Functions

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WS07.01 Applications of Sequences and Series

Task 1 Investigating Compound Interest

1. If each block represents €10, shade in €100.
2. Then, using another colour, add 20% to the original shaded area.
3. Finally, using a third colour, add 20% of the entire shaded area.
4. What is the value of the second shaded area? ______________________________
5. What is the value of the third shaded area? ______________________________
6. Why do they not have the same amount? ______________________________
7. Complete the following table and investigate the patterns which appear.

Days (Time Elapsed)	**Amount**	**Increase by %**	**Total decimal**	**Pattern/Total Amount of money received per day**
0		0		100
	€120.00	20%	1.2	100×1.2
				$100 \times 1.2 \times$

8. Can you find a way of getting the value for day 10 without having to do the table to day 10?

____

____

9. Use your whiteboard to graph amount against time. Is the relationship linear?

____

Task 2 Reducing Balance

David and Michael are going on the school tour this year. They are each taking out a loan of €600, which they hope to pay off over the next year. Their bank is charging a monthly interest rate of 1.5% on loans.

David says that with his part-time work at present he will be able to pay €100 for the first 4 months but will only be able to pay off €60 a month after that.

Michael says that he can only afford to pay €60 for the first 4 months and then €100 after that.

Michael reckons that they are both paying the same amount for the loan. Why?

____

____

Note: This problem is posed based on the following criteria:

(a) A loan is taken out
(b) after 1 month interest is added on
(c) the person then makes his/her monthly repayment.

This process is then repeated until the loan is fully paid off.

	David			Michael		
Time	Monthly Total	Interest	Payment	Monthly Total	Interest	Payment
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

1. What do David and Michael have in common at the beginning of the loan period?

____

2. Calculate the first 3 months transactions for each. (How much, in total, had they each paid back after 3 months?) David _________ Michael __________.

3. What is the total interest paid by each? David _________ Michael __________.

4. Based on your answers to the first 3 questions, when would you recommend making the higher payments and why?

____

5. Is Michael's assumption that they will eventually pay back the same amount valid?

____

6. Using your whiteboard, plot the amount of interest added each month to both David's and Michael's account.

7. Looking at the graph, who will pay the most interest overall?

____

____

WS07.02 Exponential Functions

Aim: To study the properties of exponential functions and learn the features of their graphs

Section A - Activity 1: The Exponential Function, $f(x) = 2^x$.

1. For $f(x) = 2^x$:

(i) The base of $f(x) = 2^x$ is ______________________

(ii) The exponent of $f(x) = 2^x$ is ______________________

(iii) What is varying in the function $f(x) = 2^x$? ______________________

(iv) What is constant in the function $f(x) = 2^x$? ______________________

2. For $f(x) = 2^x$:
What are the possible inputs i.e. values for x (the domain)?

Natural numbers ☐ Irrational numbers ☐
Integers ☐ Real numbers ☐
Rational numbers ☐

3. Set up a table of values and draw the graph of $f(x) = 2^x$ on your whiteboard:

X	2^x	$y = f(x)$
-4	2^{-4}	$\frac{1}{16}$
-3		
-2		
-1		
0		
1		
2		
3		
4		

4. Describe the graph of $f(x) = 2^x$:

(i) Is it a straight line? ______________________

(ii) Is $y = f(x)$ increasing or decreasing as x increases? ______________________

(iii) From the table above, find the average rate of change over different intervals.
For example from -2 to -1 and 2 to 3.

What do you notice? ______________________

____

(iv) Describe how the curvature/rate of change is changing. ______________________

____

5. For $f(x) = 2^x$:

(i) What are the possible outputs (range) for $f(x) = 2^x$. ____________________

(ii) Is it possible to have negative outputs? Explain why? ____________________

____

(iii) What happens to the output as x decreases? ____________________

____

(iv) Is an output of 0 possible? Why do you think this is? ____________________

____

(v) What are the implications of this for the x-intercept of the graph? ____________________

____

(vi) What is the y-intercept of the graph of $f(x) = 2^x$? ____________________

____

Section A – Activity 2: The Exponential Function, $g(x) = 3^x$.

1. For $g(x) = 3^x$:

(i) The base of $g(x) = 3^x$ is ______________________________

(ii) The exponent of $g(x) = 3^x$ is ______________________________

(iii) What is varying in the function $g(x) = 3^x$? ______________________________

(iv) What is constant in the function $g(x) = 3^x$? ______________________________

2. For $g(x) = 3^x$:
What are the possible inputs i.e. values for x (the domain)?

Natural numbers ☐ Irrational numbers ☐
Integers ☐ Real numbers ☐
Rational numbers ☐

3. Set up a table of values and draw the graph of $g(x) = 3^x$ on your whiteboard:

X	3^x	$y = g(x)$
-4	3^{-4}	$1/81$
-3		
-2		
-1		
0		
1		
2		
3		
4		

4. In relation to the graph of $g(x) = 3^x$:

(i) Is it a straight line? ______________________________

(ii) Is $y = g(x)$ increasing or decreasing as x increases? ______________________________

(iii) From the table above, find the average rate of change over different intervals.
For example from -2 to -1 and 2 to 3.

What do you notice? ______________________________

____

(iv) Describe how the curvature/rate of change is changing. ______________________________

____

5. For $g(x) = 3^x$:

(i) What are the possible outputs (range) for $g(x) = 3^x$. ______________________

(ii) Is it possible to have negative outputs? Explain why? ______________________

____

(iii) What happens to the output as x decreases? ______________________

____

(iv) Is an output of 0 possible? Why do you think this is? ______________________

____

(v) What are the implications of this for the x-intercept of the graph? ______________

____

(vi) What is the y-intercept of the graph of $g(x) = 3^x$? ______________________

____

Section A – Activity 3: Compare the graph of $f(x) = 2^x$ with the graph of $g(x) = 3^x$.

1. How are they similar and how do they differ? ______________________

____

____

2. Consider the relations $\{(x, y) | x \in \mathbb{R},\ y \in \mathbb{R},\ y = 2^x\}$ and $\{(x, y) | x \in \mathbb{R},\ y \in \mathbb{R},\ y = 3^x\}$.

Are they functions? Explain. ______________________

____

____

3. What name do you think is given to this type of function and why do you think it is given this name?

____

____

Section A – Activity 4: Understand the characteristics of $f(x) = a^x$, $a > 1$.

1. What is the domain of $f(x) = a^x$, $a > 1$? ______________________

2. In relation to the graph of $f(x) = a^x$, $a > 1$.

(i) Is it a straight line? ______________________

(ii) Is $y = f(x)$ increasing or decreasing as x increases? ______________________

(iii) Does it have a maximum value? ______________________

(iv) Does it have a minimum value? ______________________

(v) Describe how its curvature/rate of change is changing. ______________________

____

3. What is the range of $f(x) = a^x$, $a > 1$? ______________________

4. What is the x-intercept of the graph $f(x) = a^x$, $a > 1$? ______________________

5. What is the y-intercept of the graph $f(x) = a^x$, $a > 1$? ______________________

Section B – Activity 1: The Exponential Function, $f(x)=\left(\frac{1}{2}\right)^x$.

1. For $f(x)=\left(\frac{1}{2}\right)^x$:

(i) The base of $f(x)=\left(\frac{1}{2}\right)^x$ is ______________________

(ii) The exponent of $f(x)=\left(\frac{1}{2}\right)^x$ is ______________________

(iii) What is varying in the function $f(x)=\left(\frac{1}{2}\right)^x$? ______________________

(iv) What is constant in the function $f(x)=\left(\frac{1}{2}\right)^x$? ______________________

2. For $f(x)=\left(\frac{1}{2}\right)^x$:

What are the possible inputs i.e. values for x (the domain)?

Natural numbers	☐	Irrational numbers	☐
Integers	☐	Real numbers	☐
Rational numbers	☐		

3.

Set up a table of values and draw the graph of $f(x)=\left(\frac{1}{2}\right)^x$ on your whiteboard:

X	$\left(\frac{1}{2}\right)^x$	$y=f(x)$
-4	$\left(\frac{1}{2}\right)^{-4}$	16
-3		
-2		
-1		
0		
1		
2		
3		
4		

4. In relation to the graph of $f(x)=\left(\frac{1}{2}\right)^x$:

(i) Is it a straight line? ______________________

(ii) Is $y=f(x)$ increasing or decreasing as x increases? ______________________

(iii) From the table above, find the average rate of change over different intervals.
For example from -2 to -1 and 2 to 3.

What do you notice? ______________________

____

(iv) Describe how the curvature/rate of change is changing. ______________________

____

5. For $f(x)=\left(\frac{1}{2}\right)^x$:

(i) What are the possible outputs (range) for $f(x)=\left(\frac{1}{2}\right)^x$. ______________________

(ii) Is it possible to have negative outputs? Explain why? ______________________

____

(iii) What happens to the output as x decreases? ______________________

____

(iv) Is an output of 0 possible? Why do you think this is? ______________________

____

(v) What are the implications of this for the x-intercept of the graph? ______________________

____

(vi) What is the y-intercept of the graph of $f(x)=\left(\frac{1}{2}\right)^x$? ______________________

____

Section B – Activity 2: The Exponential Function, $g(x)=\left(\frac{1}{3}\right)^x$.

1. For $g(x)=\left(\frac{1}{3}\right)^x$:

(i) The base of $g(x)=\left(\frac{1}{3}\right)^x$ is ______________________________

(ii) The exponent of $g(x)=\left(\frac{1}{3}\right)^x$ is ______________________________

(iii) What is varying in the function $g(x)=\left(\frac{1}{3}\right)^x$? ______________________________

(iv) What is constant in the function $g(x)=\left(\frac{1}{3}\right)^x$? ______________________________

2. For $g(x)=\left(\frac{1}{3}\right)^x$:

What are the possible inputs i.e. values for x (the domain)?

Natural numbers	☐	Irrational numbers	☐
Integers	☐	Real numbers	☐
Rational numbers	☐		

3.

Set up a table of values and draw the graph of $g(x)=\left(\frac{1}{3}\right)^x$ below on your whiteboard:

X	$\left(1/3\right)^x$	$y=g(x)$
-4	$\left(1/3\right)^{-4}$	81
-3		
-2		
-1		
0		
1		
2		
3		
4		

4. In relation to the graph of $g(x)=\left(\frac{1}{3}\right)^x$:

(i) Is it a straight line? ________________________________

(ii) Is y increasing or decreasing as x increases? ________________________

(iii) From the table above, find the average rate of change over different intervals. For example from -2 to -1 and 2 to 3.

What do you notice? ______________________________

____

(iv) Describe how the curvature/rate of change is changing. __________________

____

5. For $g(x)=\left(\frac{1}{3}\right)^x$:

(i) What are the possible outputs (range) for $g(x)=\left(\frac{1}{3}\right)^x$. __________________

(ii) Is it possible to have negative outputs? Explain why? ____________________

____

(iii) What happens to the output as x decreases? ________________________

____

(iv) Is an output of 0 possible? Why do you think this is? ____________________

____

(v) What are the implications of this for the x-intercept of the graph? ____________

____

(vi) What is the y-intercept of the graph of $g(x)=\left(\frac{1}{3}\right)^x$? ____________________

____

Section B - Activity 3: Compare the graph of $f(x)=\left(\frac{1}{2}\right)^x$ with the graph of $g(x)=\left(\frac{1}{3}\right)^x$.

1. How are they the same and how do they differ? ______________________

____

____

2. Consider the relations $\left\{(x,y)\middle|\, x\in\mathbb{R},\ y\in\mathbb{R},\ y=\left(\frac{1}{2}\right)^x\right\}$ and $\left\{(x,y)\middle|\, x\in\mathbb{R},\ y\in\mathbb{R},\ y=\left(\frac{1}{3}\right)^x\right\}$.

Are they functions? Explain. ______________________

____

____

3. What name do you think we give to this type of function and why do you think it is given this name?

____

____

Section B - Activity 4: Understand the characteristics of $f(x)=a^x$, $0<a<1$.

1. What is the domain of $f(x)=a^x$, $0<a<1$? ______________________

2. In relation to the graph of $f(x)=a^x$, $0<a<1$.

(i) Is it a straight line? ______________________

(ii) Is $y=f(x)$ increasing or decreasing as x increases? ______________________

(iii) Does it have a maximum value? ______________________

(iv) Does it have a minimum value? ______________________

(iv) Describe how its curvature/rate of change is changing. ______________________

____

3. What is the range of $f(x)=a^x$, $0<a<1$? ______________________

4. What is the x-intercept of the graph $f(x)=a^x$, $0<a<1$? ______________________

5. What is the y-intercept of the graph $f(x)=a^x$, $0<a<1$? ______________________

Note: For all the following, you should assume that the domain is $\mathbb{R}$.

Section C - Activity 1: Compare the graph of $f(x) = 2^x$ with the graph of $f(x) = \left(\frac{1}{2}\right)^x$.

1. How are the graphs similar? ______________________________

____

2. How are the graphs different? ______________________________

____

3. Rewrite $f(x) = \left(\frac{1}{2}\right)^x$ in the form $f(x) = 2^k$. ______________________________

4. What transformation maps the graph of $f(x) = 2^x$ onto the graph of $f(x) = \left(\frac{1}{2}\right)^x$?

____

Section C - Activity 2: Compare the graph of $g(x) = 3^x$ with the graph of $g(x) = \left(\frac{1}{3}\right)^x$.

1. How are the graphs similar? ______________________________

____

2. How do the graphs differ? ______________________________

____

3. Rewrite $g(x) = \left(\frac{1}{3}\right)^x$ in the form $g(x) = 3^k$. ______________________________

4. What transformation maps the graph of $g(x) = 3^x$ onto the graph of $g(x) = \left(\frac{1}{3}\right)^x$?

____

1. If $f(x) = a^x$, $a \in \mathbb{R}$, $a > 1$, then the ***properties*** of the exponential ***function*** are:

2. If $f(x) = a^x$, $a \in \mathbb{R}$, $a > 1$, then the ***features*** of the exponential ***graph*** are:

3. If $f(x) = a^x$, $a \in \mathbb{R}$, $0 < a < 1$, then the ***properties*** of the exponential ***function*** are:

4. If $f(x) = a^x$, $a \in \mathbb{R}$, $0 < a < 1$, then the ***features*** of the exponential ***graph*** are:

Section C – Activity 4: Which of the following equations represent exponential functions?

Function	Is it an exponential Function? Yes/No	Reason
$f(x)=\left(\frac{1}{2}\right)^x$		
$f(x)=x^2$		
$f(x)=(-2)^x$		
$f(x)=2(3)^x$		
$f(x)=-2^x$		
$f(x)=3(x)^{\frac{1}{2}}$		
$f(x)=(0.9)^x$		

Note: Extension Activities are required to strengthen students' abilities in the following areas from the syllabus:

Level	*Syllabus*	*Page*
JCHL	$f(x) = a2^x$ and $f(x) = a3^x$, where $a \in \mathbb{N}, x \in \mathbb{R}$.	Page 31
LCFL	$f(x) = a2^x$ and $f(x) = a3^x$, where $a \in \mathbb{N}, x \in \mathbb{R}$.	Page 32
LCOL	$f(x) = ab^x$, where $a \in \mathbb{N}$, $b, x \in \mathbb{R}$.	Page 32
LCHL	$f(x) = ab^x$, where a, b, $x \in \mathbb{R}$.	Page 32

1. A cell divides itself into two every day. The number of cells C after D days is obtained from the function:

$$C = 2^D$$

(a) Draw a graph of the function for $0 \le D \le 6$.
(b) Find the number of cells after 15 days.

2. The value of a mobile phone M (in cents) after T years can be obtained from the following function:

$$M = k\left(\frac{1}{2}\right)^T, \text{where } k \text{ is a constant.}$$

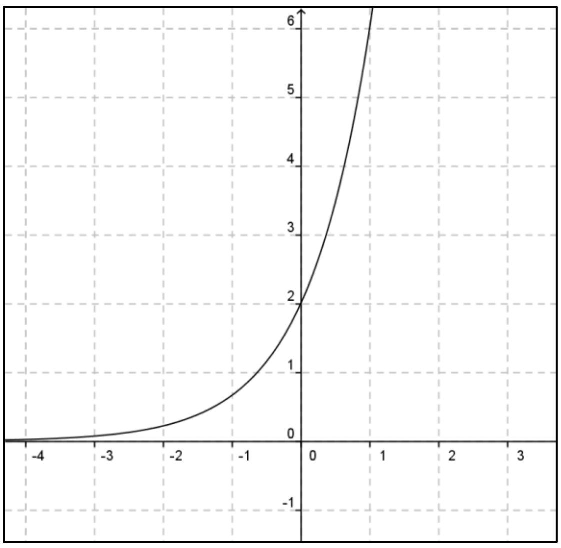
(a) Draw a graph of the function for $0 \le T \le 6$.
(b) Find the value of k given that the value of the mobile phone after 3 years is €100.
(c) Find the value of the phone after 7 years.

3. The number of bacteria B in a sample after starting an experiment for m minutes is given by:

$$B = 50(3)^{0.04m}$$

(a) Find the number of bacteria in the sample at the start of the experiment.
(b) Find the number of bacteria in the sample after starting the experiment for 3 hours.

4. The graph of $f(x) = ka^x$ is shown:
(a) Find the value of k and a.
(b) Hence find the value of $f(x)$ when $x = 8$.



Q5. Olive finds that the number of bacteria in a sample doubles every 5 hours. Originally there are 8 bacteria in the sample.

Complete the table below:

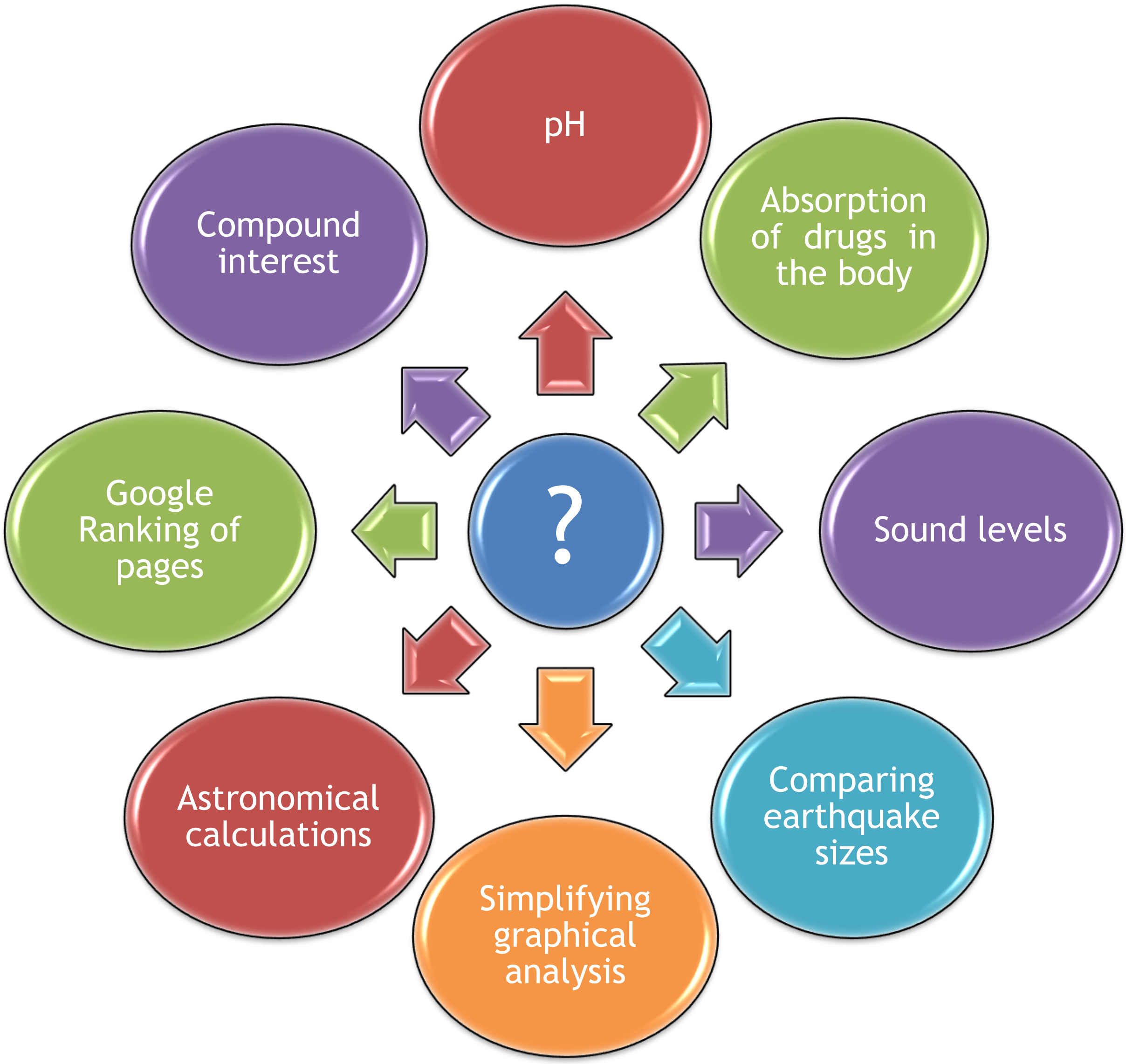
Number of hours (*hrs.*)	Number of bacteria (*b*)
0	8
5	
10	
15	

(a) Express *b* in terms of *h*.
(b) Find the number of bacteria in the sample after 13 hours.
(c) How many hours later will the number of bacteria be more than 100.

Q6. When a microwave oven is turned on for x minutes the relationship between the temperature C° inside the oven is given by $C(x) = 500 - 480(0.9)^x$ where $x \geq 0$.
(a) Find the value of $C(0)$.
(b) Explain the meaning of $C(0)$.
(c) Can the temperature inside the microwave oven reach $550C^\circ$?

Answers

Q1 (b) 32,768 cells, **Q2 (b)** €800 **(c)** €6.25, **Q3 (a)** 50 **(b)** 136,220 bacteria, **Q4 (a)** $k = 2$, $a = 3$ **(b)** 13,122, **Q5 (b)** $b = 8(2)^{\frac{h}{5}}$ **(c)** 48 bacteria **(d)** 18.22 hrs., **Q6 (a)** 20° C **(c)** No


pH
Compound interest
Absorption of drugs in the body
?
Google Ranking of pages
Sound levels
Astronomical calculations
Comparing earthquake sizes
Simplifying graphical analysis

Activity 1 Making the Most of a Euro

Invest €1 for 1 year at 100% compound Interest.

Investigate the change in the final value, if the annual interest rate of 100% is compounded over smaller and smaller time intervals.

The interest rate *i* per compounding period is calculated by dividing the annual rate of 100% by the number of compounding periods per year.

Compounding period	Final value, $F = P(1+i)^t$, where *i* is the interest rate for a given compounding period and *t* is the number of compounding periods per year. Calculate *F* correct to 8 decimal places.
Yearly $i = 1$	$F = 1(1+1)^1 = 2$
Every 6 mths. $i = \frac{1}{2}$	$F = 1\left(1+\frac{1}{2}\right)^2 = 2.25$
Every 3 mths. $i =$	
Every mth. $i =$	
Every week. $i =$	
Every day. $i =$	
Every hour. $i =$	
Every minute. $i =$	
Every second. $i =$	

What if the compounding period was 1 millisecond (10^{-3} s), 1 microsecond (10^{-6} s) or 1 nanosecond (10^{-9} s)? What difference would it make?

Will *F* ever reach 3? How about 2.8?

Activity 2 Further Exploration of Exponential Functions

1. How long will it take for a sum of money to double if invested at 20% compound interest rate compounded annually?

2. 500 mg of a medicine enters a patient's blood stream at noon and decays exponentially at a rate of 15% per hour.

(i) Write an equation to express the amount remaining in the patient's blood stream at after t hours.

(ii) Find the time when only 25 mg of the original amount of medicine remains active.

3.

x	2^x	*y*
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8
4	2^4	16
5	2^5	32
6	2^6	64
7	2^7	128
8	2^8	256
9	2^9	512
10	2^{10}	1024
11	2^{11}	2048
12	2^{12}	4096

(a) Describe the type of sequence formed by the numbers in the first column.

(b) Describe the type of sequence formed by the numbers in the second and third columns.

(c) Using the table, and your knowledge of indices, carry out the following operations of multiplication and division in the second sequence, linking the answer to numbers in the first sequence.

(i) 32×128

(ii) $4096 \div 512$

(iii) 8^4

x	2^x	y
13	2^{13}	8,192
14	2^{14}	16,384
15	2^{15}	32,768
16	2^{16}	65,536
17	2^{17}	13,1072
18	2^{18}	262,144
19	2^{19}	524,288
20	2^{20}	1,048,576
21	2^{21}	2,097,152
22	2^{22}	419,4304

x	2^x	y
23	2^{23}	838,8608
24	2^{24}	16,777,216
25	2^{25}	33,554,432
26	2^{26}	67,108,864
27	2^{27}	134,217,728
28	2^{28}	268,435,456
29	2^{29}	536,870,912
30	2^{30}	1,073,741,824
31	2^{31}	2,147,483,648
32	2^{32}	4,294,967,296

Using Different Bases

x	3^x
0	1
1	3
2	9
3	27
4	81
5	243
6	729
7	2,187
8	6,561
9	19,683
10	59,049

x	4^x
0	1
1	4
2	16
3	64
4	256
5	1,024
6	4,096
7	16,384
8	65,536
9	262,144
10	1,048,576

x	5^x
0	1
1	5
2	25
3	125
4	625
5	3,125
6	15,625
7	78,125
8	390,625
9	1,953,125
10	9,765,625

x	6^x
0	1
1	6
2	36
3	216
4	1,296
5	7,776
6	46,656
7	279,936
8	1,679,616
9	10,077,696
10	60,466,176

x	10^x
0	1
1	10
2	100
3	1,000
4	10,000
5	100,000
6	1,000,000
7	10,000,000
8	100,000,000
9	1,000,000,000
10	10,000,000,000

x	$\log_3(3^x)$
1	0
3	1
9	2
27	3
81	4
243	5
729	6
2,187	7
6,561	8
19,683	9
59,049	10

x	$\log_4(4^x)$
1	0
4	1
16	2
64	3
256	4
1,024	5
4,096	6
16,384	7
65,536	8
262,144	9
1,048,576	10

x	$\log_5(5^x)$
1	0
5	1
25	2
125	3
625	4
3,125	5
15,625	6
78,125	7
390,625	8
1,953,125	9
9,765,625	10

x	$\log_6(6^x)$
1	0
6	1
36	2
216	3
1,296	4
7,776	5
46,656	6
279,936	7
1,679,616	8
10,077,696	9
60,466,176	10

x	$\log_{10}(10^x)$
1	0
10	1
100	2
1,000	3
10,000	4
100,000	5
1,000,000	6
10,000,000	7
100,000,000	8
1,000,000,000	9
10,000,000,000	10

Formula and Tables Page 21

Séana agus logartaim | **Indices and logarithms**

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$\left(a^p\right)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a\left(x^q\right) = q\log_a x$$

$$\log_a 1 = 0$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$a^x = y \iff \log_a y = x$$

$$\log_a(a^x) = x$$

$$a^{\log_a x} = x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Switching between Exponential and logarithmic forms of Equations

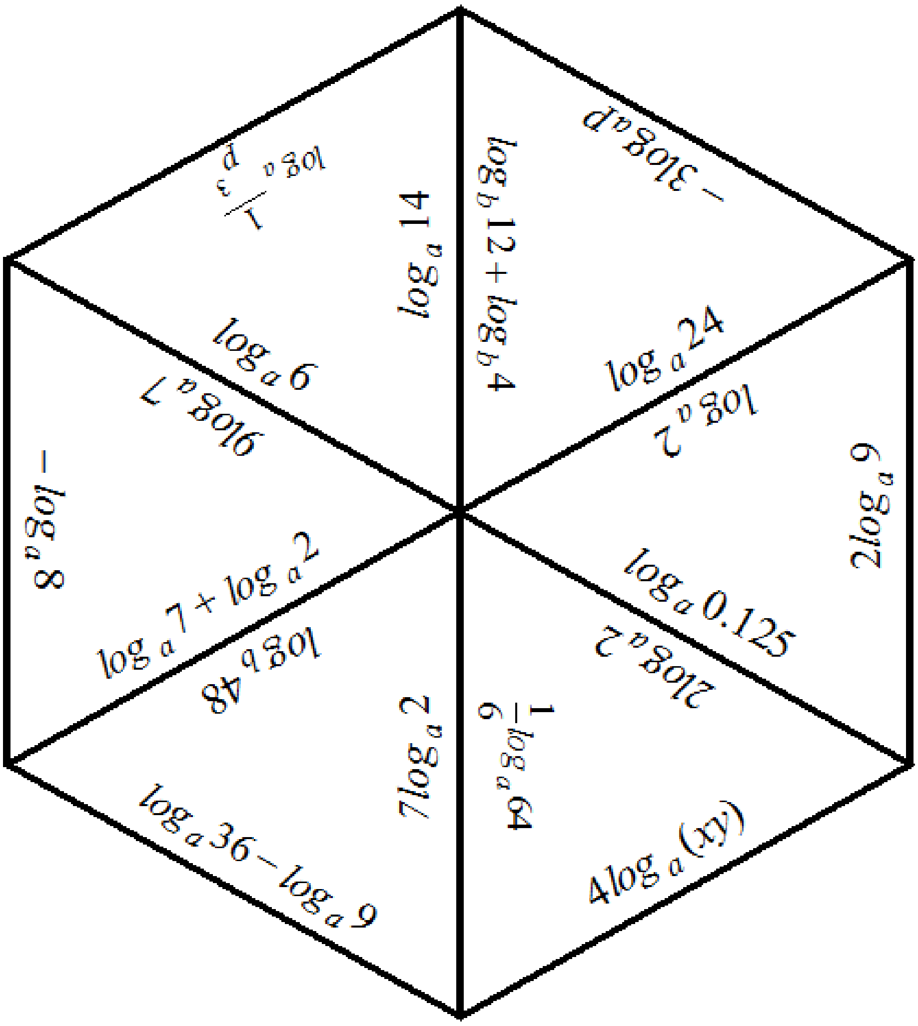
4.

Evaluate the expression below forming an equation	Write the equivalent exponential form of the equation formed from the first column
$\log_2 16 = 4$	
$\log_2\left(\frac{1}{64}\right)$	
$\log_2(1)$	
$\log_2\left(\frac{1}{8}\right)$	
$\log_e e$	
$\log_2(-4)$	

5.

Exponential form of an equation	Write the equivalent log form of the equation in the previous column
$5^2 = 25$	
$5^{-2} = \frac{1}{25}$	
$10^1 = 10$	
$9^{\frac{1}{2}} = 3$	
$27^{\frac{1}{3}} = 3$	
$b^0 = 1$	

6. Cut out the equilateral triangles and using the rules for logs, line up sides having equivalent expressions to make a hexagon.



Note: This jigsaw was created using *Tarsia*, a free software package for creating numerous types of jigsaw and matching exercises.

7A. Evaluate each of the following:

$\log_2(32\times 2)=\log_2(64)=$ ___	$\log_2(32)+\log_2(2)=$ ___ + ___ = ___
$\log_3(27\times 9)=\log_3(243)=$ ___	$\log_3(27)+\log_3(9)=$ ___ + ___ = ___
$\log_5(25\times 5)=\log_5(125)=$ ___	$\log_5(25)+\log_5(5)=$ ___ + ___ = ___
$\log_2\left(16\times\frac{1}{16}\right)=\log_2(64)=$ ___	$\log_2(16)+\log_2\left(\frac{1}{16}\right)=$ ___ + ___ = ___

What pattern seems to hold?

Can you write a rule for $\log_b(xy)$ in terms of $\log_b(x)$ and $\log_b(y)$?

7B. Evaluate each of the following:

$\log_2(64\div 4)=\log_2(16)=$ ___	$\log_2(64)-\log_2(4)=$ ___ − ___ = ___
$\log_6(216\div 6)=\log_6(36)=$ ___	$\log_6(216)-\log_6(6)=$ ___ − ___ = ___
$\log_{10}(100\div 1000)=\log_{10}\left(\frac{1}{10}\right)=$ ___	$\log_{10}(100)-\log_{10}(1000)=$ ___ − ___ = ___
$\log_5(25\div 25)=\log_5(1)=$ ___	$\log_5(25)-\log_5(25)=$ ___ − ___ = ___

What pattern seems to hold?

Can you write a rule for $\log_b\left(\frac{x}{y}\right)$ in terms of $\log_b(x)$ and $\log_b(y)$?

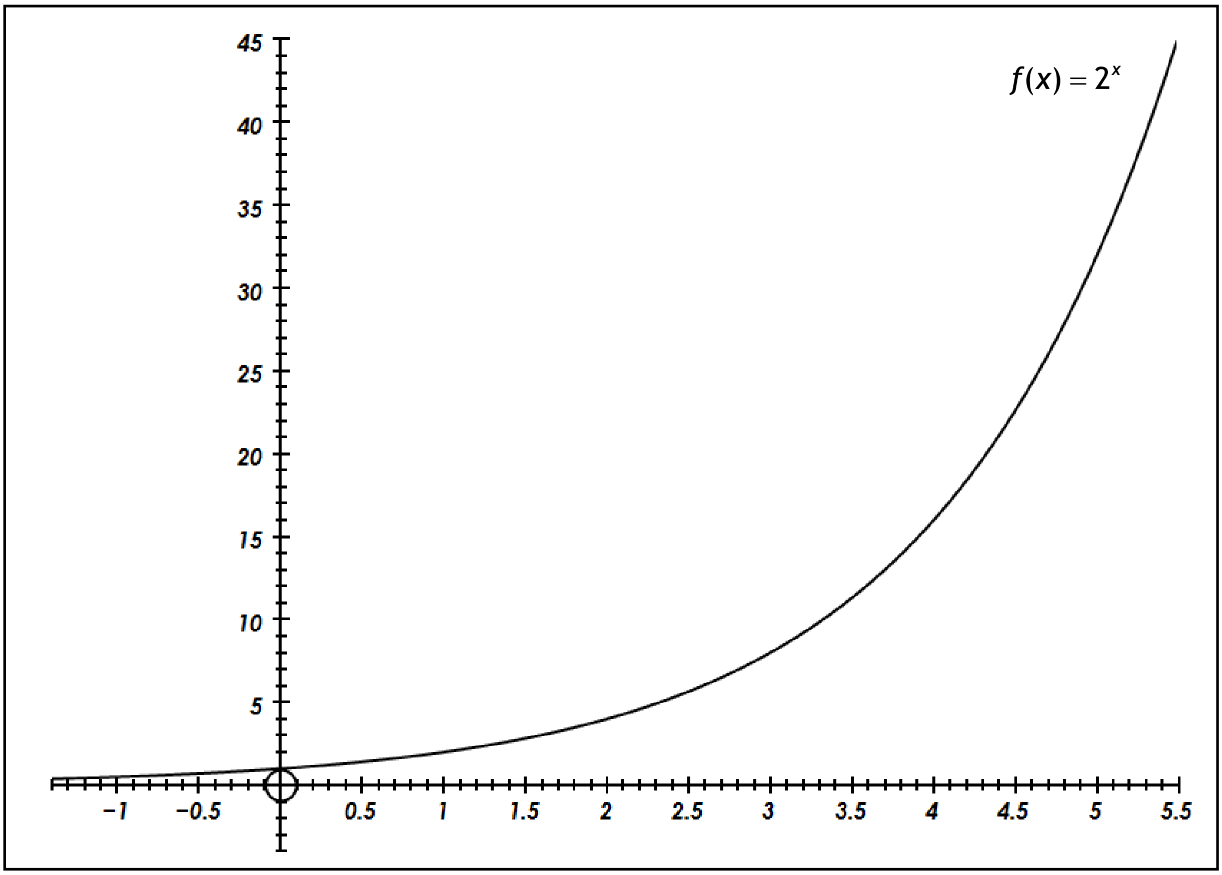
7C. Evaluate each of the following:

$\log_2(8)^3=\log_2(512)=$ ___	$3\log_2(8)=$ ___ $=3(\quad)=$ ___
$\log_2(256)^{\frac{1}{2}}=\log_2(16)=$ ___	$\frac{1}{2}\log_2(256)=$ ___ $=\frac{1}{2}(\quad)=$ ___
$\log_{10}(10)^4=\log_{10}(10{,}000)=$ ___	$4\log_{10}(10)=$ ___ $=4(\quad)=$ ___
$\log_3(27)^2=\log_3(729)=$ ___	$2\log_3(27)=$ ___ $=2(\quad)=$ ___

What pattern seems to hold?

Can you write a rule for $\log_b(x)^y$ in terms of $\log_b(x)$?

8. From discrete to continuous

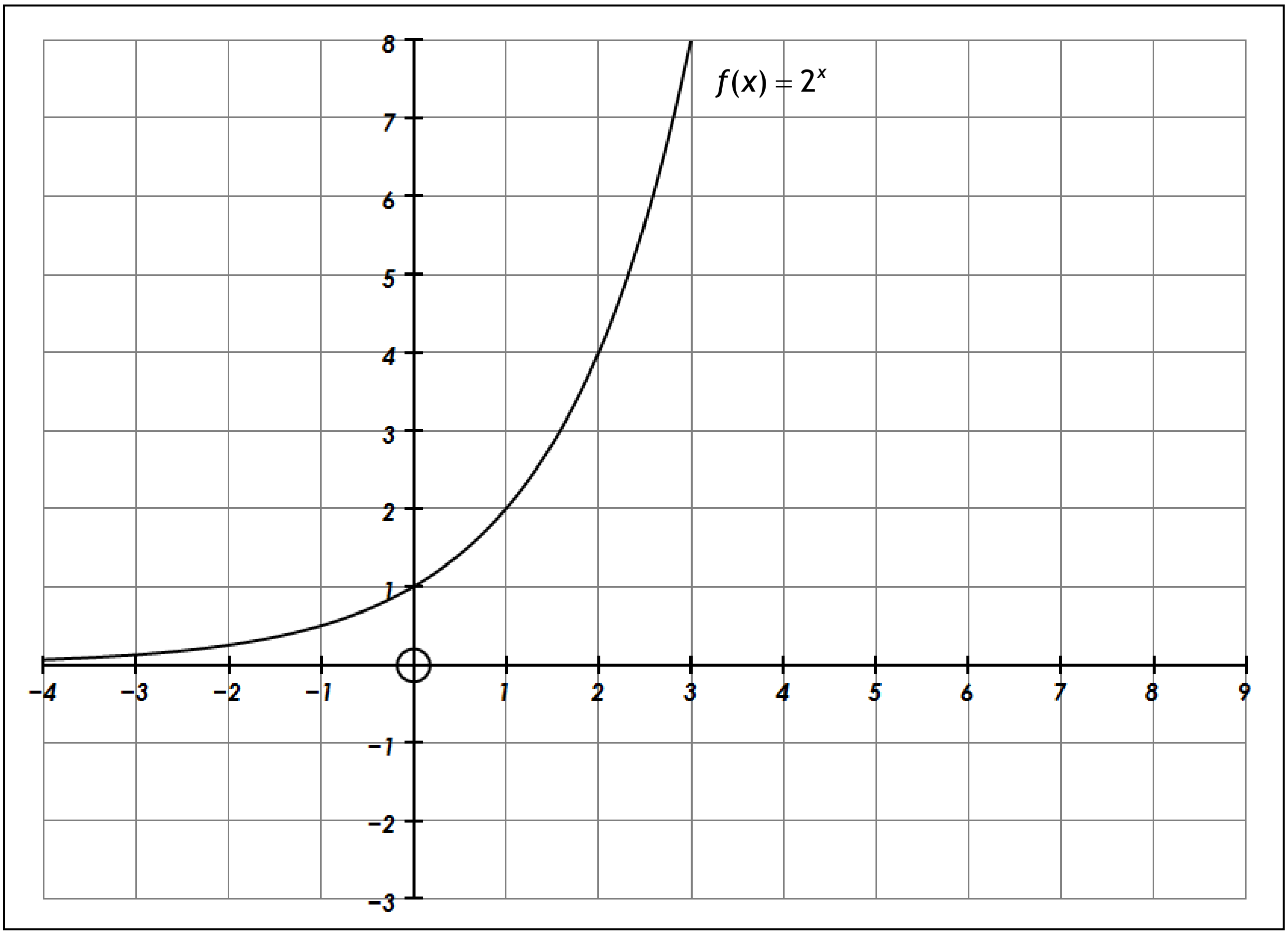


Use the graph to estimate:

(i) $\log_2(26)$

(ii) $\log_2(39.4)$

9. Drawing the graph of the inverse of $f(x) = 2^x$



(a) Fill in the table below and hence draw the graph of $g(x) = f^{-1}(x)$.

x	$f(x) = 2^x$	(x, y)
-2	$\frac{1}{4}$	$\left(-2, \frac{1}{4}\right)$
-1	$\frac{1}{2}$	$\left(-1, \frac{1}{2}\right)$
0	1	$(0, 1)$
1	2	$(1, 2)$
2	4	$(2, 4)$
3	8	$(3, 8)$

x	$g(x) = \log_2(x)$	(x, y)
$\frac{1}{4}$	-2	$\left(\frac{1}{4}, -2\right)$

(b) What is the relationship between $f(x) = 2^x$ and $g(x) = \log_2(x)$?

(c) Explain why the relation $g(x) = \log_2(x), x \in \mathbb{R}^+$ is a function.

(d) For $g(x) = \log_2(x)$:

(i) Identify the base of $g(x) = \log_2(x)$. ______________________

(ii) What is varying for the function $g(x) = \log_2(x)$. ______________________

(iii) What is constant for the function $g(x) = \log_2(x)$. ______________________

(e) For $g(x) = \log_2(x)$:

(i) What is the domain? ______________________

(ii) What is the range? ______________________

(f) In relation to the graph of $g(x) = \log_2(x)$:

(i) Is it a straight line? ______________________

(ii) Is y increasing or decreasing as x increases? ______________________

(iii) Describe how the rate of change varies as x increases. ______________________

____

____

(g) For $g(x) = \log_2(x)$:

(i) Where does the graph cross the x-axis? ______________________

(ii) What happens to the output as x decreases between 0 and 1? ______________________

____

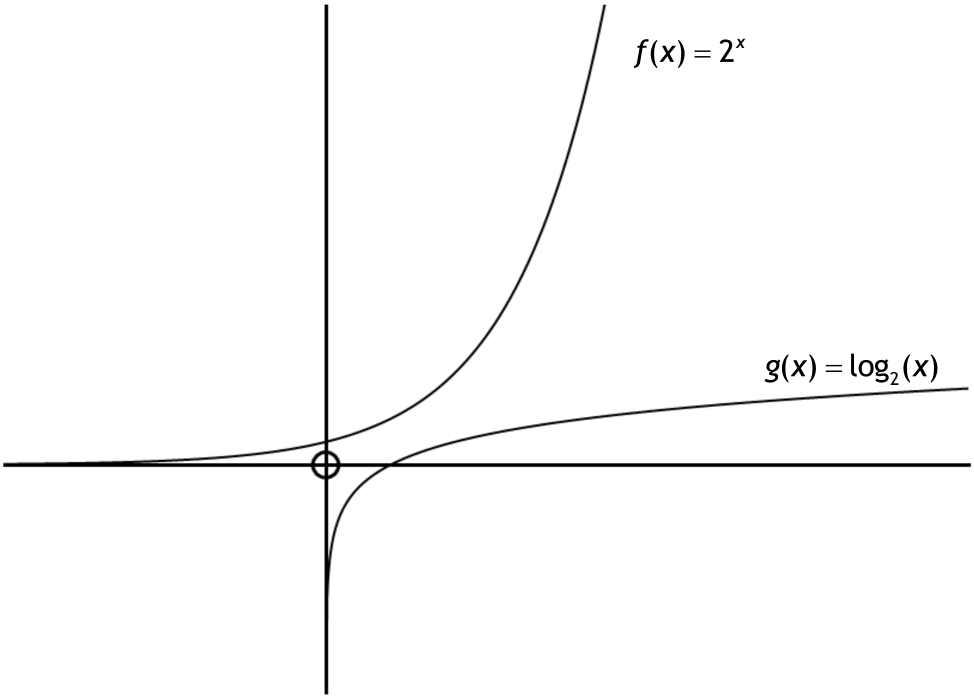
(iii) What is the y-intercept of the graph of $g(x) = \log_2(x)$. ______________________

(iv) What is the relationship between the y-axis and the graph of $g(x) = \log_2(x)$.

____

____

10. Using the graph below, sketch and label the graphs of the following functions:
$h(x)=10^x$, $k(x)=\log_{10}(x)$, $l(x)=e^x$ and $m(x)=\ln(x)$.



11. True or false discussion:

Equation	Equivalent exponential form	T/F	Correct equation (if false)
$\log_2 8 = 4$			
$\log_3 81 = 4$			
$\log_{10} 5 + \log_{10} 10 = \log_{10} 15$			
$\log_2 64 - \log_2 4 = \log_2 16$			
$\log_3\left(\frac{1}{81}\right) = -4$			
$2\log_2 8 = \log_2 16$			
$\log_2 4 + \log_2 128 = \log_2 512$			

12. Give possible numbers or variables for the blanks in the equations below.

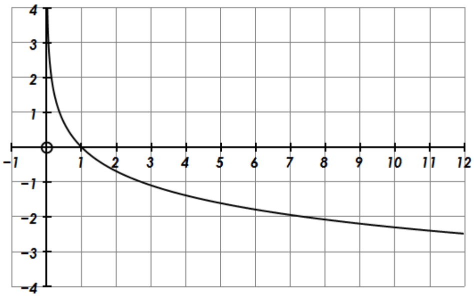
(i) $\log_{\square}\square = 3$ **(ii)** $\log_{\square}\square = \square$	**(vi)** $\log_{\square}\square - \log_{\square}\square = 3$ **(vii)** $\log_{\square}\square - \log_{\square}\square = \square$ **(viii)** $\log_{\square}\square - \log_{\square}\square = \log_{\square}\square$
(iii) $\log_{\square}\square + \log_{\square}\square = 7$ **(iv)** $\log_{\square}\square + \log_{\square}\square = \square$ **(v)** $\log_{\square}\square + \log_{\square}\square = \log_{\square}\square$	**(ix)** $\square\log_{\square}\square = \log_{\square}\square$

13. **(i)** What is the relationship between $\log_b(x)$ and $\log_{\frac{1}{b}}(x)$?

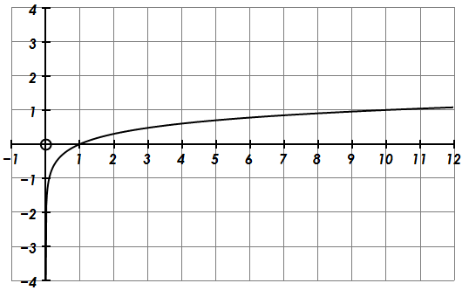
(ii) Cut out the following and match each graph to its function.

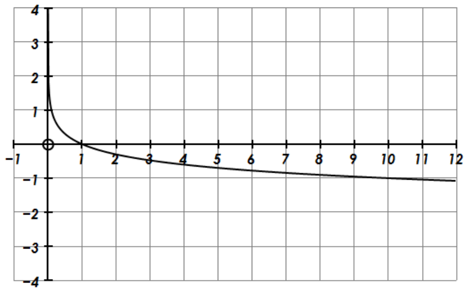
$$j(x) = \log_{\frac{1}{10}}(x)$$

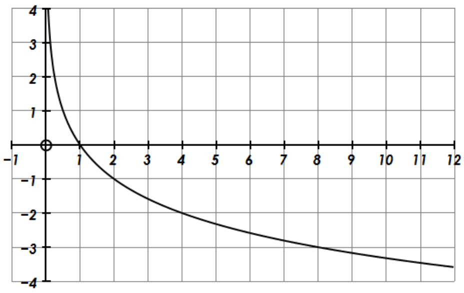
$$w(x) = \log_{10}(x)$$



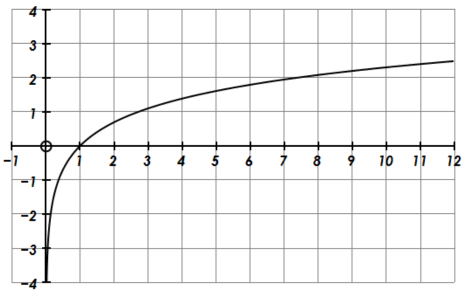
$$n(x) = \ln(x)$$

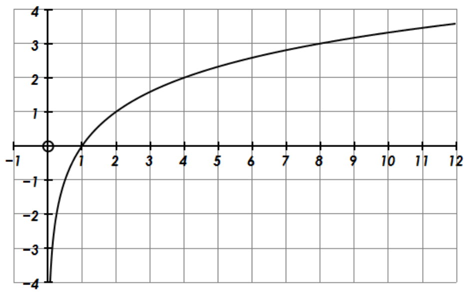






$$p(x) = \log_2(x)$$





$$r(x) = \log_{\frac{1}{2}}(x)$$

$$u(x) = \log_{\frac{1}{e}}(x)$$

14. **LCFL 2012 Paper 1 Q7 (b)**
A scientist is growing bacteria in a dish. The number of bacteria starts at 10 000 and doubles every hour.

(i) Complete the table below to show the number of bacteria over the next five hours

Time in hours	0	1	2	3	4	5
Number of bacteria (in thousands)	10					

(ii) Draw a graph to show the number of bacteria over the five hours.

(iii) Use your graph to estimate the number of bacteria in the dish after $2\frac{1}{2}$ hours.

(iv) The scientist is growing the bacteria in order to do an experiment. She needs at least 250 000 bacteria in the dish to do the experiment. She started growing the bacteria at 10:00 in the morning. At what time is the dish of bacteria ready for the experiment?

15. M_1 and M_2 are the magnitudes of two earthquakes on the Richter magnitude scale. If A_1 and A_2 are their corresponding amplitudes, measured at equal distances from the earthquakes, then $M_1 - M_2 = \log_{10}\left(\frac{A_1}{A_2}\right)$.
An earthquake rated 6.3 on the Richter magnitude scale in Iran on 26th Dec 2003 killed 40,000 people. The earthquake on Banda Aceh on 26th Dec 2004 was rated at 9.2 on the Richter magnitude scale.
How many times greater in amplitude of ground motion was the earthquake in Banda Aceh compared to the earthquake on Iran?

16. pH is a measure of the acidity. The pH is defined as follows:

$pH = -\log_{10}[H^+]$ where $[H^+]$ is the hydrogen ion concentration in an aqueous solution.

A pH of 7 is considered neutral. For bases: $pH > 7$. For acids: $pH < 7$. [at 25°C]
A substance has $[H^+] = 2.7 \times 10^{-5}$ moles/litre.
Determine the pH and classify the substance as an acid or a base.

17. Sound levels are measured in decibels (dB).
If we are comparing two sound levels, B_1 and B_2 measured in dB,

$$B_2 - B_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$

The sound levels at The Who concert in 1976 at a distance of 46 m in front of the speakers was measured as B_2 = 120 db. What is the ratio of the intensity I_2 of the band sound at that spot compared to a jackhammer operating at a sound level of B_1 = 92 db at the same spot.

18. €140,000 was deposited at the beginning of January 2005 into an account earning 7% compound interest annually.
When will the investment be worth €200,000?

Verify and justify that the following two formulae give the same answer.

$$F = 140{,}000(1.07)^t$$

$$F = 140{,}000e^{0.0676586485t}$$

Comment on that answer in each case.

Activity 1: A to G

1. Fill in the tables for the activity you are completing.
2. On your white boards, using the same axes and scales, plot the graphs of the given functions for your activity. Label your graphs clearly.
   (i.e. all the graphs for activity A should be on the same graph.

LOOK AT THE *X* AND *Y* COUPLES TO HELP YOU SCALE YOUR AXES

ACTIVITY A

x	$f(x) = x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$g(x) = 2x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$p(x) = 3x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$k(x) = 0.5x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		

ACTIVITY B

x	$f(x) = x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$g(x) = -x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$p(x) = -3x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$k(x) = -0.5x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		

Activity A

By considering the graph of $f(x) = x^2$ what effect does "a" have on $g(x) = af(x) = ax^2$.

Activity B

By considering the graph of $f(x) = x^2$ what effect does "a" have on $g(x) = af(x) = ax^2$.

ACTIVITY C

x	$f(x) = x^2$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$g(x) = x^2 + 1$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$p(x) = x^2 + 3$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$k(x) = x^2 - 4$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

ACTIVITY D

x	$f(x) = -x^2$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$g(x) = -x^2 + 1$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$p(x) = -x^2 + 3$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$k(x) = -x^2 - 4$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		

Activity C

By considering the graph of $f(x) = x^2$ what effect does "c" have on $g(x) = f(x) + c = x^2 + c$.

Activity D

By considering the graph of $f(x) = -x^2$ what effect does "c" have on $g(x) = f(x) + c = -x^2 + c$.

ACTIVITY E

x	$f(x)=x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$g(x)=(x+1)^2$	(x, y)
–4		
–3		
–2		
–1		
0		
1		
2		
x	$p(x)=(x+3)^2$	(x, y)
–6		
–5		
–4		
–3		
–2		
–1		
0		
x	$k(x)=(x+0.5)^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		

ACTIVITY F

x	$f(x)=x^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		
x	$g(x)=(x-1)^2$	(x, y)
–2		
–1		
0		
1		
2		
3		
4		
x	$p(x)=(x-3)^2$	(x, y)
0		
1		
2		
3		
4		
5		
6		
x	$k(x)=(x-0.5)^2$	(x, y)
–3		
–2		
–1		
0		
1		
2		
3		

Activity E

By considering the graph of $f(x)=x^2$ what effect does "a" have on $g(x)=f(x+a)=(x+a)^2$.

Activity F

By considering the graph of $f(x)=x^2$ what effect does "a" have on $g(x)=f(x+a)=(x+a)^2$.

ACTIVITY G

x	$f(x)=x^2$	(x, y)
-3		
-2		
-1		
0		
1		
2		
3		
x	$g(x)=(x+1)^2$	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2		
x	$p(x)=(x+1)^2+2$	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2		
x	$k(x)=(x+1)^2-2$	(x, y)
-4		
-3		
-2		
-1		
0		
1		
2		

Activity G

By considering the graph of $f(x)=x^2$ what effect do "*a*" and "c" have on $g(x)=f(x+a)\pm c=(x+a)^2\pm c$?

Activity 2: Review of Graphs of Functions

Write down the function which represents the graphs shown on the slides.

Graph	*Function*	*Local Maximum/Minimum*
1	$f(x) = x^2$	
2	$f(x) =$	
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

Activity 3: Different forms of the Quadratic

x	$y = x^2 - 4x - 5$	(x, y)
-2		
-1		
0		
1		
2		
3		
4		
5		
6		
x	$y = (x-5)(x+1)$	(x, y)
-2		
-1		
0		
1		
2		
3		
4		
5		
6		
x	$y = (x-2)^2 - 9$	(x, y)
-2		
-1		
0		
1		
2		
3		
4		
5		
6		

1. Fill in the tables opposite.

2. Plot the points and draw the graph for each of the functions in the table.

3. What do you notice about all the graphs and all of the three functions you have plotted in this activity?

____

____

____

4. What items of information from each of the functions can help us if sketching the graph of a function?

____

____

____

____

____