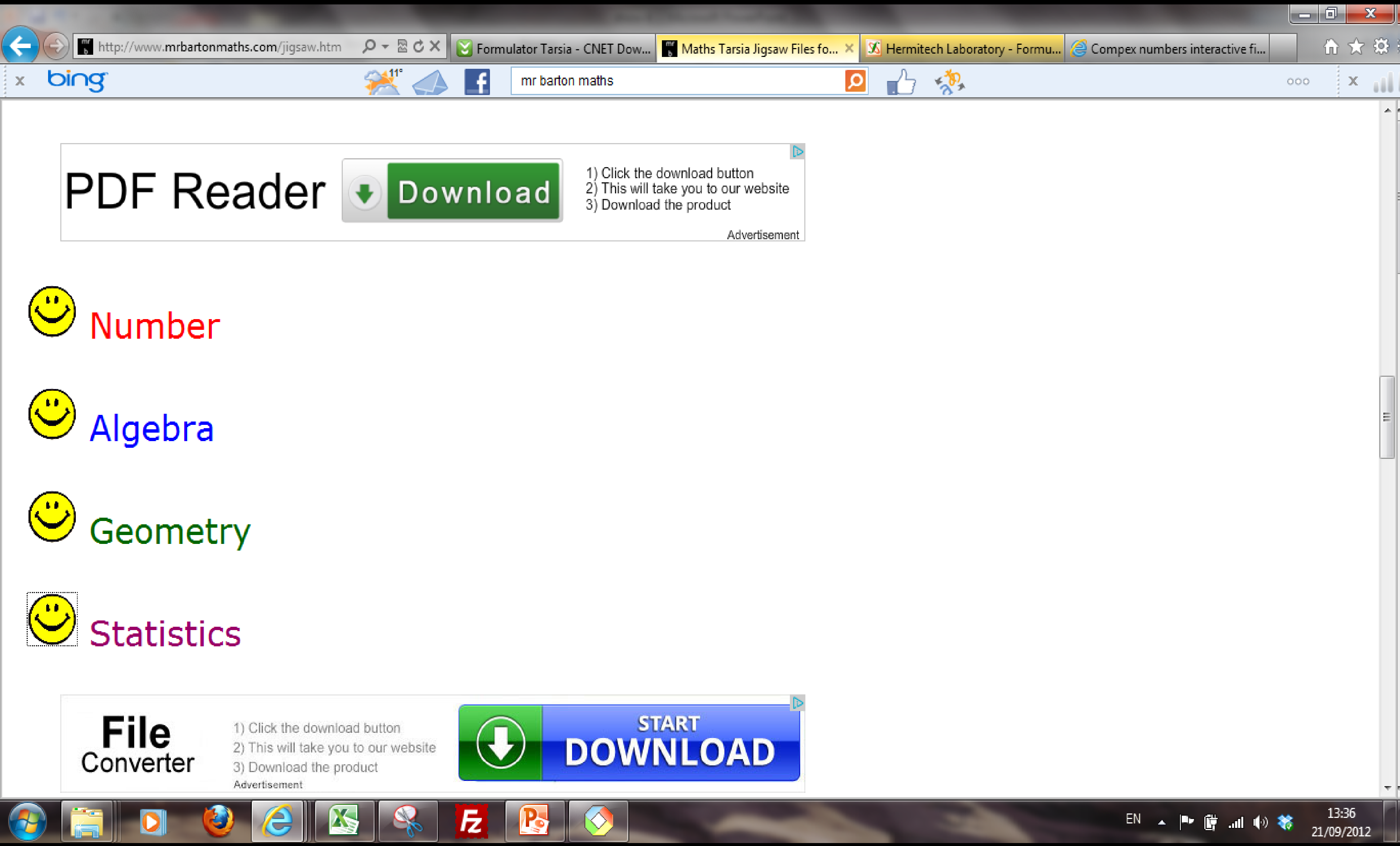
Natural logs are powers of base e

Formulator Tarsia





Tarsia - [Indices puzzle]
File View Window Help
Full version
Simplified version
$\frac{1}{4}$
36
$\sqrt{64-2^3}$
0
$1/3$
$2^3 \div \sqrt{16}$
1
$3^3 \times 3^3$
$(3^3)^3$
3^9
2
$\sqrt{144} \div \sqrt{16}$
$1/2$
7
30
$\sqrt{25} \times \sqrt{36}$
3
$\frac{1}{3^3}$
Input Table Output Solution Back Side
Page: 1 Zoom: 100 %
EN
12:17
24/09/2012


http://www.mrbartonmaths.com/jigsaw.htm
Formulator Tarsia - CNET Dow...
Maths Tarsia Jigsaw Files fo...
Hermitech Laboratory - Formu...
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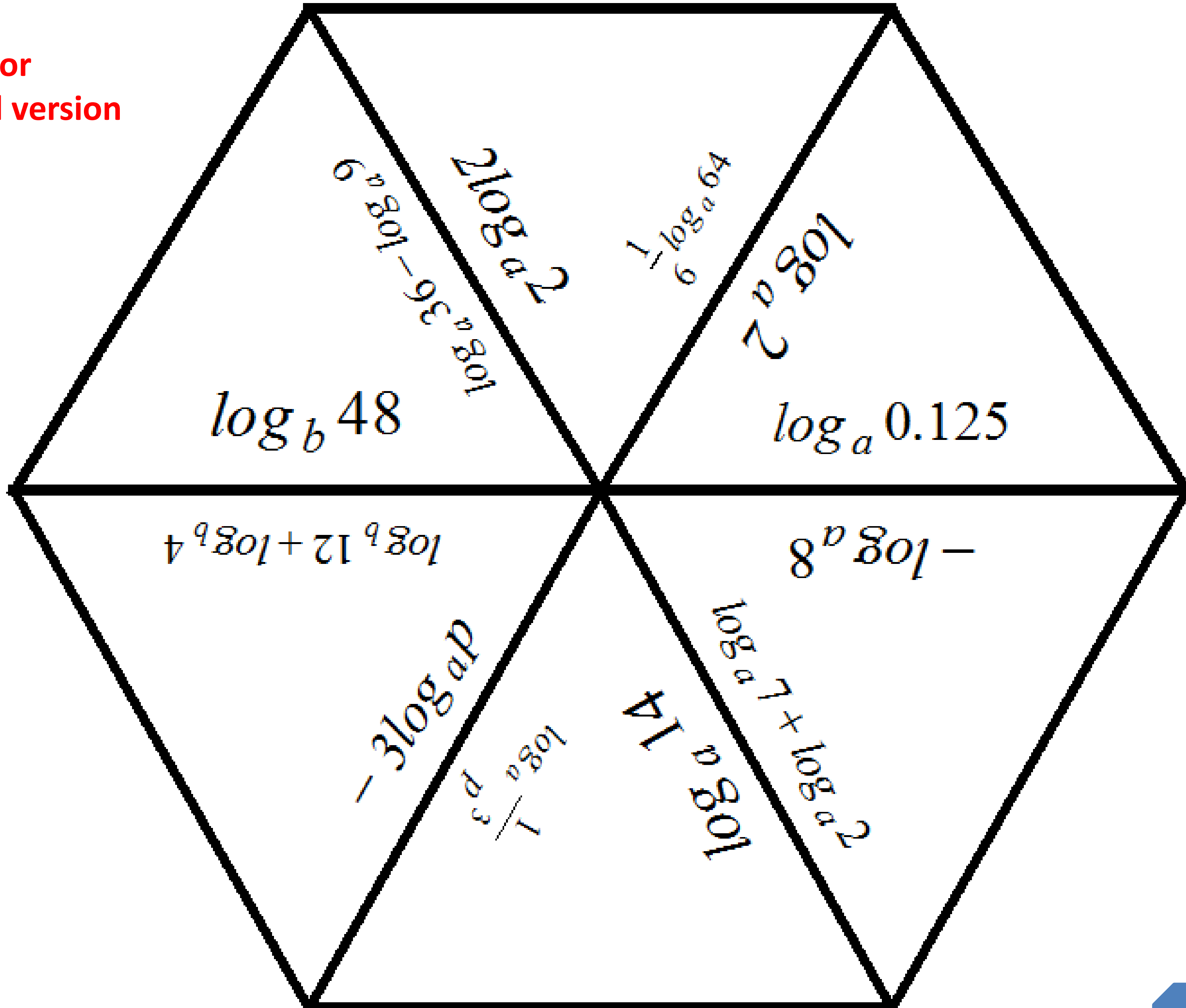
21/09/2012

Solution for simplified version



Switching between Exponential and logarithmic forms of Equations

4.

Evaluate the expression below forming an equation	Write the equivalent exponential form of the equation formed from the first column
$\log_2 16 = 4$	$2^4 = 16$
$\log_2\left(\frac{1}{64}\right)$	$2^{-4} = \frac{1}{64}$
$\log_2(1)$	$2^0 = 1$
$\log_2\left(\frac{1}{8}\right)$	$2^{-3} = \frac{1}{8}$
$\log_e e$	$e^1 = e$
$\log_2(-4)$	*Not possible*

5.

Exponential form of an equation	Write the equivalent log form of the equation in the previous column
$5^2 = 25$	$log_5 25 = 2$
$5^{-2} = \frac{1}{25}$	$log_5\left(\frac{1}{25}\right) = -2$
$10^1 = 10$	$log_{10} 10 = 1$
$9^{\frac{1}{2}} = 3$	$log_9 3 = \left(\frac{1}{2}\right)$
$27^{\frac{1}{3}} = 3$	$log_{27} 3 = \left(\frac{1}{3}\right)$
$b^0 = 1$	$log_b 1 = 0$

7A. Evaluate each of the following:

$\log_2(32 \times 2) = \log_2(64) = \underline{\ 6\ }$	$\log_2(32) + \log_2(2) = \underline{\ 5\ } + \underline{\ 1\ } + = \underline{\ 6\ }$
$\log_2(27 \times 9) = \log_2(243) = \underline{\ 5\ }$	$\log_2(27) + \log_2(9) = \underline{\ 3\ } + \underline{\ 2\ } + = \underline{\ 5\ }$
$\log_2(25 \times 5) = \log_2(64) = \underline{\ 3\ }$	$\log_2(25) + \log_2(5) = \underline{\ 2\ } + \underline{\ 1\ } + = \underline{\ 3\ }$
$\log_2\left(16 \times \frac{1}{16}\right) = \log_2(64) = \underline{\ 0\ }$	$\log_2(16) + \log_2\left(\frac{1}{16}\right) = \underline{\ 4\ } + \underline{\ -4\ } + = \underline{\ 0\ }$

What pattern seems to hold?

Can you write a rule for $\log_b(xy)$ in terms of $\log_b(x)$ and $\log_b(y)$?

7B. Evaluate each of the following:

$\log_2(64 \div 4) = \log_2(16) = \underline{\ 4\ }$

$\log_6(216 \div 6) = \log_6(36) = \underline{\ 2\ }$

$\log_{10}(100 \div 1000) = \log_{10}\left(\frac{1}{10}\right) = \underline{-1}$

$\log_5(25 \div 25) = \log_5(1) = \underline{\ 0\ }$

$\log_2(64) - \log_2(4) = \underline{\ 6\ } - \underline{\ 2\ } = \underline{\ 2\ }$

$\log_6(216) - \log_6(6) = \underline{\ 3\ } - \underline{\ 1\ } = \underline{\ 2\ }$

$\log_{10}(100) - \log_{10}(1000) = \underline{\ 2\ } - \underline{\ 3\ } = \underline{-1}$

$\log_5(25) - \log_5(25) == \underline{\ 1\ } - \underline{\ 1\ } = \underline{\quad}$

What pattern seems to hold?

Can you write a rule for $\log_b\left(\frac{x}{y}\right)$ in terms of $\log_b(x)$ and $\log_b(y)$?

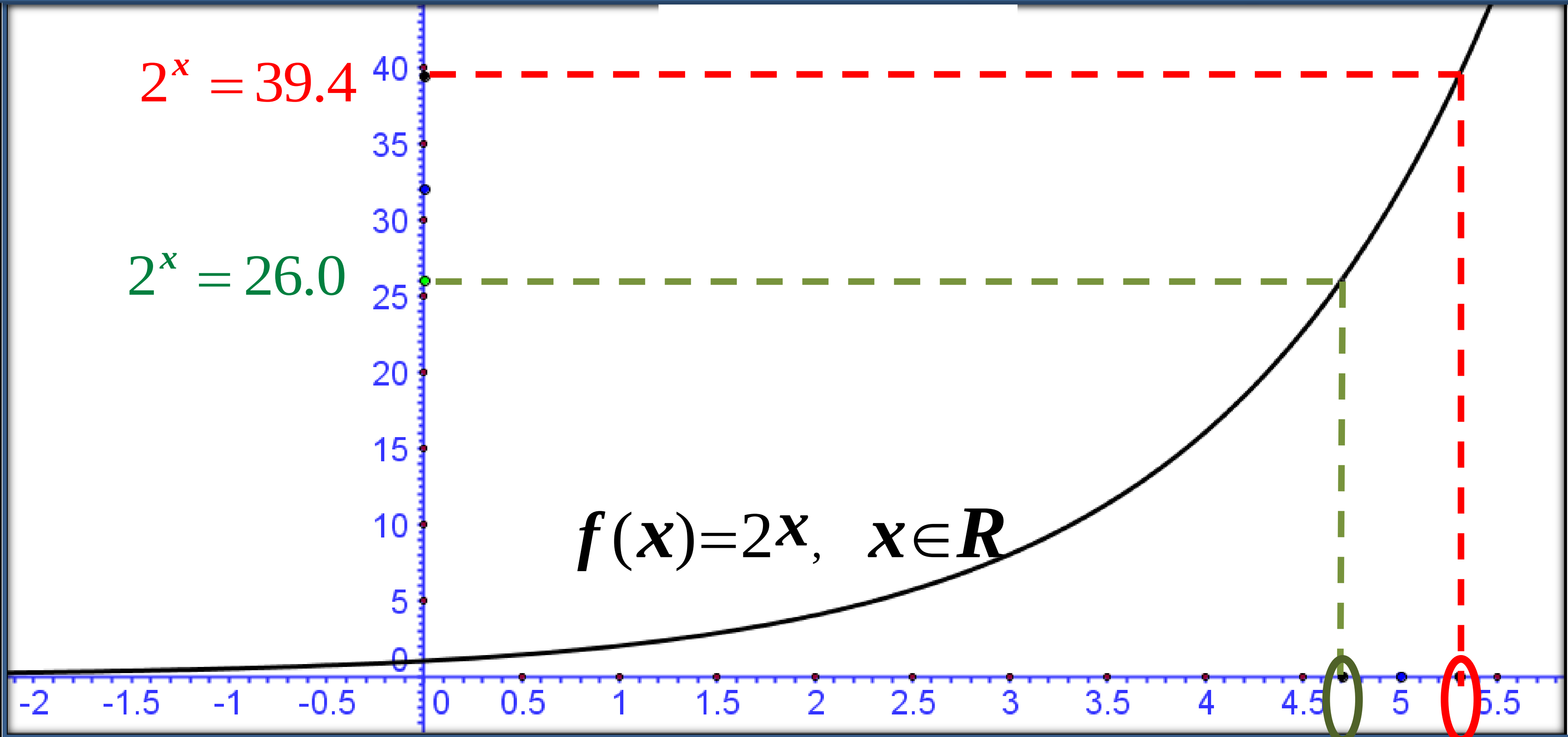
7C. Evaluate each of the following:

$\log_2(8)^3 = \log_2(512) = \underline{\ 9\ }$	$3\log_2(8) = 3(3) = \underline{\ 9\ }$
$\log_2(256)^{\frac{1}{2}} = \log_2(16) = \underline{\ 4\ }$	$\frac{1}{2}\log_2(256) = \frac{1}{2}(8) = \underline{\ 4\ }$
$\log_{10}(10)^4 = \log_{10}(10{,}000) = \underline{\ 4\ }$	$4\log_{10}(10) = 4(1) = \underline{\ 4\ }$
$\log_3(27)^2 = \log_3(729) = \underline{\ 6\ }$	$2\log_3(27) = 2(3) = \underline{\ 6\ }$

What pattern seems to hold?

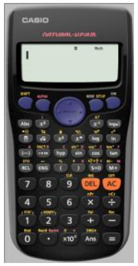
Can you write a rule for $\log_b(x)^y$ in terms of lo

Use the graph of $y=2^x$ to estimate (i) $\log_2 26$ (ii) $\log_2 39.4$?



$$2^{4.7} \approx 26 \quad \Rightarrow \log_2(26) \approx 4.7$$

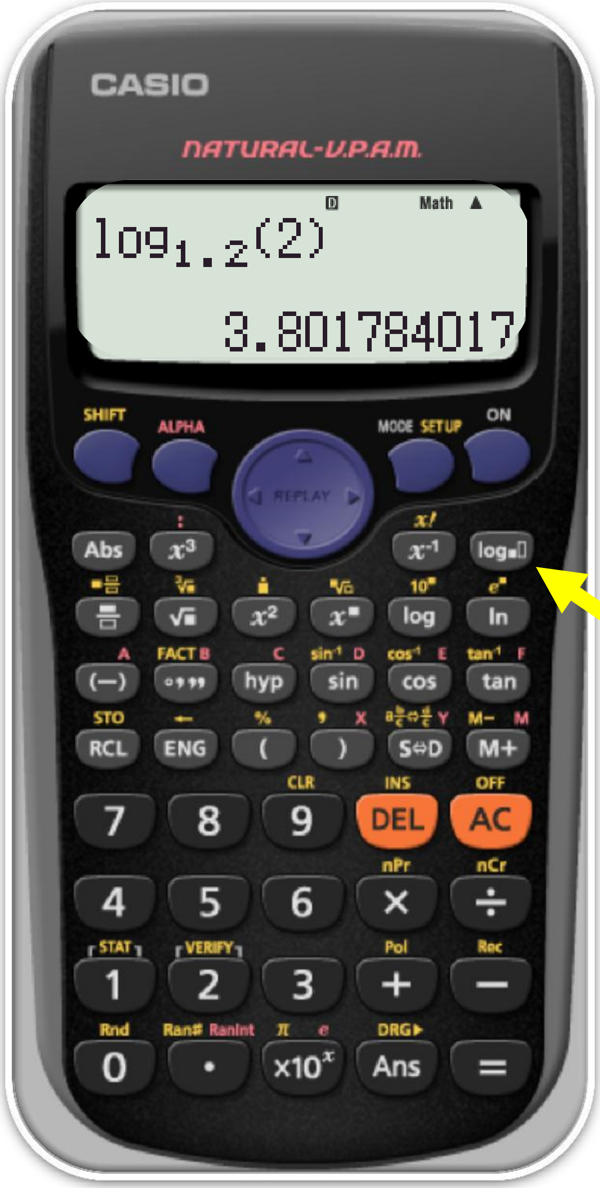
$$2^{5.3} \approx 39.4 \quad \Rightarrow \log_2(39.4) = 5.3$$



Logs give the input for some output; the cause for some effect

$$1.2^t = 2$$

$$t = log_{1.2}2$$



What power do I put on 1.2, to get 2?

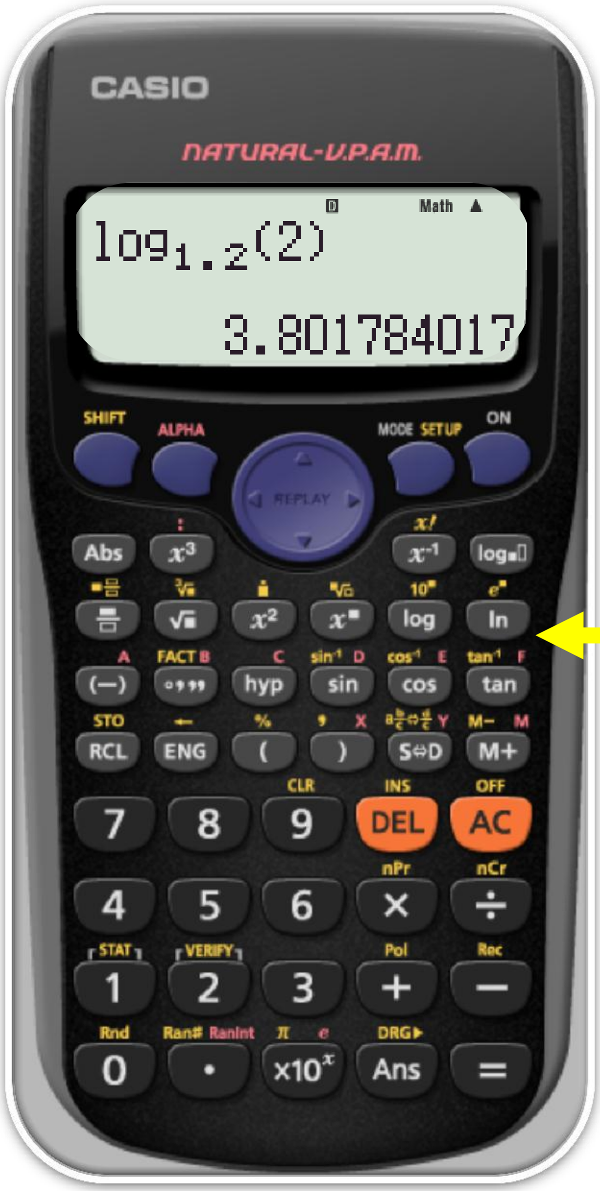
log▪□ 1 • 2 ▶ 2 =

Logs

Answer: 4 years

$$t = log_{1.2}2$$

What power do I put on 1.2, to get 2?

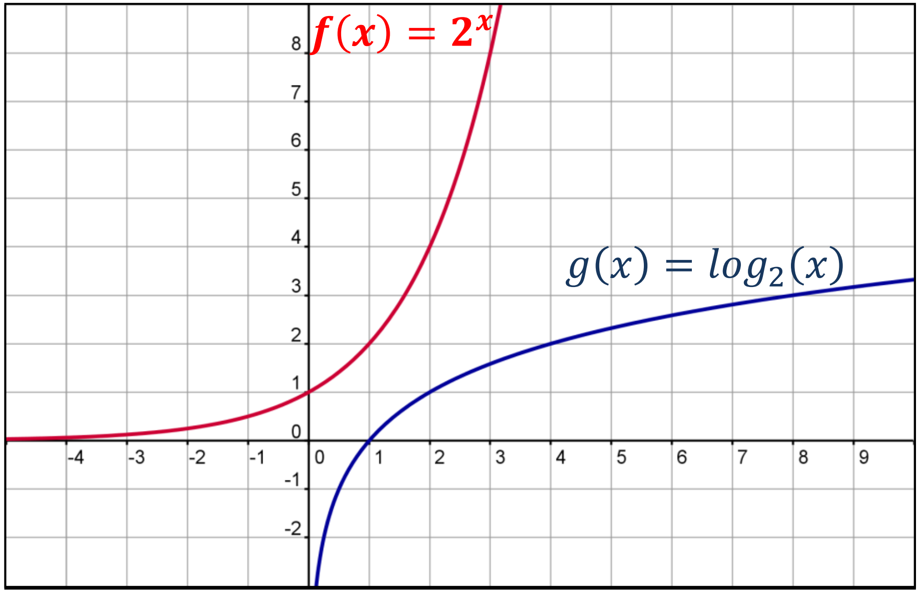


[log■□] [1] [•] [2] [▶] [2] [=]

Logs

Answer: 4 years

Fill in the table and hence draw the graph of $g(x) = f^{-1}(x)$



x	$f(x) = 2^x$	(x, y)
-2	$\frac{1}{4}$	$\left(-2, \frac{1}{4}\right)$
-1	$\frac{1}{2}$	$\left(-1, \frac{1}{2}\right)$
0	1	$(0,1)$
1	2	$(1,2)$
2	4	$(2,4)$
3	8	$(3,8)$

x	$g(x) = log_2(x)$	(x, y)
$\frac{1}{4}$	-2	$\left(\frac{1}{4}, -2\right)$
$\frac{1}{2}$	-1	$\left(\frac{1}{2}, -1\right)$
1	0	$(1,0)$
2	1	$(1,2)$
4	2	$(4,2)$
8	3	$(8,3)$

(b) What is the relationship between $f(x) = 2^x$ and $g(x) = log_2(x)$

(c) Explain why the relation $g(x) = log_2(x), x \in \mathbb{R}^+$ is a function

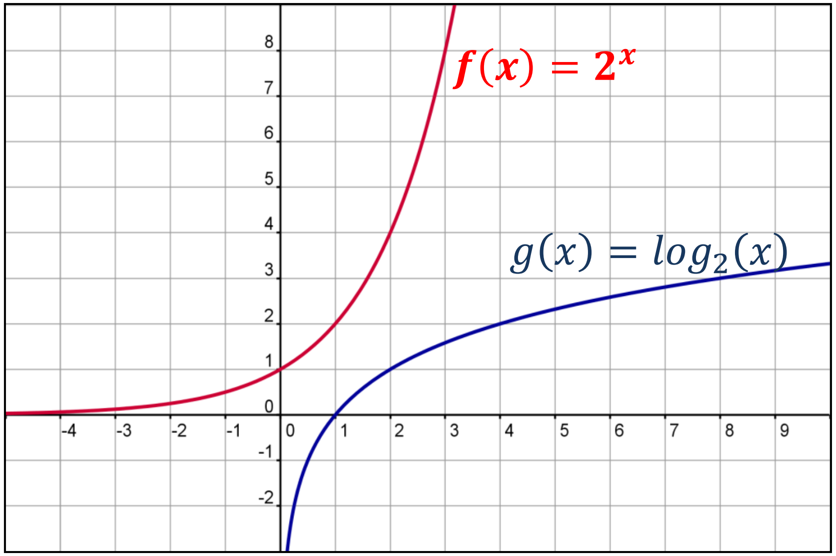
(d) For $\boldsymbol{g(x) = log_2(x)}$

(i) Identify the base of $g(x) = log_2(x)$

(ii) What is varying for the function $g(x) = log_2(x)$

(iii) What is constant for the function $g(x) = log_2(x)$

(iv) What is constant in the function $f(x) = 2^x$



(e) For $\boldsymbol{g(x) = log_2(x)}$

(i) What is the domain?

(ii) What is the range?

(f) Describe the graph of $\boldsymbol{g(x) = log_2(x)}$

(i) Is it a straight line?

(ii) Is y increasing or decreasing as x increases?

(iii) Describe how the rate of change varies as x increases.

(g) For $\boldsymbol{g(x) = log_2(x)}$

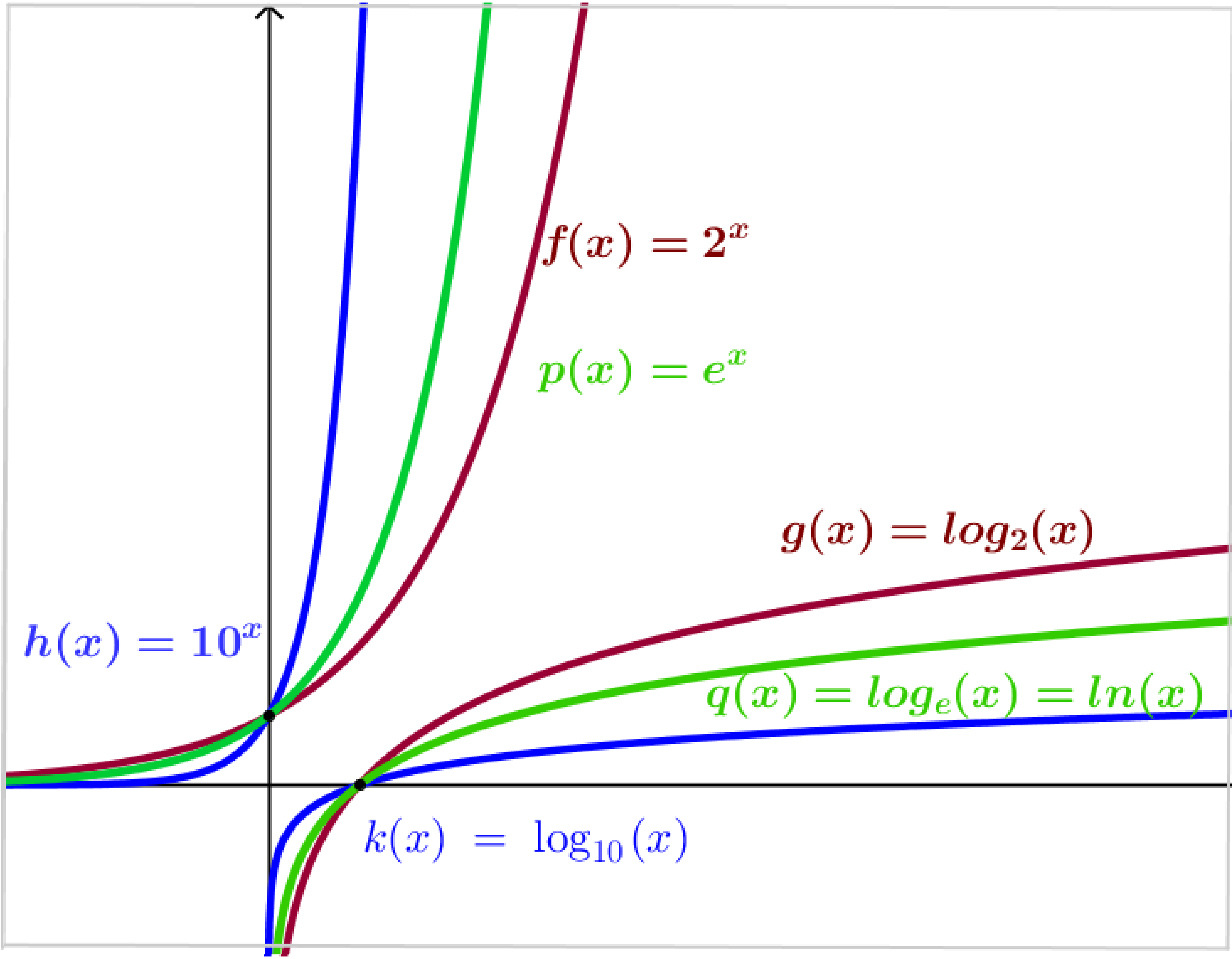
(i) Where does the graph cross the x–axis?

(ii) What happens to the output as x decreases between 0 and 1?

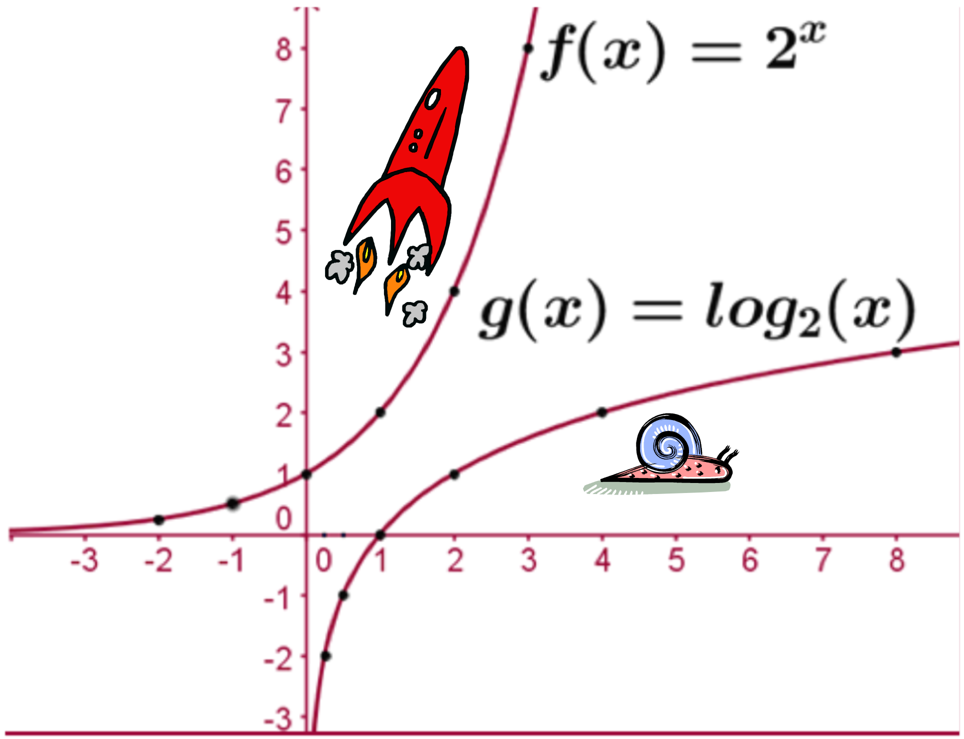
(iii) What is the y–intercept of the graph of $g(x) = log_2(x)$

(iv) What is the relationship between the y–axis and the graph of $g(x) = log_2(x)$

Sketching...........



Graphs of $f(x) = 2^x$ and $f^{-1}(x) = \log_2(x)$



$$f^{-1}(x) = \log_b(x), x \in R^+$$

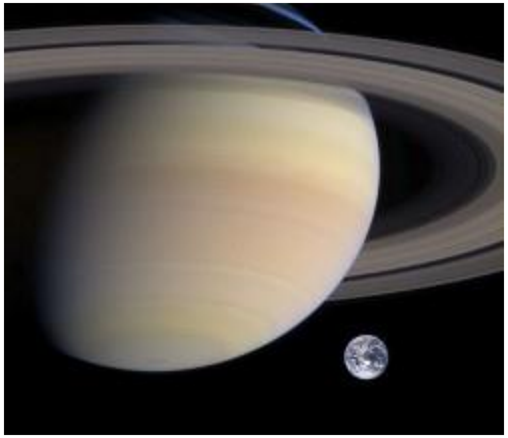
$\log_b b^x$?

$b^{\log_b x}$?

Concept of logarithms is everywhere

In Defense of Six Figure Salaries

order of magnitude



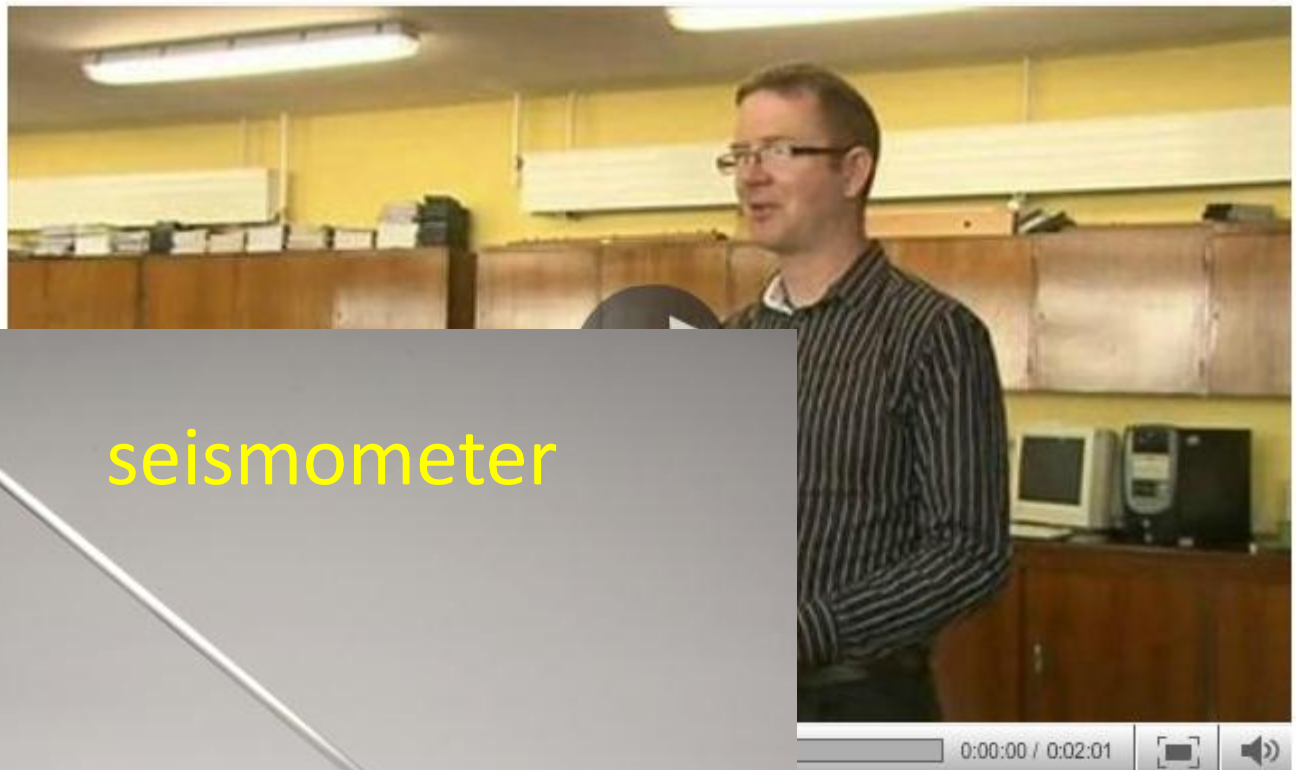
Saturn is two orders of magnitude more massive than the earth.

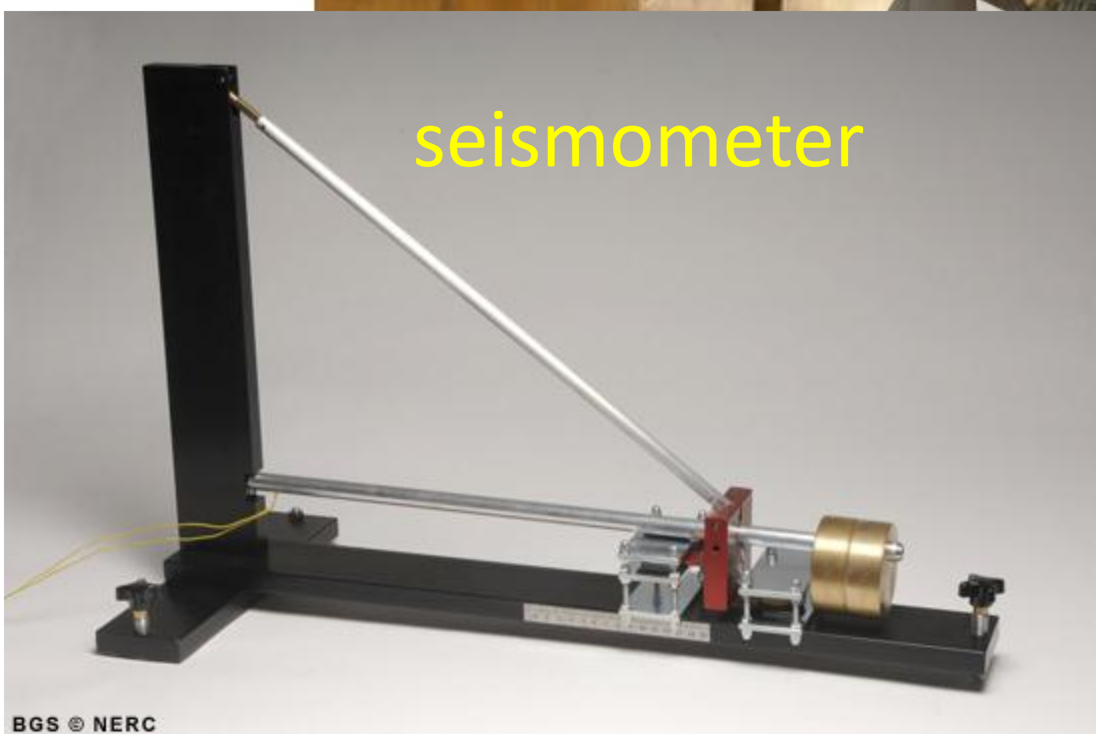
Biologists refer to the growth period of bacteria as their **"log phase"** because of the connection between their repeated doublings and binary logs.

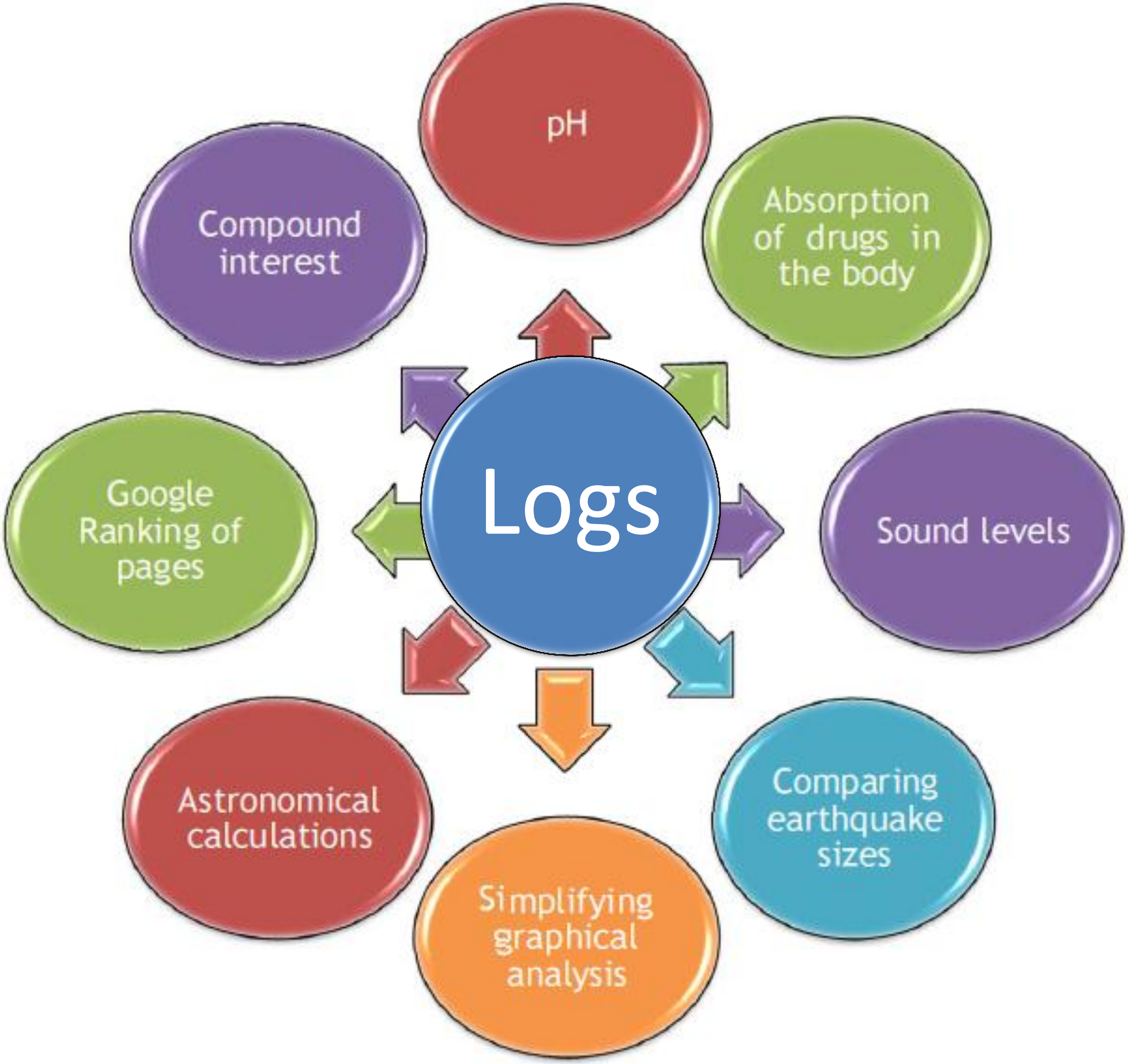
Minor earthquake felt in north Donegal

Updated: 19:18, Thursday, 26 January 2012

http://www.rte.ie/news/2012/0126/donegal.html






Logs
pH
Absorption of drugs in the body
Sound levels
Comparing earthquake sizes
Simplifying graphical analysis
Astronomical calculations
Google Ranking of pages
Compound interest

Charles Richter
1900 -1985

An Interview with Charles F. Richter

by

Henry Spall

U.S. Geological Survey, Reston, Va.

"I found a paper by Professor K. Wadati of Japan in which he compared large earthquakes by plotting the maximum ground motion against distance to the epicenter. I tried a similar procedure for our stations,

but the range between the largest and smallest magnitudes seemed unmanageably large.

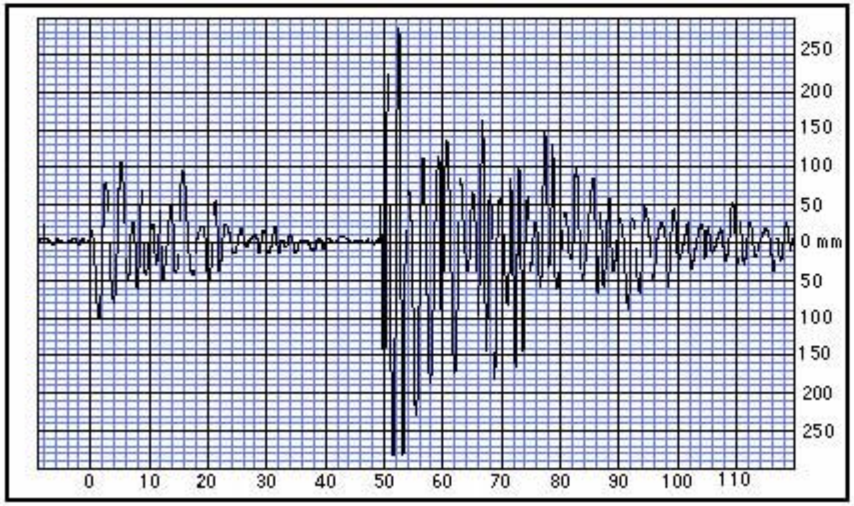
Dr. Beno Gutenberg then made the natural suggestion to plot the amplitudes logarithmically. I was lucky because logarithmic plots are a device of the devil".

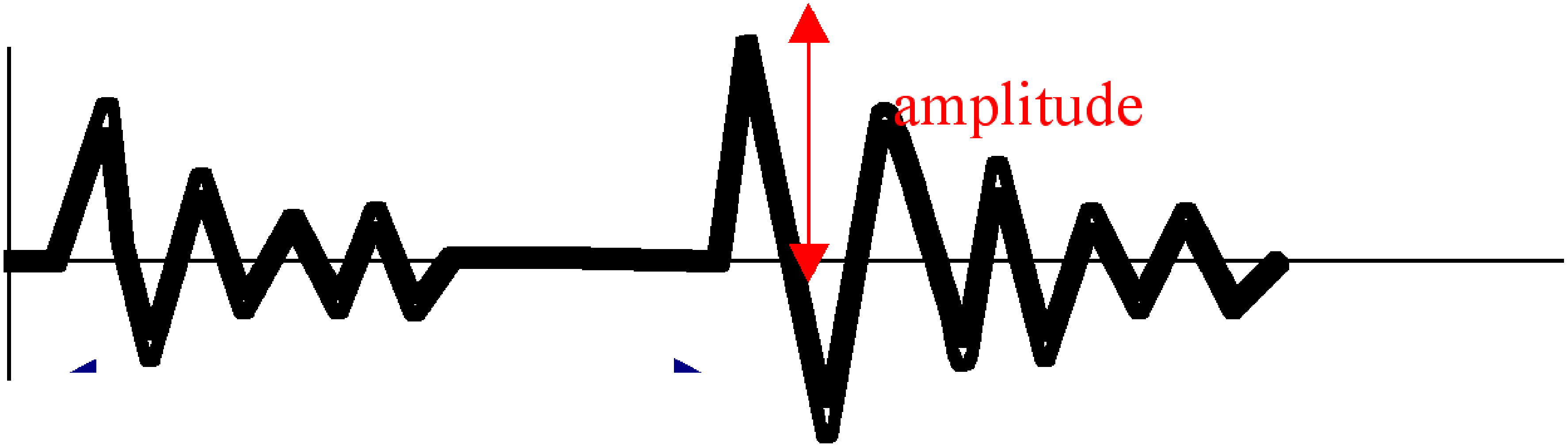
http://earthquake.usgs.gov/learn/topics/people/int_richter.php

Richter Magnitude Scale (1934)

Seismograph – An instrument that detects and measures vibrations of Earth's surface.

Seismogram – The record made by a seismograph.





An earthquake rated 6.3 Richter magnitude in Iran on 26 Dec 2003 killed 40,000 people.

The earthquake in Aceh in 2004 was rated 9.2 Richter magnitude.

How much greater in amplitude of ground motion was the earthquake in Aceh compared to the one in Iran?

Solution:

$$9.2 - 6.3 = 2.9$$

$$2.9 = \log_{10} \frac{A_2}{A_1}$$

$$10^{2.9} = \frac{A_2}{A_1} = 794$$

Hence the Aceh earthquake was 794 times greater in amplitude.

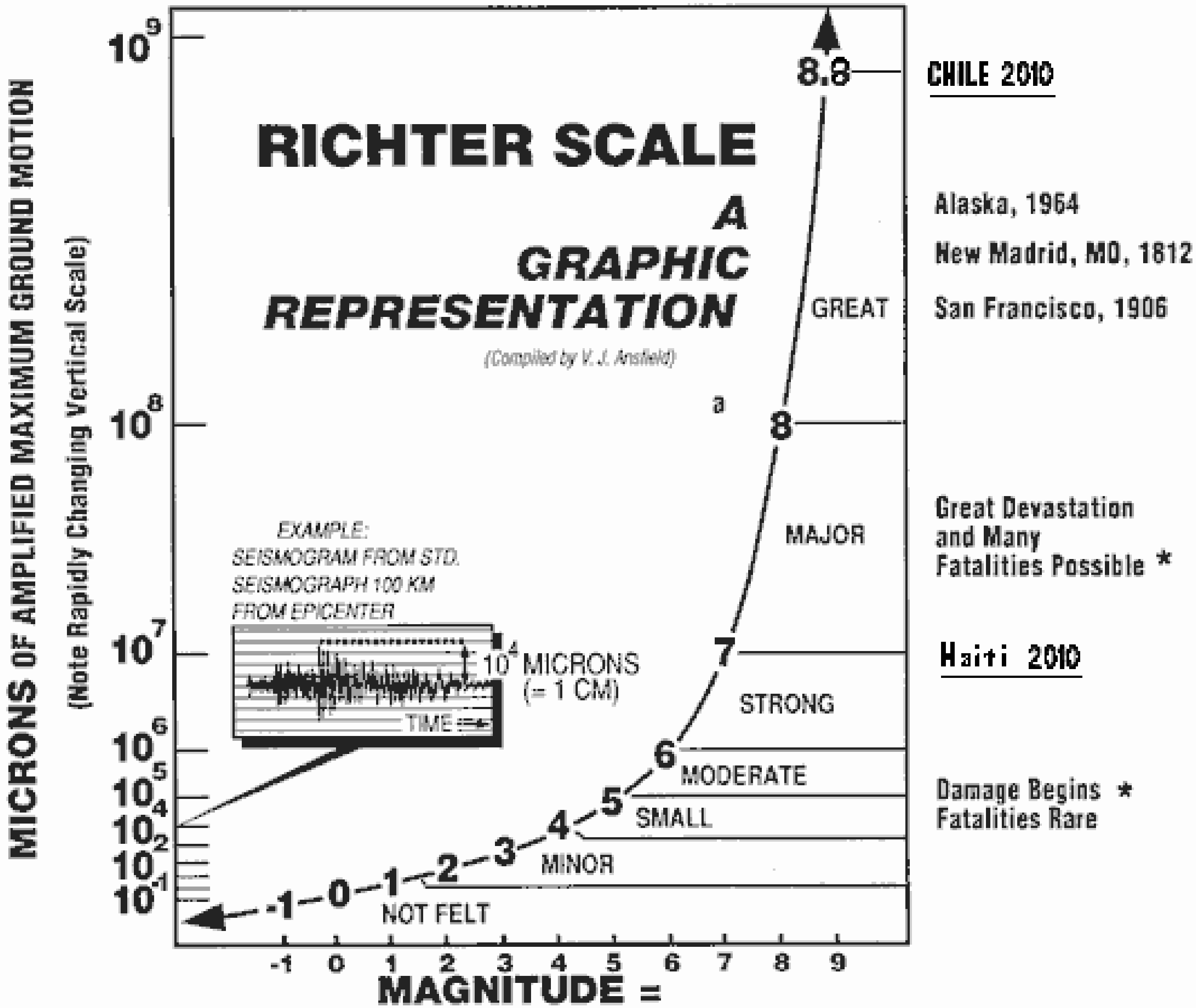
The earthquake in Yunnan, China, on 26 Nov 2003 was 10000 times weaker than the 9.0 magnitude earthquake in Aceh. What was the magnitude of this earthquake on the Richter scale?

Solution:

$$9.0 - M_2 = \log_{10} \frac{A_1}{A_2} = \log_{10} \frac{10000}{1}$$

$$9.0 - M_2 = 4$$

$$M_2 = 5$$



LOGARITHM (BASE 10) OF MAXIMUM AMPLITUDE MEASURED IN MICRONS **

* EFFECTS MAY VARY GREATLY DUE TO CONSTRUCTION PRACTICES, POPULATION DENSITY, SOIL DEPTH, FOCAL DEPTH, ETC.

** MICRON = A MILLIONTH OF A METER

*** EQUIVALENT TO A MOMENT MAGNITUDE OF 9.5

http://quakes.globalincidentmap.com/

Richter Magnitude Scale (1934)


Charles Richter
1900 -1985

Richter's definition of earthquake magnitude

The magnitude $\boldsymbol{M}$ of an earthquake of amplitude $\boldsymbol{A}$ is

$$\boldsymbol{M} = \log_{10} \frac{\boldsymbol{A}}{\boldsymbol{A}_0}$$

where $\boldsymbol{A}_0$ is the magnitude of the "standard earthquake" measured at the same distance.

Magnitude Comparison Formula

f $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ are the magnitudes of two earthquakes, and if A_1 and A_2
re their amplitudes,
neasured at equal distances, then

$$\boldsymbol{M}_1 - \boldsymbol{M}_2 = \log_{10}\left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_0}\right) - \log_{10}\left(\frac{\boldsymbol{A}_2}{\boldsymbol{A}_0}\right) = \log_{10} \frac{\frac{\boldsymbol{A}_1}{\boldsymbol{A}_0}}{\frac{\boldsymbol{A}_2}{\boldsymbol{A}_0}} = \log_{10}\left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_2}\right) \Rightarrow \frac{\boldsymbol{A}_1}{\boldsymbol{A}_2} = \mathbf{10}^{M_1 - M_2}$$

- Logarithms are particularly useful when the data extends from the very small to the very large.

- The most important feature of the log which makes it so useful is that it moves big values closer together and small values farther apart.

  Hence logs increase the range over which numbers can be seen in a meaningful way.

- Logs “tame” big numbers. They reduce a wide range to a more manageable size.

- Logs make very big numbers and very small numbers more human-friendly.

- Log_2: A one unit increase in the log scale is equivalent to multiplying by 2 in the original scale

- Log_{10}: A one unit increase in the log scale is equivalent to multiplying by 10 in the original scale

- There is a constant ratio (the base) between consecutive numbers on a log scale.

- There is a constant difference between consecutive numbers on a linear scale.

Continuous versus discrete growth

€140,000 was deposited at the beginning of January 2005 into an account earning 7% compound interest annually.
When will the investment be worth €200,000?
Verify and justify that the following two formulae give the same answer.

$$F = 140,000(1.07)^t$$

$$F = 140,000e^{0.0676586485t}$$

Comment on that answer in each case.

Solution

Verifying

$$(i)\quad 200,000 = 140,000(1.07)^t$$

$$\frac{200,000}{140,000} = \frac{20}{14} = (1.07)^t$$

$$\log\left(\frac{20}{14}\right) = \log(1.07)^t = t\log(1.07)$$

$$\frac{\log\left(\frac{20}{14}\right)}{\log(1.07)} = 5.27168\ldots \text{ years}$$

Using base e

$$200000 = 140,000e^{0.0676586485t}$$

$$\ln\frac{20}{14} = t(0.0676586485)$$

$$t = \frac{\ln\frac{20}{14}}{0.0676586485} = 5.27168 \text{ years}$$

Justifying

$$F = 140,000(1+i)^t$$

We know that $\ln\left(e^x\right) = x = e^{\ln(x)}$

$$\Rightarrow (1+i)^t = e^{\ln\left((1+i)^t\right)} = e^{t\ln(1+i)}$$

$$F = 140,000(1+i)^t = 140,000e^{t\ln(1.07)} = 140,000e^{t(0.0676586485)}$$

Note: $\ln(1.07) = 0.0676586485 \approx 0.07$

$\ln(1+i) \approx i$ for small i

Comment

The two models are equivalent and are interchangeable.

However only a whole number of years makes sense in a compound interest formula if the interest is compounded annually.

Measuring acidity

In an aqueous solution, at 25° C, no matter what it contains,

$$[H^+][OH^-] = 1x10^{-14}$$

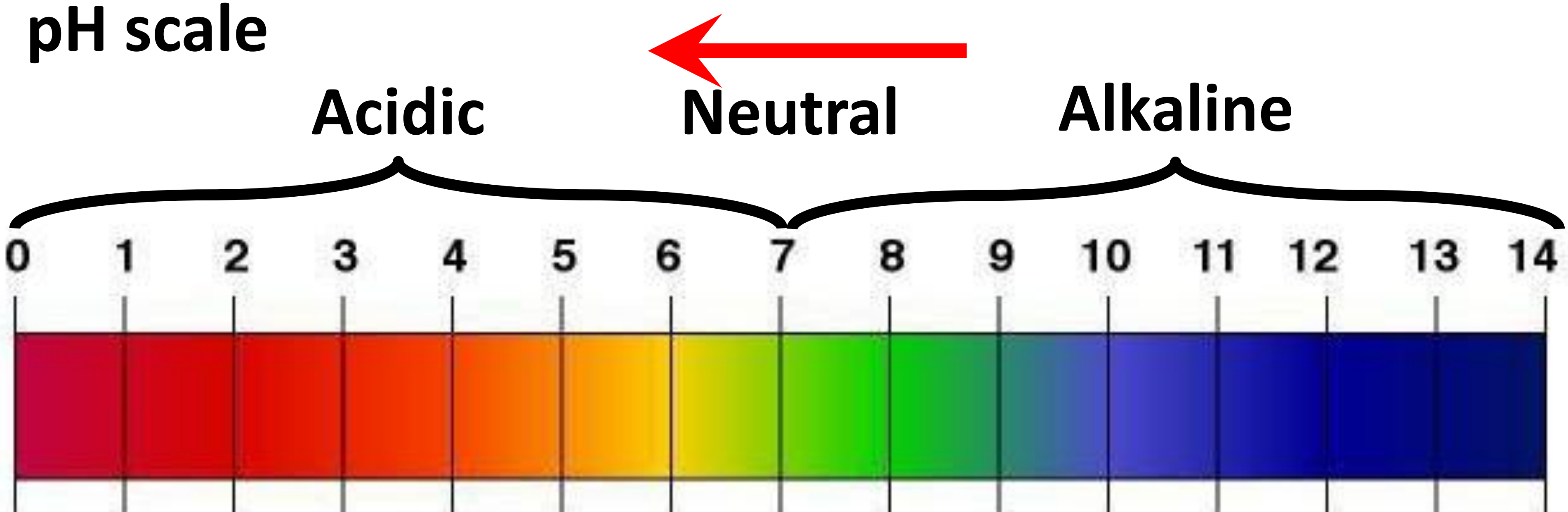
There are 3 possible situations:

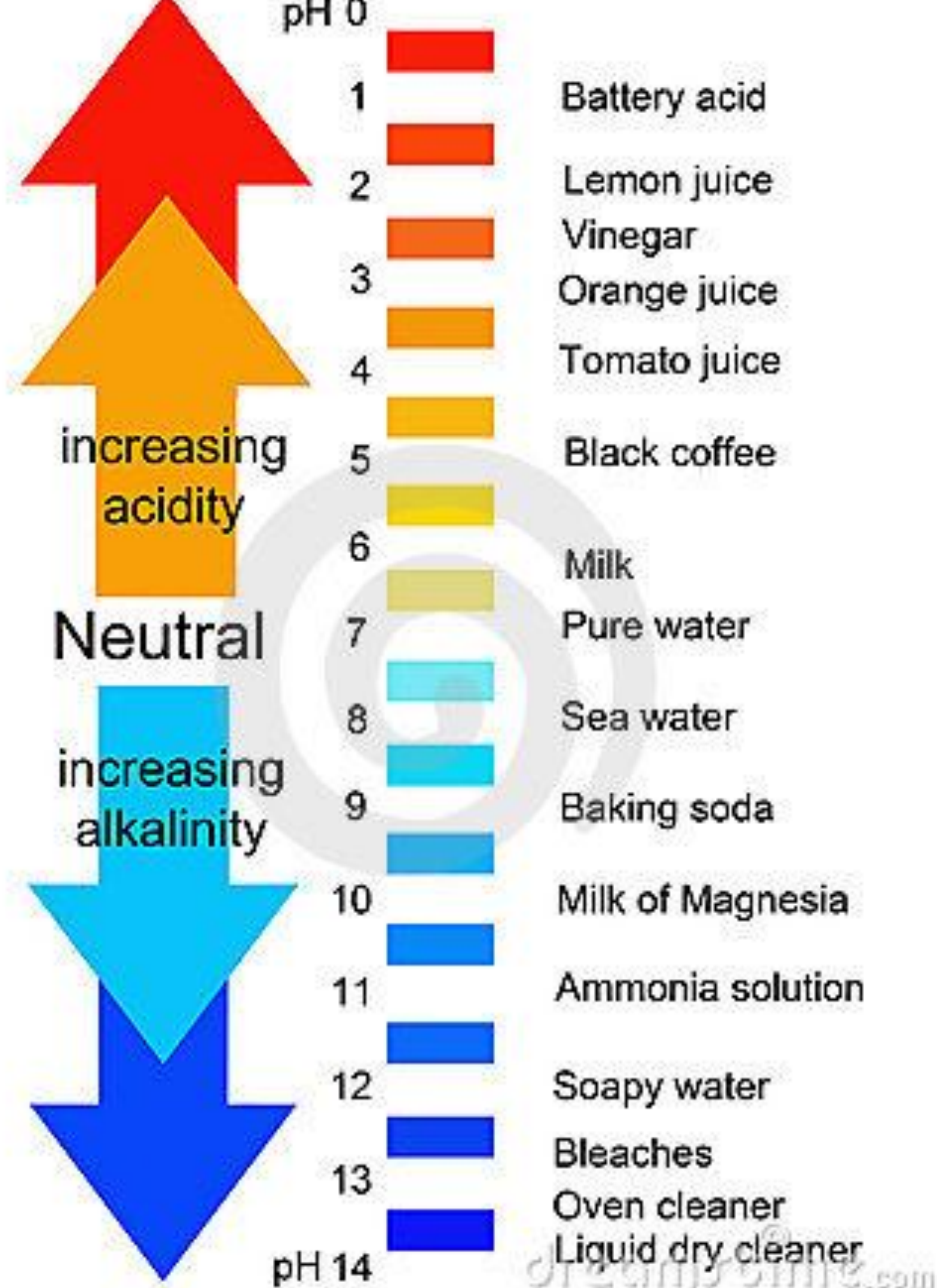
1. **A neutral solution where** $[H^+]=[OH^-]=1x10^{-7}$
2. **An acidic solution where** $[H^+]>[OH^-]$
3. **A basic/alkaline solution where** $[H^+]<[OH^-]$

Increasing hydrogen ion concentration

pH 6 is 10 times more acidic than pH 7 and 100 times more acidic than pH 8

pH scale




pH 0
1
2
3
4
5
6
7
8
9
10
11
12
13
pH 14
increasing acidity
Neutral
increasing alkalinity
Battery acid
Lemon juice
Vinegar
Orange juice
Tomato juice
Black coffee
Milk
Pure water
Sea water
Baking soda
Milk of Magnesia
Ammonia solution
Soapy water
Bleaches
Oven cleaner
Liquid dry cleaner

$$pH = -\log[H]^{+}$$

A substance has a hydrogen ion concentration of

$\left[H^{+}\right] = 2.7 \times 10^{-5}$ moles per litre.

Determine the pH and classify the substance as an acid or a base.

Solution:

$$\begin{aligned} pH &= -\log\left[H^{+}\right] \\ &= -\log\left(2.7 \times 10^{-5}\right) \\ &= 4.6 \end{aligned}$$

Hence the substance is an acid.

DECIBEL LEVEL AT KATIE'S MATCH HIT 113.7

The decibel level at Katie's Olympic semi final was 113.7 dB

A jackhammer operates at sound level of 92 dB

Difference in dB = 113.7 – 92

Difference in B = 21.7

Difference in B=

Solution $\beta_2 - \beta_1 = \mathbf{120} - \mathbf{92} = \mathbf{28}\text{dB} = \mathbf{2.8}\text{B}$

$\mathbf{10^{2.8} \approx 630}$

The Who are (were) loud!!

rd for the loudest concert.

nt of the speakers was

The decibel scale for sound intensity level

- Your ears can hear everything from your fingertip brushing lightly over your skin to a loud jet engine.

- In terms of power, the sound of the jet engine is about 1,000,000,000,000 times more powerful than the smallest audible sound. **That's a big range!**
- **We need a log scale**.

- The **ear responds to the ratio of the intensities of sounds** (measured in watts/m^2) and not to their differences.

Decibel Scale

If an intensity changes from I_1 to I_2 we define the change in the number of decibels as follows :

Number of **decibels(dB)** change = $$10\log_{10}\frac{I_2}{I_1}$$

Intensity levels in decibels for some common sounds:

Near total silence	0 dB
A whisper	15dB
Normal conversation	60 dB
A lawnmower	90 dB
A car horn	110 dB
A rock concert or a jet engine	120 dB

Show that an increase of 3 dB represents a doubling of sound intensity.

The loudest concert

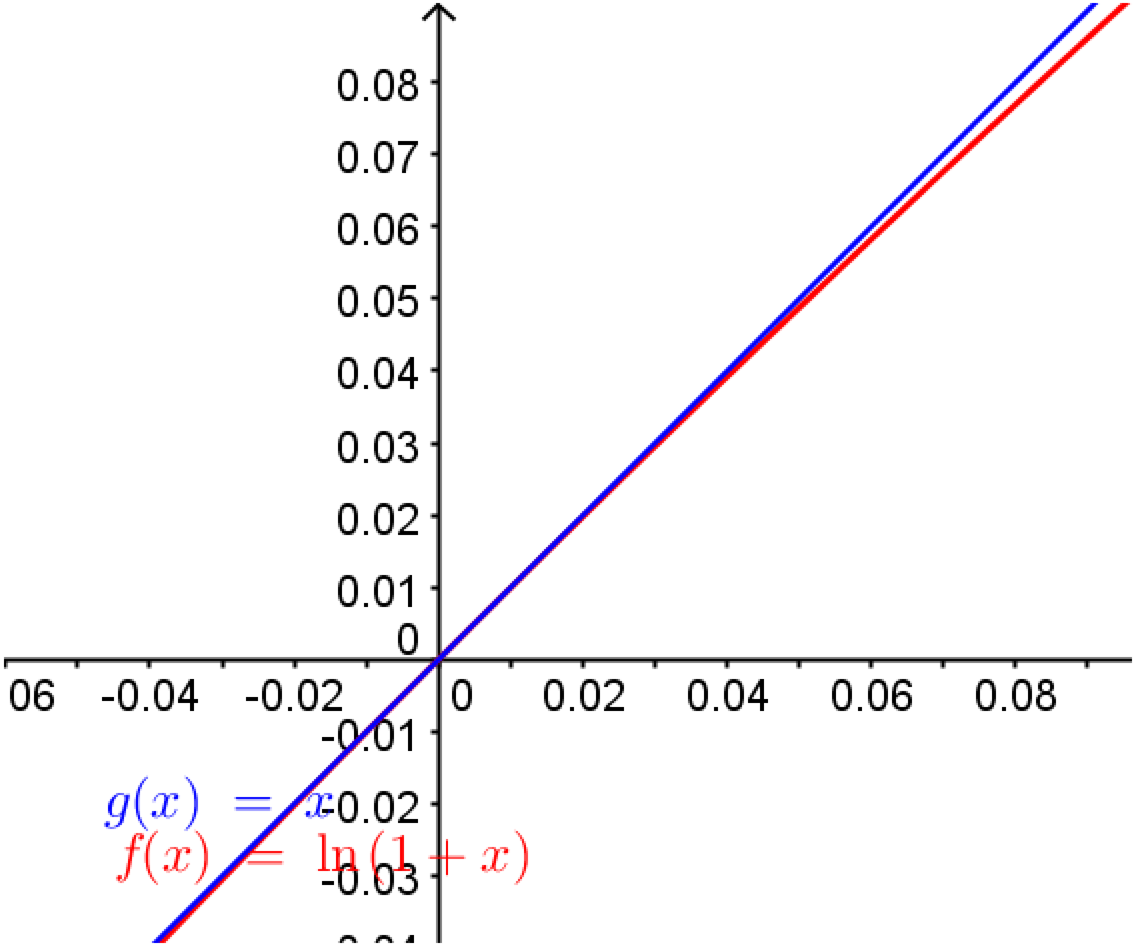
In 1976 the Who set a record for the loudest concert.
The sound level 46 m in front of the speakers was
$\beta_2 = 120$dB.
What is the ratio of the intensity I_2 of the band sound at that spot compared to the intensity I_1 of the sound from a jackhammer operating at sound level of $\beta_1 = 92$ dB?

Solution $\beta_2 - \beta_1 = 120 - 92 = 28\text{dB} = 2.8\text{B}$

$$10^{2.8} \approx 630$$

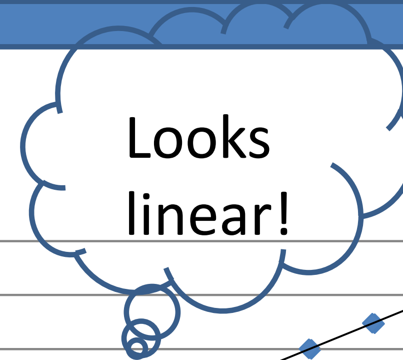
The Who are (were) loud!!

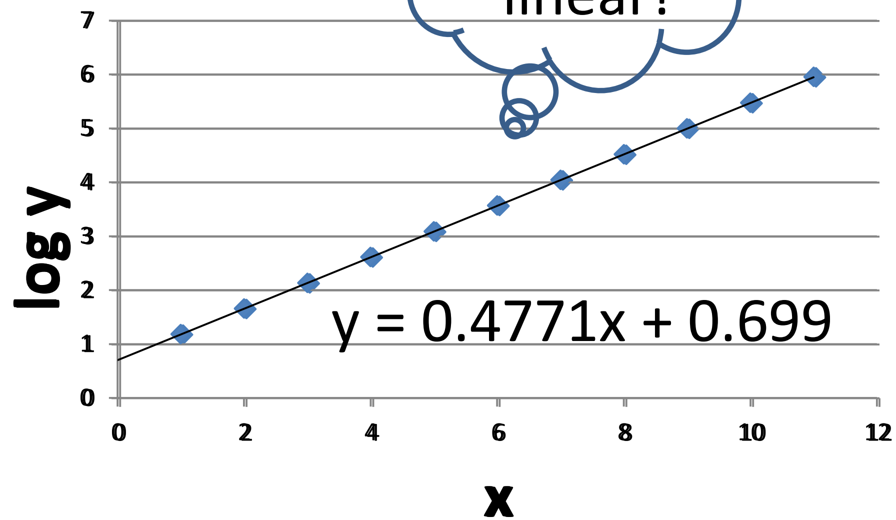
$\ln(1 + \boldsymbol{x}) \cong \boldsymbol{x}$, **for small** $\boldsymbol{x}$



What is the relationship between x and y?

x	y	log y
1	15	1.176091
2	45	1.653213
3	135	2.130334
4	405	2.607455
5	1215	3.084576
6	3645	3.561698
7	10935	4.038819
8	32805	4.51594
9	98415	4.993061
10	295245	5.470183
11	885735	5.947304





$$y = 5(3)^x$$

$$y = ab^x$$

$$\log_{10} y = \log_{10} a + x \log_{10} b$$

$$Y = c + x\, m$$

$$Y = 0.699 + x\,(0.4771)$$

$$a = 10^{0.699} = 5$$

$$b = 10^{0.4771} = 3$$

Transforming exponential graphs into linear graphs using logarithms.

Consider the function $y = 5(3)^x$.

$$y = 5(3)^x$$

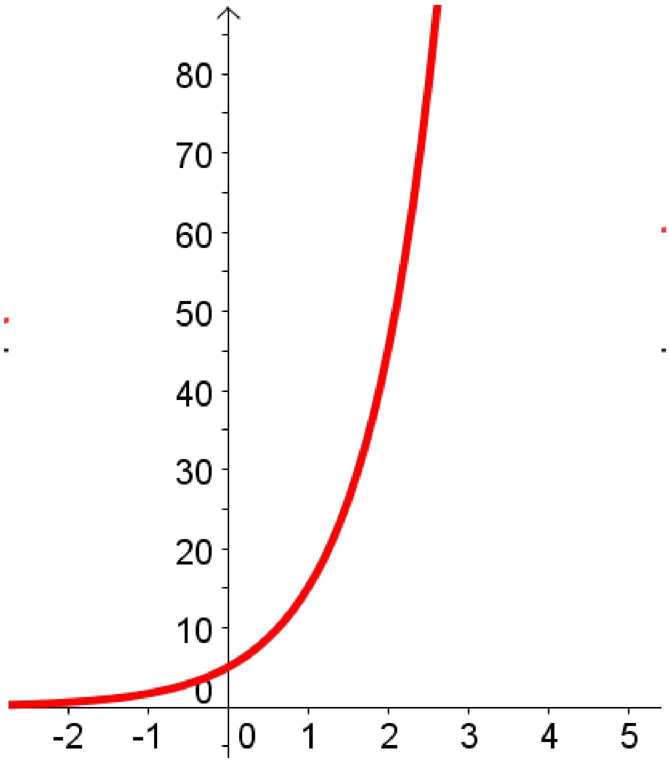
$$\log y = \log(5) + x\log(3)$$

↓ ↓ ↓ ↓

$$Y = C + X\,M$$

This is a line with slope = log (3) and

y- intercept = log (5)



In general for $y = ab^x$, if we plot $\log(y)$ against x:

y-intercept = $\log(a)$

Slope = $\log(b)$

Have a chat about rules of logs and some activity to reinforce them.

Séana agus logartaim — Indices and logarithms

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$\left(a^p\right)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a\left(x^q\right) = q\log_a x$$

$$\log_a 1 = 0$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$a^x = y \Leftrightarrow \log_a y = x$$

$$\log_a(a^x) = x$$

$$a^{\log_a x} = x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Most commonly used log bases

Base 10

- Engineering fields
- Decibel Scale
- Richter scale
- Google Page Rank
- pH

Base 2

- Computer Science
- Information Theory
- Mathematics

Base e

- Mathematical Analysis
- Physics and Chemistry
- Statistics
- Economics
- Engineering fields

Your turn....

A base is always a base

A log is a power for a base

Exponential form	Equivalent log form
$5^2 = 25$	$\log_5 25 = 2$
$(5)^{-2} = \frac{1}{25}$	$\log_5\left(\frac{1}{25}\right) = -2$
$10^1 = 10$	$\log_{10} 10 = 1$
$9^{\frac{1}{2}} = 3$	$\log_9 3 = \frac{1}{2}$
$27^{\frac{1}{3}} = 3$	$\log_{27} 3 = \frac{1}{3}$
$b^0 = 1$	$\log_b 1 = 0$

The input for the log is the output for the exponential.

$$\log_2 a = c \Rightarrow 2^c = a$$

Your turn….

A base is always a base

A log is a power for a base

Evaluate	Equivalent exponential form
$\log_2 16 = \mathbf{4}$	$2^4 = \mathbf{16}$
$\log_2 8 = \mathbf{3}$	$2^3 = \mathbf{8}$
$\log_2 1 = \mathbf{0}$	$2^0 = \mathbf{1}$
$\log_2 1024 = \mathbf{10}$	$2^{10} = \mathbf{1024}$
$\log_2 2 = \mathbf{1}$	$2^1 = \mathbf{2}$
$\log_2 \boldsymbol{a} = \boldsymbol{c}$	$2^{c} = \mathbf{a}$

The input for log is the output for the exponential.

$$\log_2 \boldsymbol{a} = \boldsymbol{c} \Rightarrow 2^{c} = \boldsymbol{a}$$

Q7 b 2012 Paper 1 LCFL Phase 3

Adapt for OL and HL

(b) A scientist is growing bacteria in a dish. The number of bacteria starts at 10 000 and doubles every hour.

(i) Complete the table below to show the number of bacteria over the next five hours.

Time in hours	0	1	2	3	4	5
Number of bacteria (in thousands)	10					

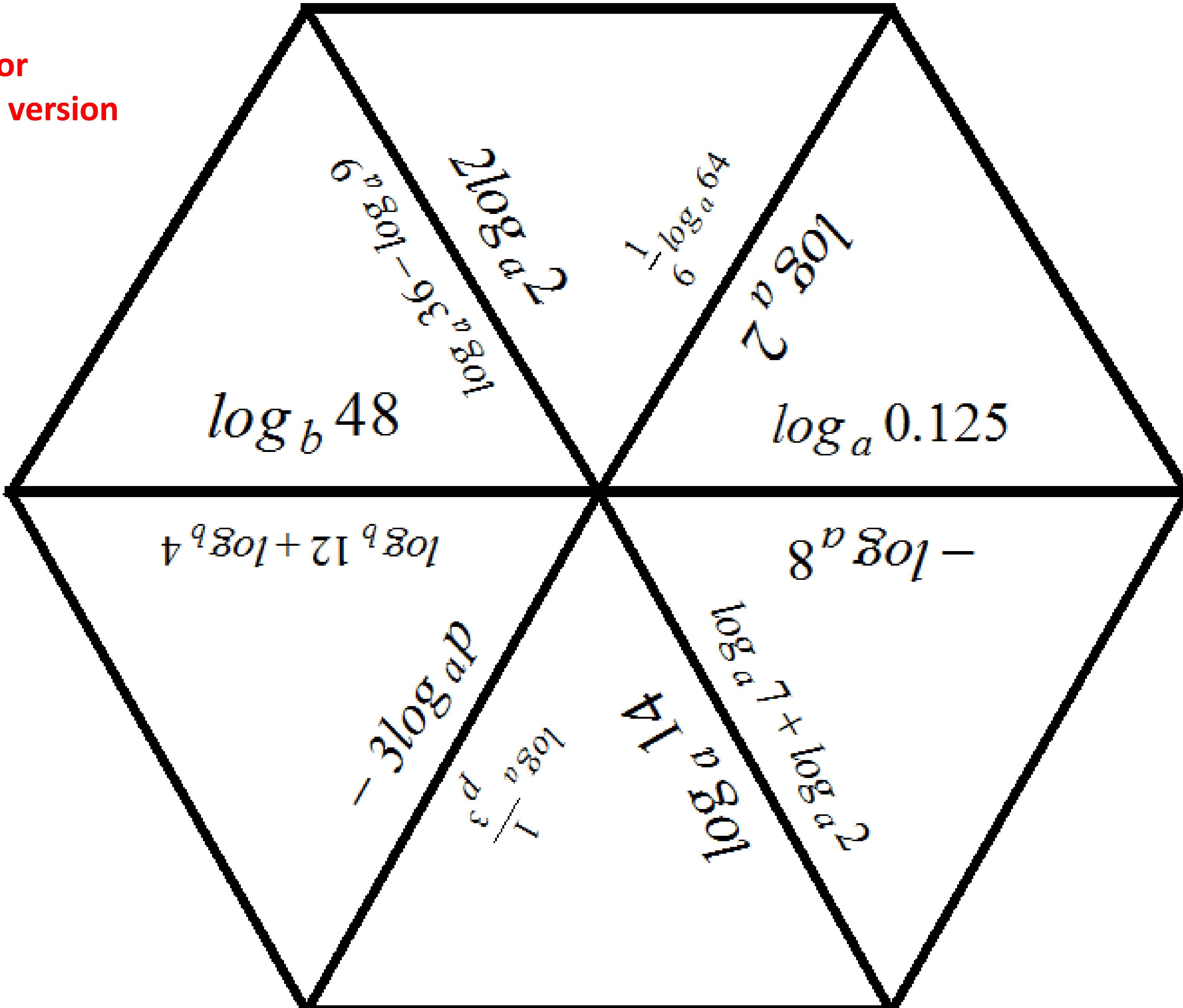
(ii) Draw a graph below to show the number of bacteria over the five hours.

(iii) Use your graph to estimate the number of bacteria in the dish after $2\frac{1}{2}$ hours.

Answer: ________________

(iv) The scientist is growing the bacteria in order to do an experiment. She needs at least 250 000 bacteria in the dish to do the experiment. She started growing the bacteria at 10:00 in the morning. At what time is the dish of bacteria ready for the experiment?

Solution for simplified version



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Answer: ____________

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We started here…….

$$f(x) = 2^x$$

$$x = 6$$

in

$$2^6 = 64$$

out

64

A new function is born whose input ………

$$g(x) = \log_2(x) = f^{-1}(x)$$

$$x =$$

in

$$\log_2(64) = \log_2(2^6) = 6$$

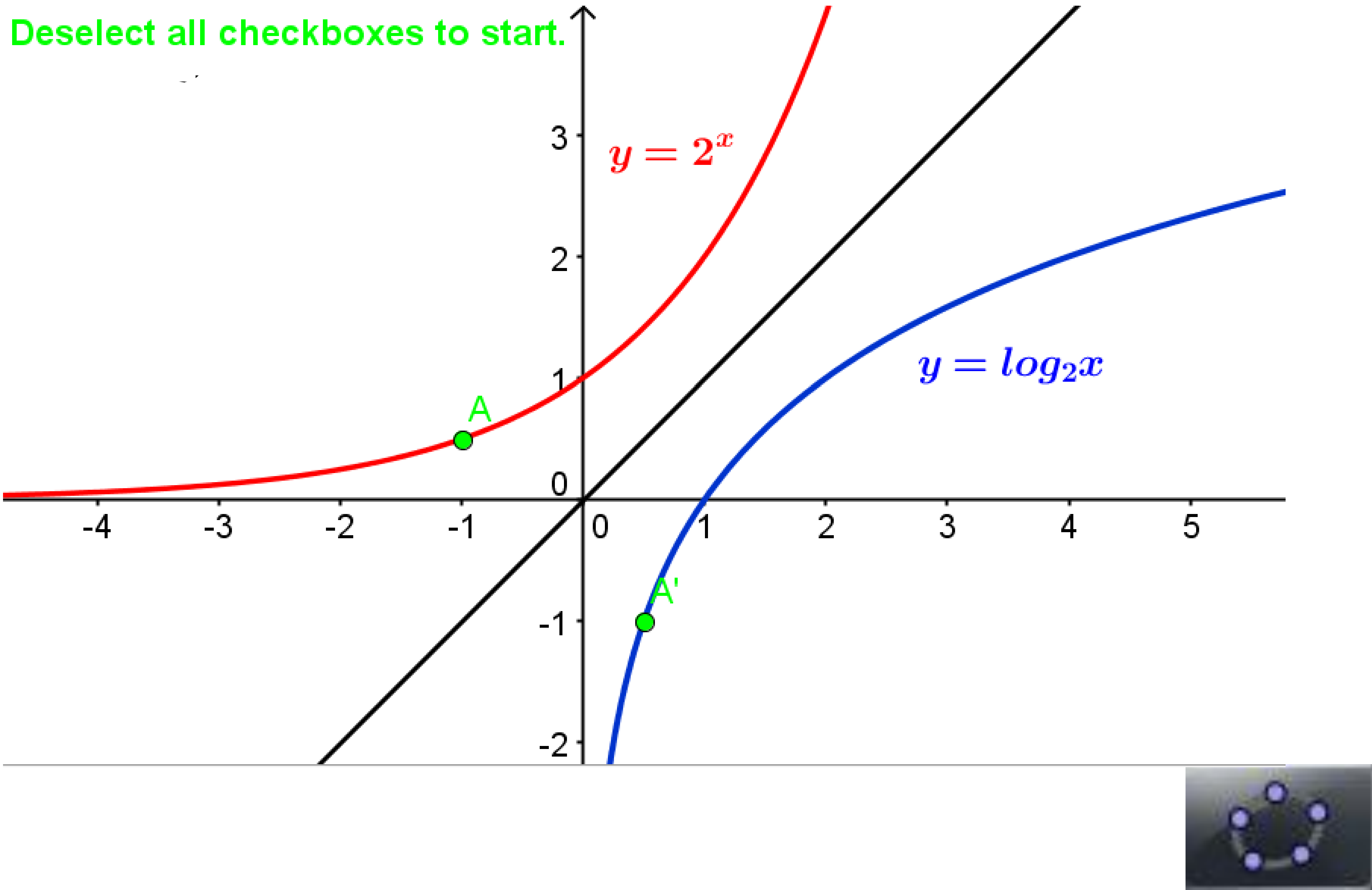
out

6

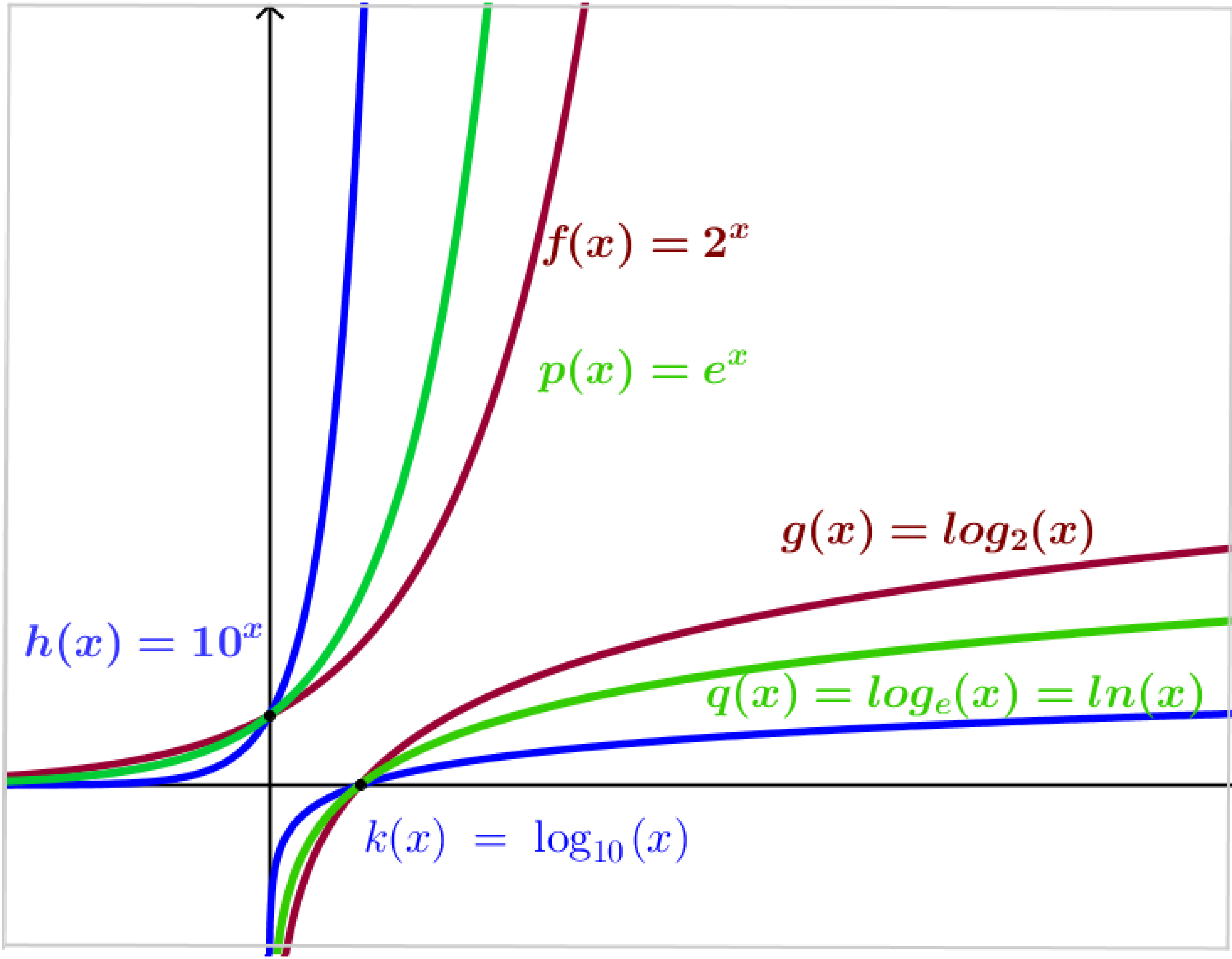
$$\log_2(64) = \log_2(2)^6 = 6$$

- **What type of function is f(x) = 2^x?**
- **What does this mean for the inverse relation of f(x) = 2^x?**

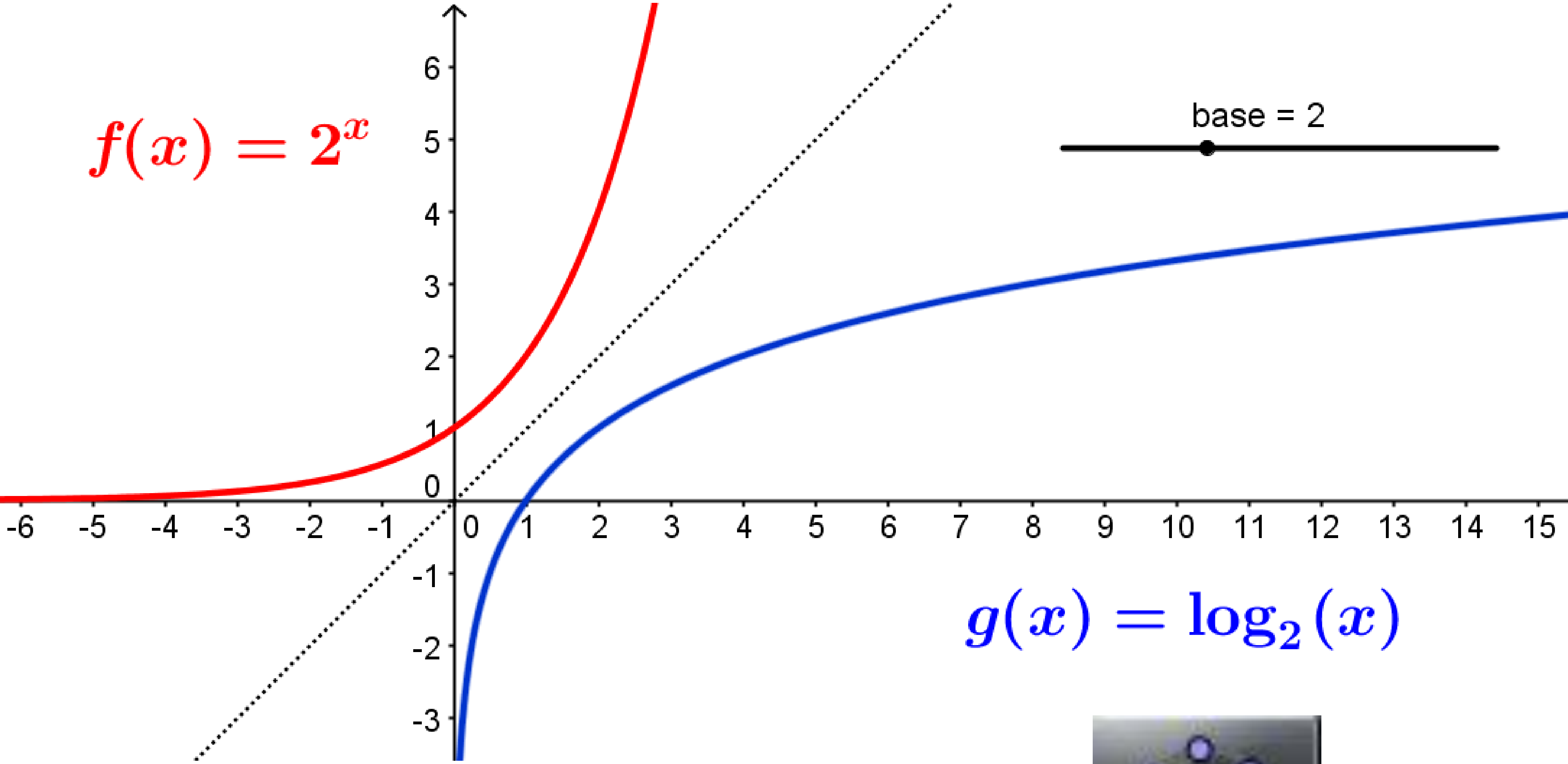
Graphs of $f(x) = 2^x$ and $f^{-1}(x) = \log_2(x)$



Sketching...........



Graphs of $y = b^x$ and $y = \log_b(x)$, variable base



$y = \log_{\frac{1}{a}}(x)$ is a reflection in the x-axis of the graph of $y = \log_a(x)$.

Justify.

$$y = \log_a(x)$$

$$\Rightarrow a^y = x$$

$$\Rightarrow \left(\frac{1}{a}\right)^{-y} = x$$

$$\Rightarrow \log_{\frac{1}{a}} x = -y$$

$$\Rightarrow y = -\log_{\frac{1}{a}} x$$

Hence the graph of $y = \log_{\frac{1}{a}} x$ is the image of $y = \log_a(x)$ by reflection in the x-axis

This slide is needed for the graph matching

Activities

1. Mesopotamia and Drug absorption (identifying gaps in our knowledge)
2. Using 2^x table to reduce multiplication to addition etc.
3. Comparing two earthquake amplitudes given their Richter magnitude ratings.
4. Familiarity with the notation activity – logs to indices and indices to logs.
5. Finding $\log_2$ of a number not in the table from the graph and using the log to any base button on the calculator
6. Completing Mesopotamia and drug absorption using the calculator
7. Adapting LC FL paper
8. Drawing graph of $g(x) = \log_2(x)$ and describing it
9. Geogebra file showing log and exponential as inverses
10. Geogebra file showing variable base for log and exponential
11. Sketching graphs of log and exp for base 2,10 and e

Other activities

1. Rules for logs (seeing patterns)
2. True/False discussion
3. Common misconceptions
4. Fill in the blanks
5. Approximating discrete growth using e (**not done on Thursday but important given LCHL 2012)**

On slides but not done on Thursday

1. Comparing sound level of the Who concert with sound level from a jackhammer
2. Finding pH

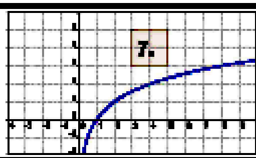
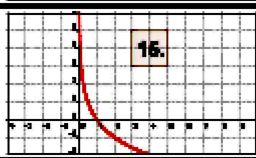
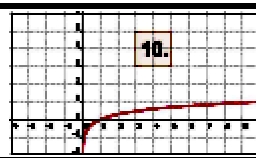
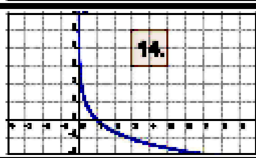
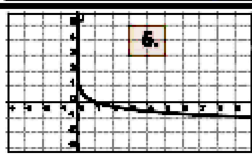
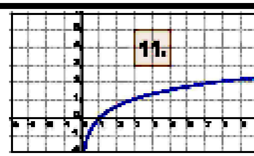
Graph matching

Tarsia hexagon – matching expressions

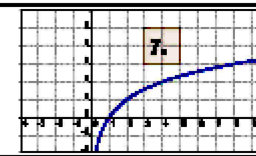
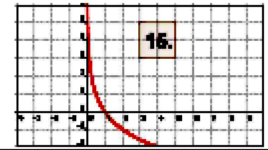
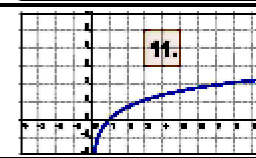
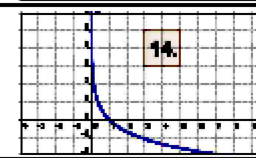
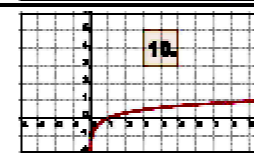
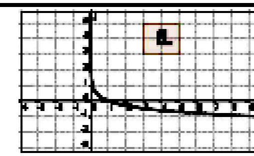
LCFL 2012 Q7(b) - adapt of LCOL and LCHL

Matching Exercise

Student Output

$j(x) = log_{\frac{1}{10}}(x)$	$w(x) = log_{10}(x)$
	$n(x) = ln(x)$
	
	$p(x) = log_2(x)$
	
$r(x) = log_{\frac{1}{2}}(x)$	$u(x) = log_{\frac{1}{e}}(x)$

Solution

$p(x) = log_2(x)$	
$r(x) = log_{\frac{1}{2}}(x)$	
$n(x) = ln(x)$	
$u(x) = log_{\frac{1}{e}}(x)$	
$w(x) = log_{10}(x)$	
$j(x) = log_{\frac{1}{10}}(x)$	

http://www.mmlsoft.com/index.php?option=com_content&task=view&id=11&Itemid=12

Link to oodles more Tarsias: http://www.mrbartonmaths.com/jigsaw.htm

True or false discussion

Equation	Equivalent form	True /False	Correct equation (if false)
$\log_2 8 = 4$			
$\log_3 81 = 4$			
$\log_{10} 5 + \log_{10} 10 = \log_{10} 15$			
$\log_2 64 - \log_2 4 = \log_2 16$			
$\log_3 \left(\frac{1}{81}\right) = -4$			
$2\log_2 8 = \log_2 16$			
$\log_2 4 + \log_2 128 = \log_2 512$			

Equation	True /False	Explanation
$\frac{\log_2 64}{\log_2 4} = \log_2 16$		
$\frac{\log_2 64}{\log_2 4} = 16$		
$\frac{\log_2 64}{\log_2 16} = \log_2 (64 - 16)$		
$(\log_b a)^c = c(\log_b a)$		

Equation	True /False	Explanation
$\frac{\log_2 64}{\log_2 4} = \log_2 16$		
$\frac{\log_2 64}{\log_2 4} = 16$		
$\frac{\log_2 64}{\log_2 16} = \log_2 (64 - 16)$		
$(\log_b \boldsymbol{a})^c = \boldsymbol{c}(\log_b \boldsymbol{a})$		

Give possible numbers or variables for the blanks in the equations below

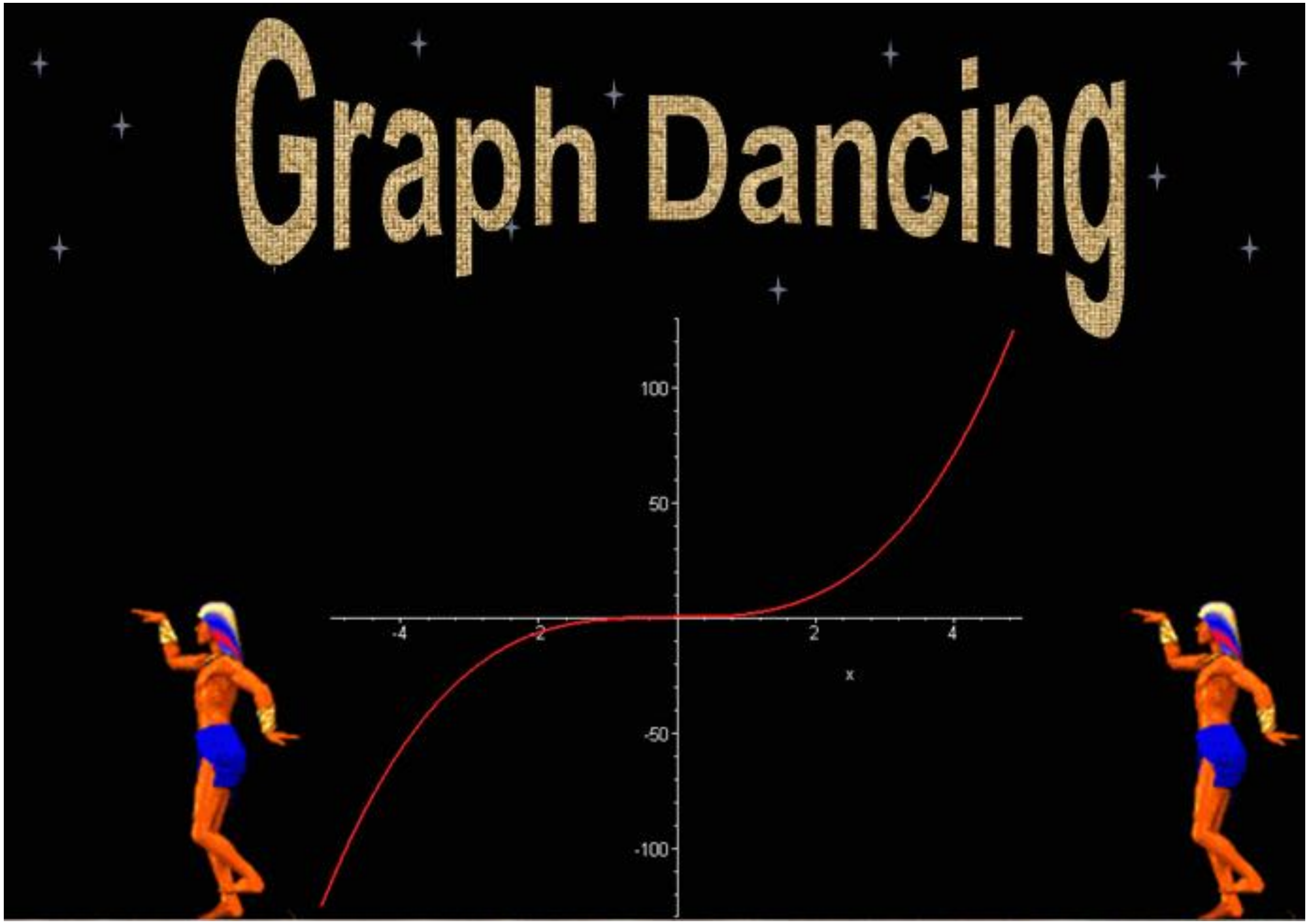
$$\log_{\square} \square = 3$$

$$\log_{\square} \square + \log_{\square} \square = 8$$

$$\square \log_{\square} \square = \log_{\square} \square$$

$$\log_{\square} \square - \log_{\square} \square = 6$$

http://www.tes.co.uk/teaching-resource/Egyptian-Graph-Dancing-6020550/



Click on the image to link to the powerpoint.