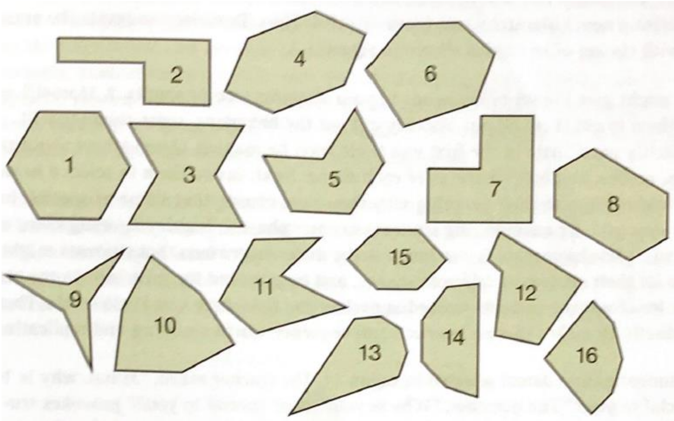
Extract from "Putting Essential Understanding of Geometry into Practice in Grades 9 -12"

The set of hexagons used in the Hierarchy of Hexagons activity(Danielson 2014)



You might give the side hexagons to your students and ask them to cut them all out. As they cut out the hexagons some students will actively participate in the first van Hiele level by making observations about the sides, angles and other features of each shape.

Next invite them to select a hexagon and explain to their neighbour why they have chosen that shape as special. In this step, you are encouraging students to enter phase 1, inquiry/ investigation, of the van Hiele hierarchy. Try to imagine reasons that students might give different reasons for their choices of different shapes, and be prepared for surprises.

In the class that we observed, students worked at each of the first three van Hiele levels. That productivity makes this an instructional sequence worth studying and replicating.

A student named Jamal selected hexagon 14. The teacher asked "Why is 14 special to you?". The question "Why is your shape special to you?" provokes students to visualise the hexagons in an act of directed orientation. Jamal responded "Because when you reflect it, you get a decagon. I thought it was weird that when you reflect a six-sided shape you get a shape with ten sides."

The teacher asked the class "Does anybody have any questions about the definition? Depending on the cultural norms that you have established, your students may be uncomfortable critiquing and questioning other students thinking. But the ability to engage in this kind of argumentation and critique is one element of the third practice identified in the Standards of Mathematical Practice in the Common Core State Standards (CCSSM)

In Jamal's class, the teacher emphasized that no one's definitions are completely precise on their first attempt, that questions and criticism are a necessary and positivity part of this process, and that in offering his definition' Jamal " did us an enormous favour by giving us something to talk about."
A student then asked Jamal, "Where are you reflecting it? Where's the reflection line?" Jamal had to think about this for a minute and then replied, "Ok, I've got it. You reflect it across its longest side."

The teacher said “Ok, this feels like a really clear property now. I can test it. I can find out which shapes match it or don’t match it. What do you want to call this kind of shape , jamal?” Jamal answered “A reflector”, and the teacher wrote the name on the board in large letters. The teacher asked, “Does anybody still need clarification on Jamal’s definition?” Another student said, “OK, what about hexagon 5, where it seems like there’s more than one longest side?”.
The class was silent as students considered the hexagon. A student at the back of the classroom pointed out that hexagon 13 had the same feature as hexagon 5 (see fig. 1.3). She asked Jamal whether it was a Reflector.

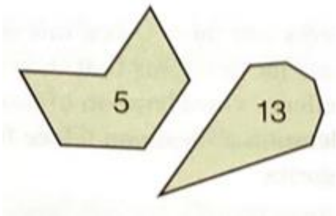


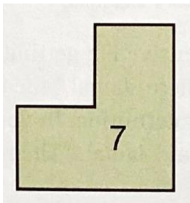
Fig 1.3 Reflector Hexagons

When Jamal said, "I can't believe how defensive I'm getting about this," the class laughed. "We're trying to understand you here, Jamal," the teacher said. "You have a lot of power. This is your shape and your definition. In your heart of hearts, do you think 13 and 5 are reflectors?" "No," said Jamal. "Then we need to adjust our definition," the teacher suggested.

The class eventually decided to define a reflector as "a hexagon that has only one longest side and becomes a decagon when it's reflected across that longest side." This formulation seemed clever to the students, and even a bit sneaky. They had identified two shapes (5 and 13) that complicated Jamal's definition, so they found a way to write them out of the definition. The teacher told the class that mathematicians often call this action "monster barring"—a way of rescuing a definition or conjecture from monsters like hexagons 13 and 5. You simply bar them from the classification (Lakatos 1976).

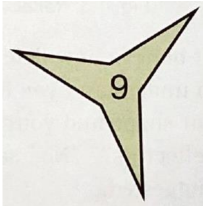
Danielson facilitated this process two more times, one for hexagons that students dubbed ***squeakies*** and one for hexagons that they termed ***stacies.***

The students decided that squeakies were hexagons all of whose angles are right angles.

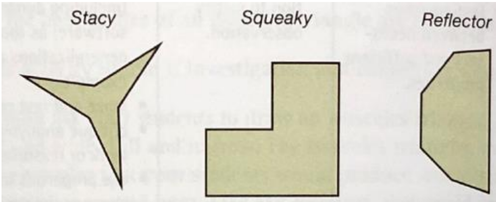


The teacher pressed students to attend to precision at this point, leading them to note that one of the angles was 270 degrees, not 90 degrees.

The students also decided that stacies were hexagons with three congruent acute angles.

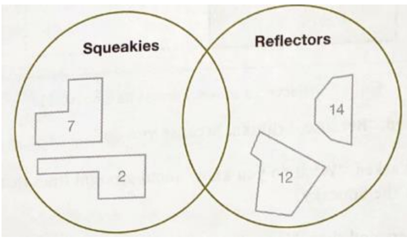


Next Danielson asked his students to draw Venn diagrams for the different shapes. Can stacies be squeakies? Students should be encouraged to draw a stacy and a squeaky. Can reflectors be stacies? Students should draw a reflector.



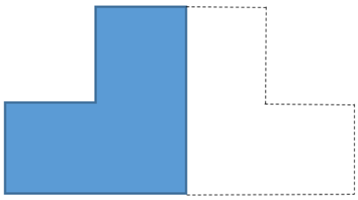
At different points in the process, students may make a claim of the form of “All_______are also__________.” or “No ___________can be a _________”.

In Danielson’s classroom a student came to the conclusion “No squeaky can be a reflector”



“Is this true?” the teacher asked. “How do you know? Is this a theorem? Can you prove it?”. He gave the class time to think about a response.

One student said, “Well when you reflect a Squeaky across its longest side you only get an eight sides, not ten.” The teacher asked, “Why do we only get eight sides?



The teacher asked “What do you know about straight lines that becomes true when you reflect the Squeaky?”

The student understood then. “Lines have 180 degrees. The squeaky’s angles here are 90 degrees and when you reflect them they add to 180 degrees. They make the side that was there a little longer but they don’t create a new one.” The students reference to definition was the beginning of a move to the deductive level, but not the end of it.

In Danielson's Hierarchy of Hexagon activity, the teacher gave the students many hexagons, all of them intentionally chosen to feature similarities and differences. However, the *students* decided which ones were special. The *students* developed definitions for their favourite hexagons. The *students* named them. The *students* developed conjectures about the hexagons - conjectures that they could not yet say were true or false because they had just made them, creating the necessity for them to prove whether or not the conjectures were true. The cognitive demand of the hexagon task was high. Student interest was high also.

The Hierarchy of Hexagons activity is an example of a task that can provide opportunities for students to engage in level 1 analysis.