





**Task 1.1 – Modelling**

*A bank offers a yearly compound interest rate of 20%. Paul wishes to invest €100. What will the value of this investment be at the end of n years?*

Each Box represents €10, colour in Paul’s initial investment.

In a different colour, add in his interest received after 1 year.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Now repeat the steps above using the same two colours but also add in his interest received after 2 years in a new colour. (Any fractional parts should be represented by a partially completed box)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Repeat the above steps adding in his interest received after 3 years.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

What do you notice about the pattern?

Fill in the blanks.

This is a/an \_\_\_\_\_\_\_\_\_\_ sequence with a common \_\_\_\_\_\_\_\_\_\_\_\_ of \_\_\_\_\_\_\_\_.

In the orange boxes provided, write an expression to describe each of John’s investments in the form .

Complete the table to represent Paul’s investment for the first five years.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **Starting Amount** | **Interest** | **Total Amount**  **at end of year** |  |  |
| 1 | 100 | 100 x 0.2 = 20 | 100 + 20 = 120 | 100 + 100 x 0.2 | 100(1 + 0.2) |
| 2 | 120 | 120 x 0.2 = 24 | 120 + 24 = 144 | 120 + 120 x 0.2 | 120(1 + 0.2) |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |
| 5 |  |  |  |  |  |

Complete the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Starting Amount** | **Total**  **Amount** |  |  |
| 1 | 100 | 100(1 + 0.2) | 100(1 + 0.2) | 100(1 + 0.2) |
| 2 | 120 | 120(1 + 0.2) | 100(1 + 0.2)(1 + 0.2) | 100(1 + 0.2)2 |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

Can you predict the amount the investment is worth at the end of n years?



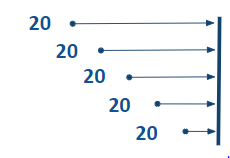
Additional Notes



**Task 1.2 – Investments**

*Jenny wishes to invest some money. She invests €20 at the start of each month for five months, in a savings account which offers a monthly rate of 0.5%. What will the value of this investment be at the end of five months?*







**Task 1.3 - Modelling**

*Jenny wishes instead to invest her money for 5 years She invests €20 at the start of each month for five years, into a savings account which offers a monthly rate of 0.5%. Model this investment as a group of individual deposits.*



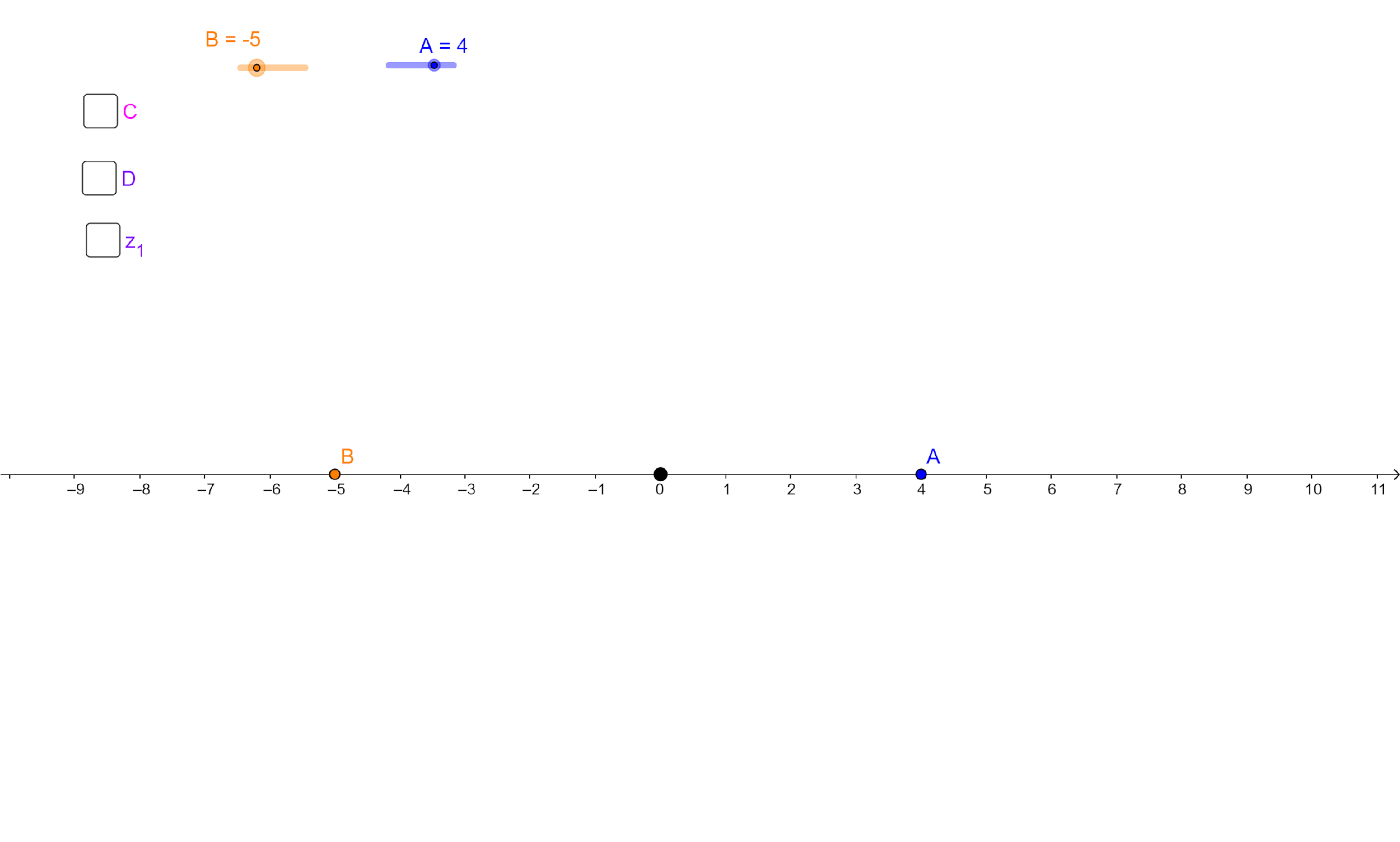
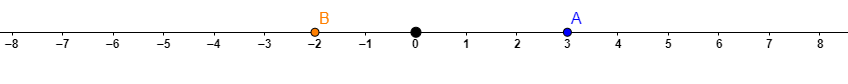
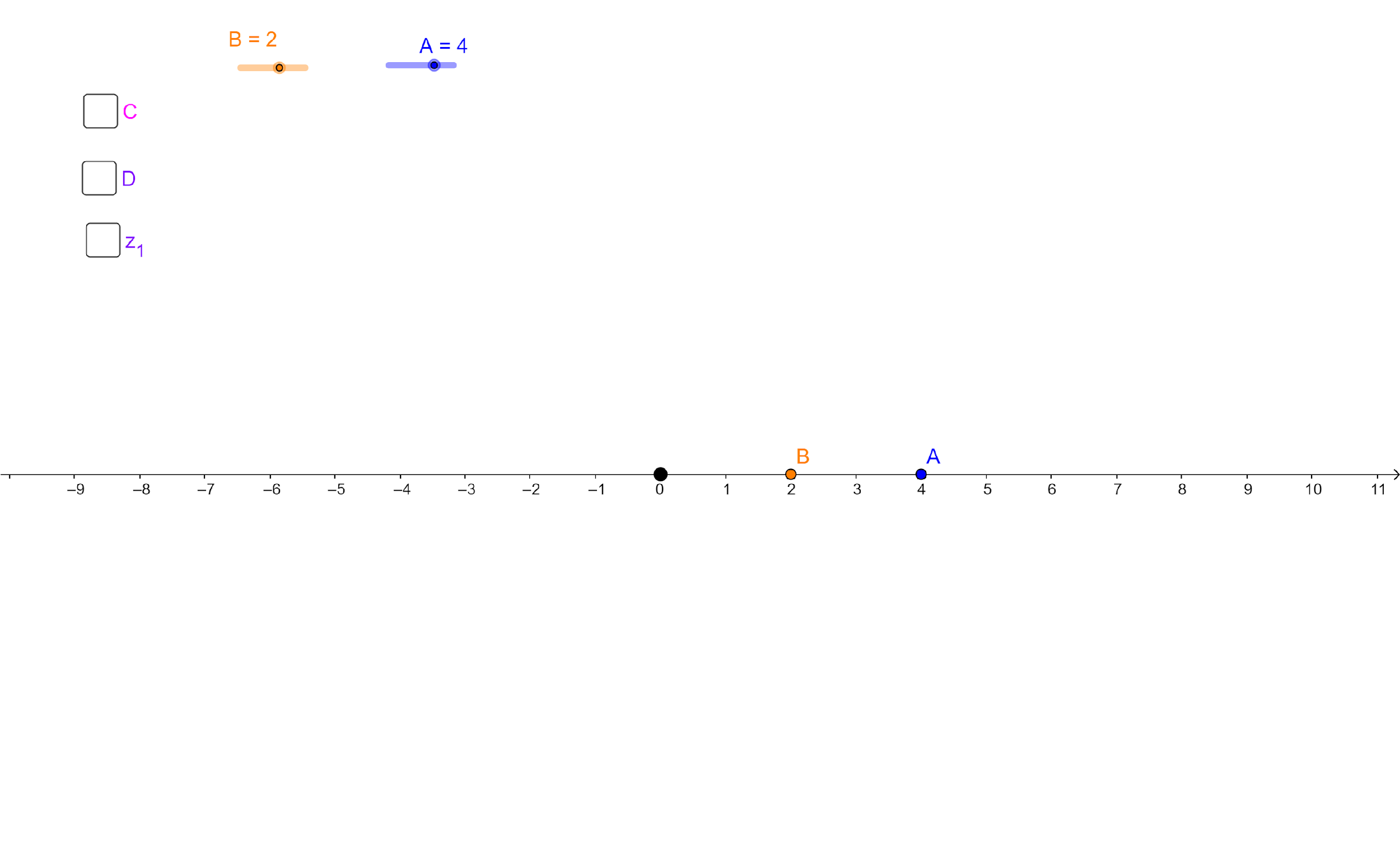
**Task 1.3 - Extension**

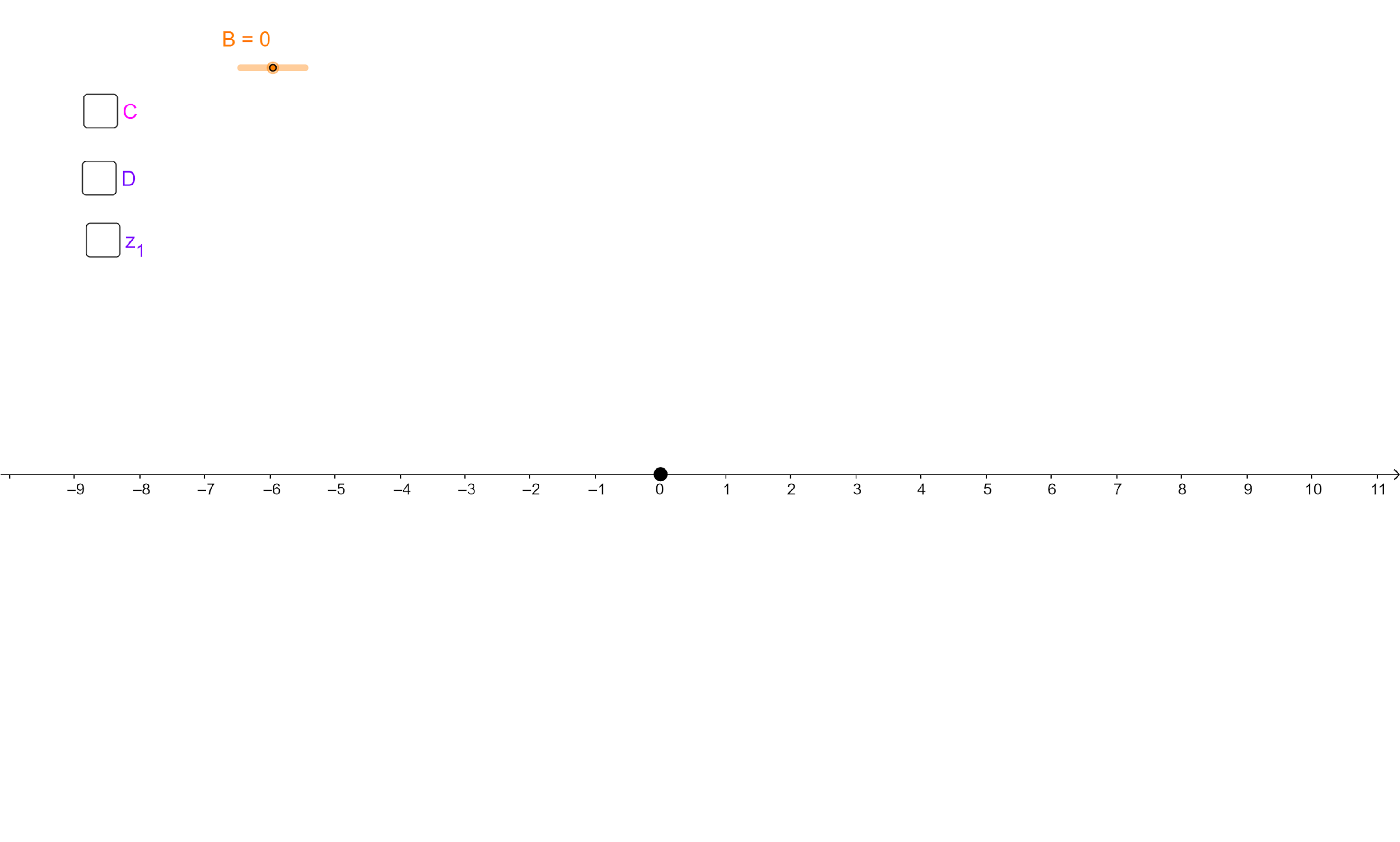
*Jenny invests €20 at the start of each month for three years. She then reduces her investment to €15 for the remaining two years. What will the value of this investment be at the end of five years?*

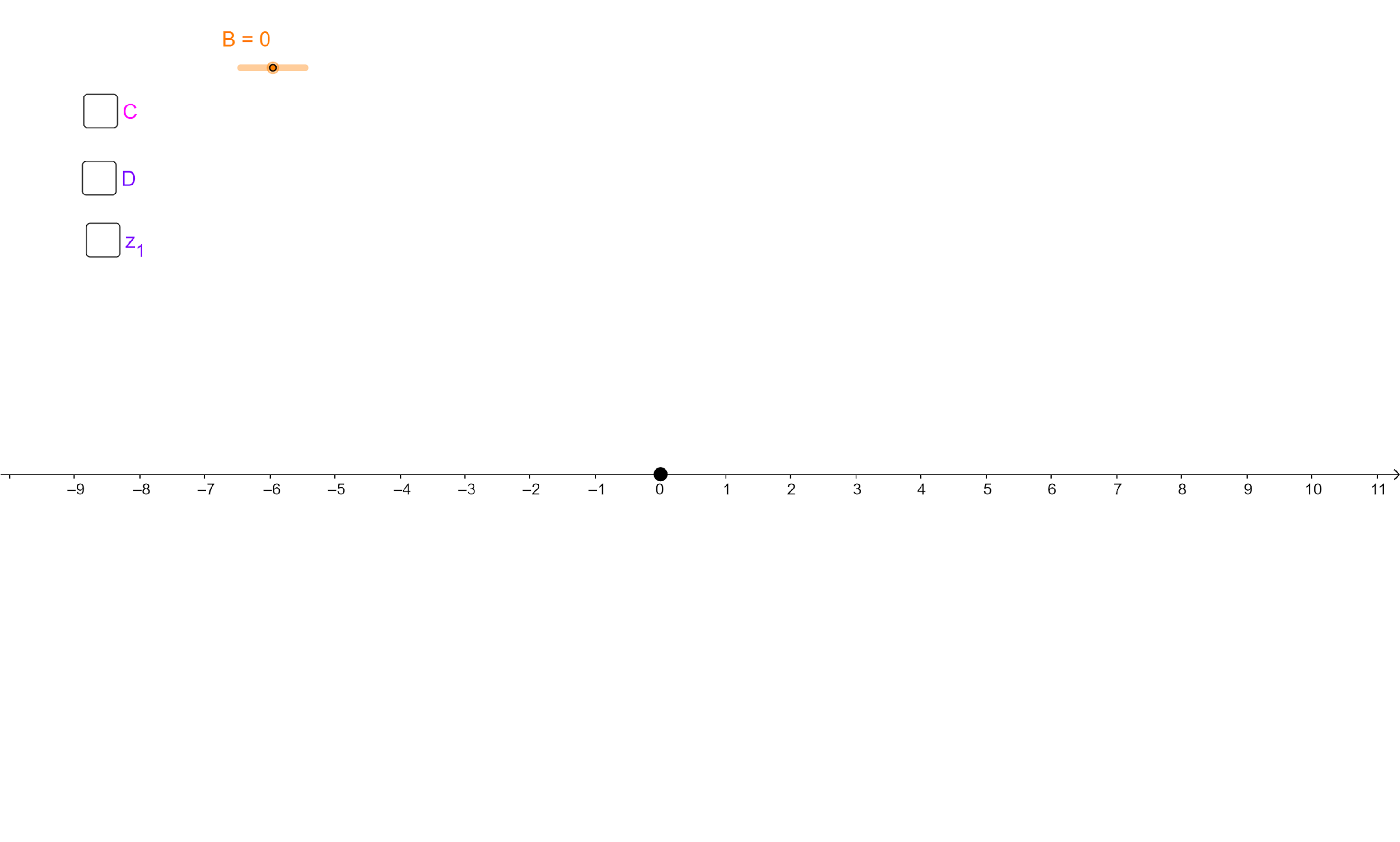


**Task 2.1 – Multiplication of Integers**

*A is a natural number. If B is a positive integer, what is the effect on A of multiplying by B?* *Describe the effect of multiplication on A with particular reference to the location of the product and the scale factor.*









*Describe the effect of multiplication on a real number by another real number. Refer to the scale factor, location and rotation.*



*A is an integer, B= -3 What is the effect on A of multiplying by each of the following?*

|  |  |  |  |
| --- | --- | --- | --- |
| **Problem** | **Rotation** | **Number of rotations** | **Scale factor** |
| **A x (-3)1** | **180°** | **1** | 3 |
| A x (-3)2 | **180° + 180°** | **2** | 32 |
| A x (-3)3 |  |  |  |
| A x (-3)4 |  |  |  |
| A x (-3)5 |  |  |  |



*What is the effect of raising A to the power of n?*



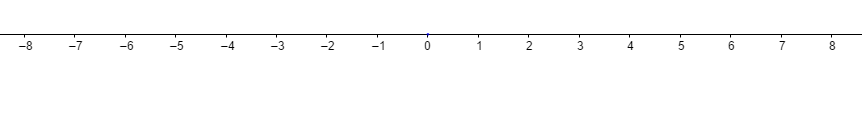
*A is an integer, B= -1 What is the effect on A of multiplying by each of the following?*

|  |  |  |  |
| --- | --- | --- | --- |
| **Problem** | **Rotation** | **Number of rotations** | **Scale factor** |
| **A x (-1)1** |  |  |  |
| A x (-1)2 |  |  |  |
| A x (-1)3 |  |  |  |
| A x (-1)4 |  |  |  |
| A x (-1)5 |  |  |  |

What would be the effect of multiplying A x (-1)1/2?



*If A = 1, B = A x i, can you suggest a suitable position for B on the diagram below?*



**Notes**



**Task 2.2 – Multiplication of Complex Numbers**

Open the GeoGebra file:

<https://tinyurl.com/WS4task2-2>

*z1 and z2 are complex numbers with a product z3. By moving z1 and observing z3 consider the effect on z1 of multiplication by z2. Use the check boxes to support this investigation.*

*Can you suggest in words a rule for multiplying complex numbers?*



**Extensions**

Exponents and complex numbers.

*z1 = -1+i is a complex number with a modulus of 2 and argument of 135. By setting z1 = z2 = -1+i, now z3 = (-1 + i)2. Describe the effect of squaring z1, referring to the moduli and the angles.*

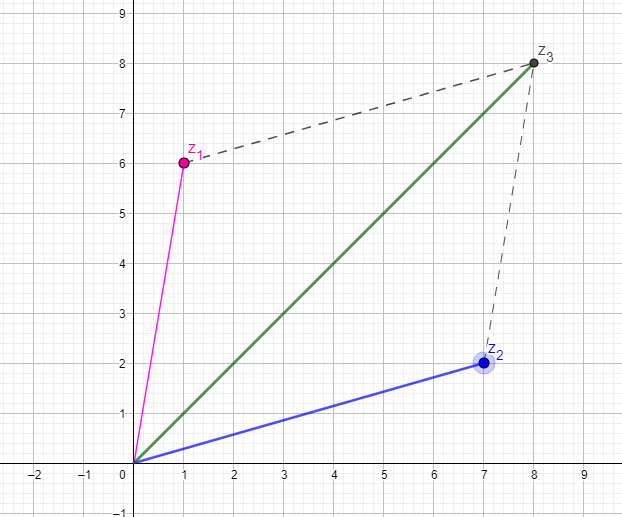


*Can you describe the effect on a complex number z, of raising z to the power of n?*



**Task 2.3 Addition of Complex Numbers**

<https://www.geogebra.org/classic/hapyyrhd>



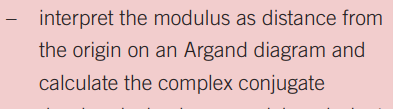
**Task 3: Take Home Task**

This task should be approached from the perspective of a higher level 5th year student.

Student Prior Knowledge:

* JC Number Operations
* Modulus of a complex number
* Multiplication of complex numbers using modulus and angles.

Link to Syllabus (LC OL):

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Link to Syllabus (LC HL):

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**https://lh3.googleusercontent.com/grnkgXgMAvO4SnzzoFQBGL6bn7TYxAa9cbXmzfwiNV6wJA0bceYGY79gW2-LaGn5nX8DaWeRan8v5iR9IqHC7GLspngTQ1Ou-PJHER-Wns-EF9aULoi8l0Jnix1jEEJFC594RU2F**

**Investigation 3.1**

Open the GeoGebra file “complex Conjugate”

<https://tinyurl.com/WS4Takehome>

*z1, z2 and z3 are complex numbers. Move z1. What is the effect on z2 of moving z1?*



*z3 is the product of z1 and z2. Why does z3 always lie on the real axis?*



Click the checkbox |z|.

Choose a value for z1, and record the modulus of z1, z2 and z3.

Repeat this process with a different value for z1.

*Can you describe the relationship between the moduli?*



Click the checkbox “Angle”

Change the position of z1 and note the effect on the angles α, β and γ.

*Write down in words any relationship you can find between α, β and γ.*



*Why might this investigation help students to understand the role of the conjugate in complex division?*



**Investigation 3.2**

Construct a line segment from z1 to z2.

*What questions could you ask students to connect Pythagoras’ Theorem to the modulus of a complex number?*



Using the trigonometric ratios, write down the coordinates of the point z1 and z2?

*Why might this investigation help students understand their study of De Moivre’s Theorem?*



**Feedback**

Please log onto:

<https://tinyurl.com/WS4Home>

and provide some feedback for this task.