Complex Numbers

Making Connections across Different Strands of Mathematics

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Overview of the Workshop

Session 1	Welcome & Introduction **Breakout Session 1** Teaching and Learning approach to Number Introducing the Argand Diagram
Session 2	Multiplication of Complex Numbers **Breakout Session 2**
Stretch Break	
Session 3	DeMoivre’s Theorem **Breakout session 3** Conclusion

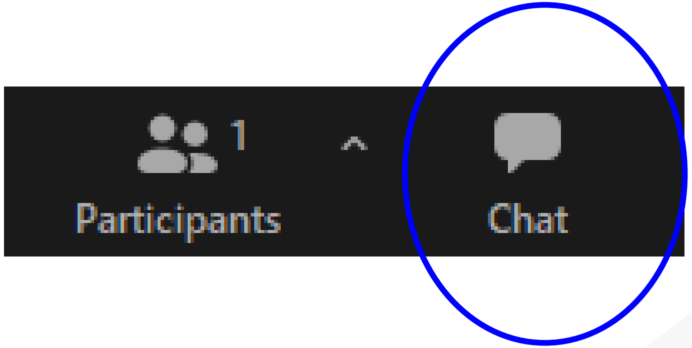
Introduction

Protocols & Technology

Chat function

https://tinyurl.com/complexnumbers-AD

Polls



Key Messages

Understand the importance of using a geometric view to make sense of complex numbers.

Understand the importance of extending prior knowledge by using a constructivist approach.

By the end of this workshop, participants will

- engage with constructivist tasks that will encourage students to become more confident in their mathematical ability and develop their critical-thinking skills.
- recognise the value of making the connection between the geometric view and the algebraic view of complex numbers.
- appreciate the importance of reflecting on practice, the value of effective collaboration and the long-term benefits of engaging with continual professional development.

Breakout Session 1

Introductions and Reflection

Feedback

The Argand Diagram

A constructivist approach

Introduction of Complex Numbers

Prior Content Knowledge

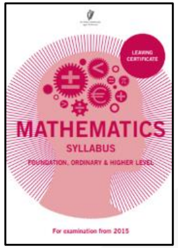
Investigation of multiplication using models including the number line.

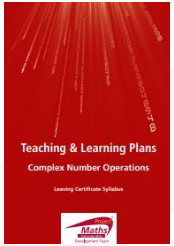
Select and use suitable strategies for finding solutions to quadratic equations.

Junior Cycle Specification pages 15, 19, Junior Certificate Syllabus pages 22, 29.

Work with complex numbers in rectangular form to solve quadratic equations

Leaving Certificate Syllabus page 39



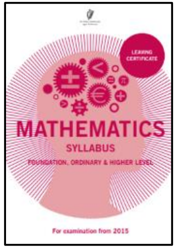


Introduction of Complex Numbers

Key Content for this task

Students should be able to illustrate complex numbers on an Argand diagram.

Leaving Certificate Syllabus page 29



Pedagogy

Constructivism

Methodologies

Effective Questioning

Group Work

Constructivism

Social and collaborative learning activities

Learning activities set in a **meaningful context**

Learners given the freedom to understand and construct meaning at their **own pace**

Chuang & Rosenbusch (2005)

Constructivism

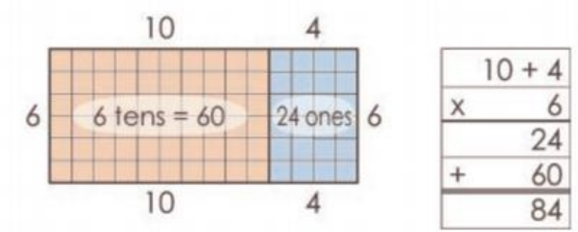
*"A constructivist pedagogical orientation supports teachers in **effectively** using digital technologies with their students i.e. learners are **actively involved** in a process of **determining** meaning and knowledge for themselves"*

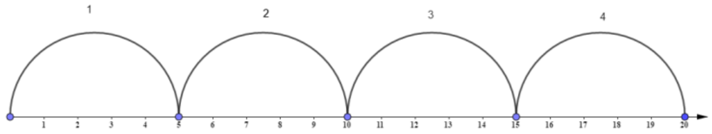
DES (2015)

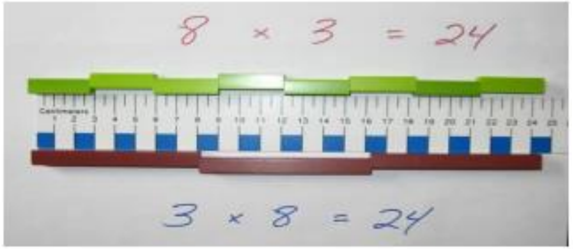
Prior Knowledge: Primary Approach

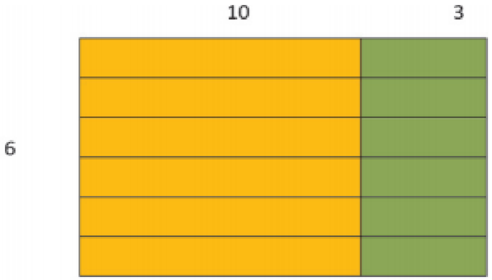
What approaches from primary school might a first year student have taken to explore multiplication?

Linear method
Distribution
Array method
Skip counting method









Prior Knowledge: Exploration of Multiplication

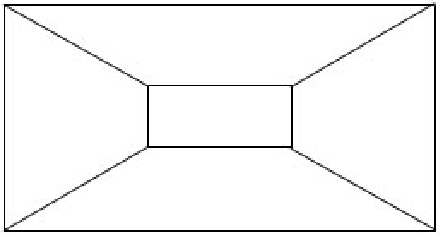
Student Tasks:

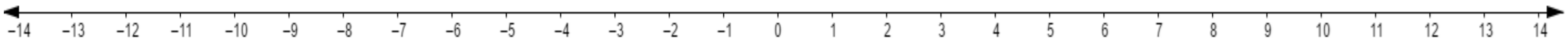
Open Question

Think-Pair-Share

Group Discussion

Concluding Statement





Prior Knowledge: Exploration of Multiplication

Class Discussion

Present concluding statements

Question/Critique

Verify/Justify

Summary of key learning

Individual Reflection

Prior Knowledge: Exploration of Multiplication

Student Task:

If $A, B \in \mathbb{R}$, Investigate the effect on A of multiplying by B.

$A, B \in \mathbb{R}^+$

$A \in \mathbb{R}^+, B \in \mathbb{R}^-$

$A \in \mathbb{R}^-, B \in \mathbb{R}^+$

$A, B \in \mathbb{R}^-$

$(A \text{ or } B = 0)$

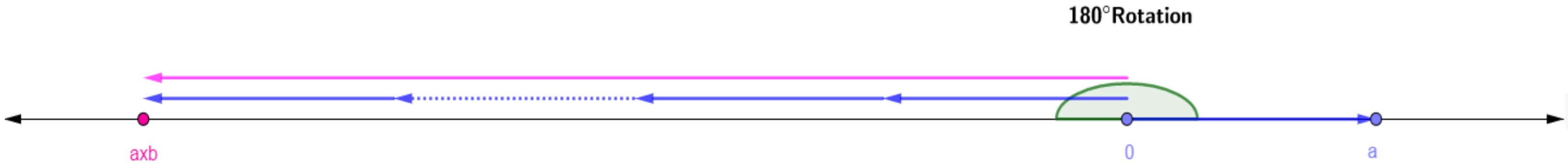
Prior Knowledge:
Exploration of Multiplication

Summary of Key Learning

For A,B ∈ $\mathbb{R}$,

If B>0 there is a **scaling** by a factor of B and a 0° **rotation**.

If B<0 there is a **scaling** by a factor of B and a 180° **rotation**.



Prior Knowledge:
Exploration of Multiplication

Student Task:

If $A \in \mathbb{R}$, $B \in \mathbb{R}^+$, $n \in \mathbb{N}$

What is the effect on A of multiplying by $(B)^n$?

A is **scaled** by a factor of B^n and there is a **rotation** of $0°$

Extension:

What is the effect on a real number A of multiplying by $(-B)^n$?

A is **scaled** by a factor of B^n and there is are $n \times 180°$ **rotations.**

Prior Knowledge:
Approach to Exploring Functions

Junior Cycle Functions:

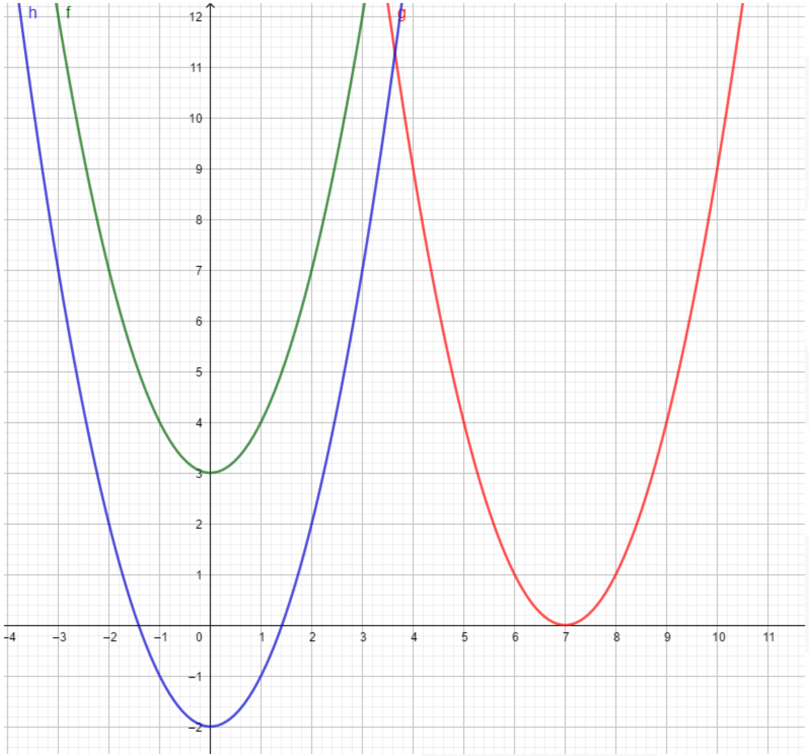
'no roots in the real number system'

Senior Cycle Functions:

$$b^2 - 4ac < 0$$

$$x = \pm\sqrt{-1} \qquad x^2 - 6x + 13 = 0$$

$$x = \pm i \qquad x = 3 \pm 2i$$



Introducing the Imaginary Axis

What is the effect on a real number A of multiplying by $(-B)^n$?

A is **scaled** by a factor of B^n and there are n x 180° **rotations.**

What is the effect on a real number A of multiplying by $(-1)^{½}$?

A is scaled by a factor of 1

½ rotation of 180° ≔ 90° rotation

Summary of Key Learning

What is the effect on a real number A of multiplying by i?

There is a rotation of 90°.

Q & A

Session 2

Multiplication of Complex Numbers

Multiplication of Complex Numbers

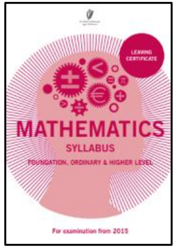
Prior Content Knowledge

Illustrate complex numbers on an Argand diagram.

Interpret the modulus as distance from the origin.

Multiplication of complex numbers of the form $z=ai$.

Leaving Certificate Syllabus page 29



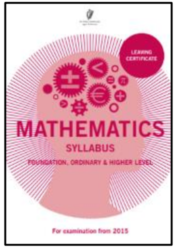
Multiplication of Complex Numbers

Key Content for this task

Investigate multiplication and division with complex numbers in rectangular form a+bi.

Calculate the complex conjugate.

Leaving Certificate Syllabus page 29



Pedagogy

Constructivism

Methodology

Effective Questioning

Group Work

Multiplication of Complex Numbers

Task:

$$z_1 = 1 + 2i,\ z_2 = 2 + 3i,\quad find\ z_1 \times z_2$$

Distributive Law

$$\begin{aligned} z_1 \times z_2 &= (1 + 2i)(2 + 3i) \\ &= 1(2 + 3i) + 2i(2 + 3i) \\ &= 2 + 3i + 4i + 6i^2 \\ &= 2 + 7i - 6 \\ &= -4 + 7i \end{aligned}$$

The Array Model

	2	$3i$
1	2	$3i$
$2i$	$4i$	$6i^2$

$$\begin{aligned} z_1 \times z_2 &= 2 + 3i + 4i + 6i^2 \\ &= 2 + 7i - 6 \\ &= -4 + 7i \end{aligned}$$

Multiplication of Complex Numbers (Geometric Approach)

Breakout Session 2

Multiplication of Complex Numbers

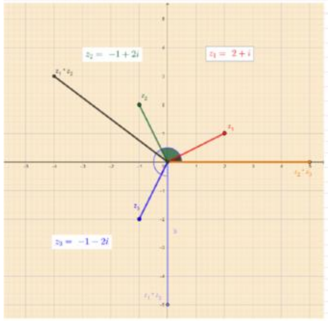
Multiplication of Complex Numbers

Collaborative Task:

Choose a role, complete the investigation and record the results in the master table.

Group Roles:
Record (Input to table)
Method 1 (Geogebra File)
Method 2 (Trigonometry)
Verifier (Confirm Results)

Activity 1



Multiplication

Activity 1	Breakout room 1						
	Complex number	2 + i	-1 + 2i	-1 - 2i	(2+i)(-1+2i)	(2+i)(-1-2i)	(-1+2i)(-1-2i)
Geogebra	Length						
	Angle						
Trigonometry	Length						
	Angle						
Rulers	Length						
	Angle						

Breakout room 1
After you have completed the table above try to answer the questions below.

What Can you conclude about the length of the product of two complex numbers after multiplication?

What can you conclude about the argument of the product of two complex numbers after multiplication?

Summary of Key Learning

Student key learning:

When two complex numbers are multiplied their moduli are multiplied and their arguments are added.

Reflection:

What elements of this task make a good learning experience for students?

Stretch Break

Session 3

DeMoivre's Theorem

Multiplication of Complex Numbers

Prior Content Knowledge

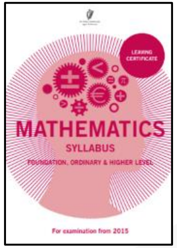
Trigonometric Ratios and Pythagoras' Theorem.

Junior Cycle Specification page 17, Junior Certificate Syllabus page 20.

Multiplication of complex numbers.

Polar form of a complex number.

Leaving Certificate Syllabus page 29, 39.



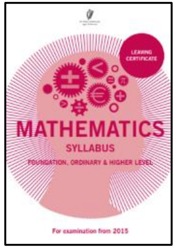
Multiplication of Complex Numbers

Key Content for this task

DeMoivre’s Theorem.

Polar form of a complex number

Leaving Certificate Syllabus page 39



Pedagogy

Constructivism

Methodology

Effective Questioning

Structured Problem Solving

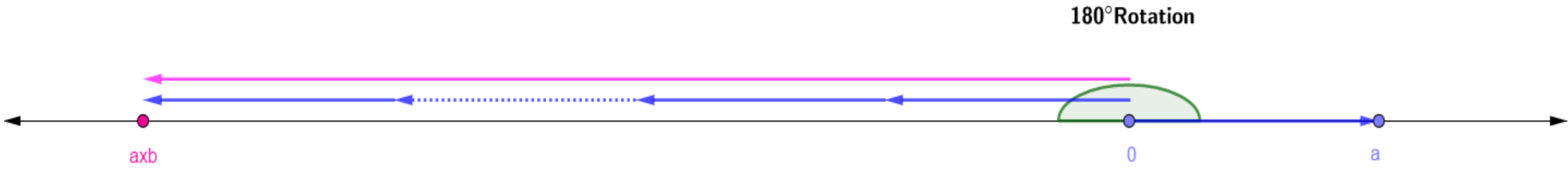
Prior Knowledge: Geometric View of Multiplication

Task 1

Multiplication of Real Numbers $a \times b$, where $a, b \in \mathbb{R}$

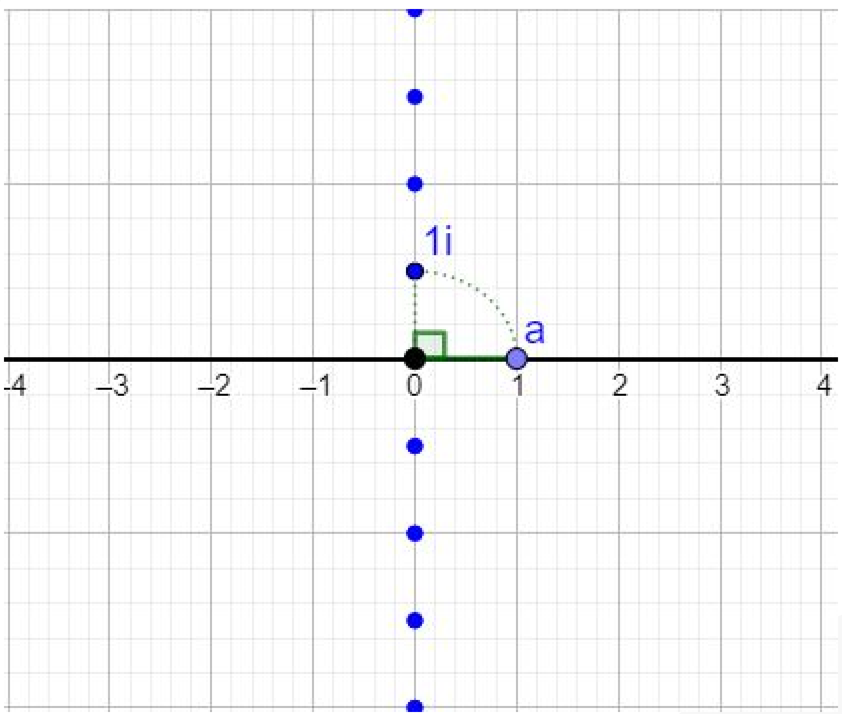
If b>0 there is a scaling by a factor of b and 0° rotation.

If b<0 there is a scaling by a factor of b and 180° rotation.



Prior Knowledge:
Geometric View of Multiplication

Multiplication by $(-1)^{½}$
Scaling by a factor of 1
Rotation of 90°



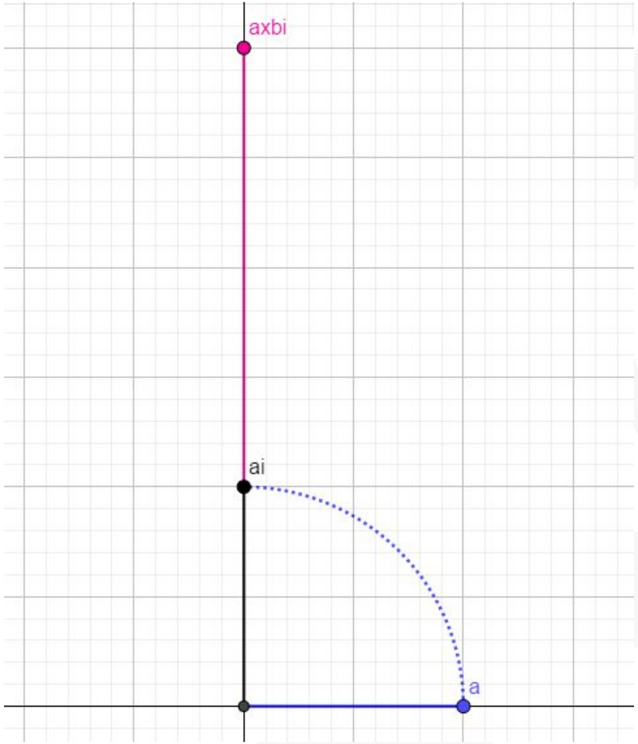
Prior Knowledge: Geometric View of Multiplication

Multiplication of Complex Numbers of the form bi

Scale factor of b

If b>0 Rotation of 90°

If b<0 Rotation of 270°



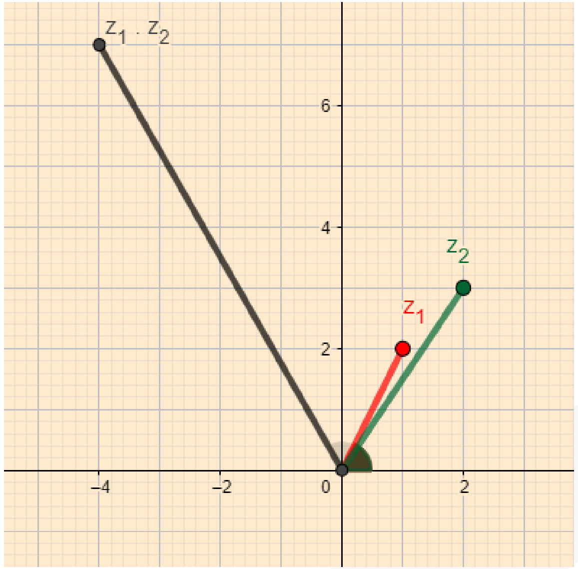
Prior Knowledge:
Multiplication of Complex Numbers

Task 2

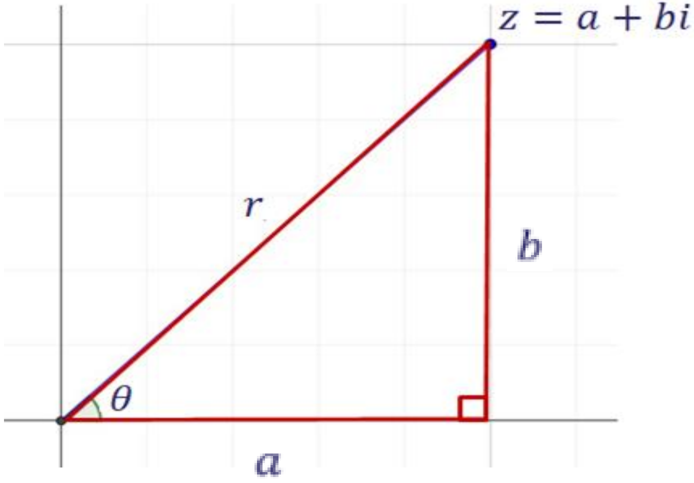
What are the effects of multiplying z_1 by z_2, if both z_1 and z_2 are complex numbers in the form $(a+bi)$, where $a, b \in \mathbb{R}$?

$|z_1|$ is scaled by a factor of $|z_2|$
(Product of the Moduli)

z_1 is rotated by the argument of z_2
(Sum of the Arguments)



Prior Knowledge: Trigonometry



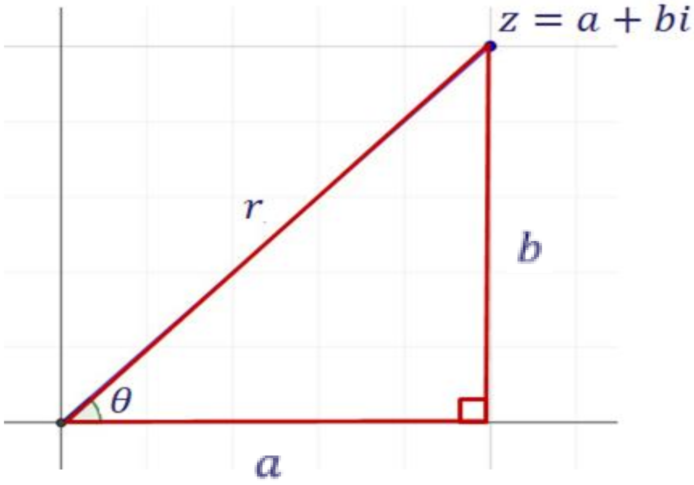
$$Sin\theta = \frac{b}{r}$$

$$Cos\theta = \frac{a}{r}$$

$$rSin\theta = b$$

$$rCos\theta = a$$

Prior Knowledge:
Polar Form of a Complex Number



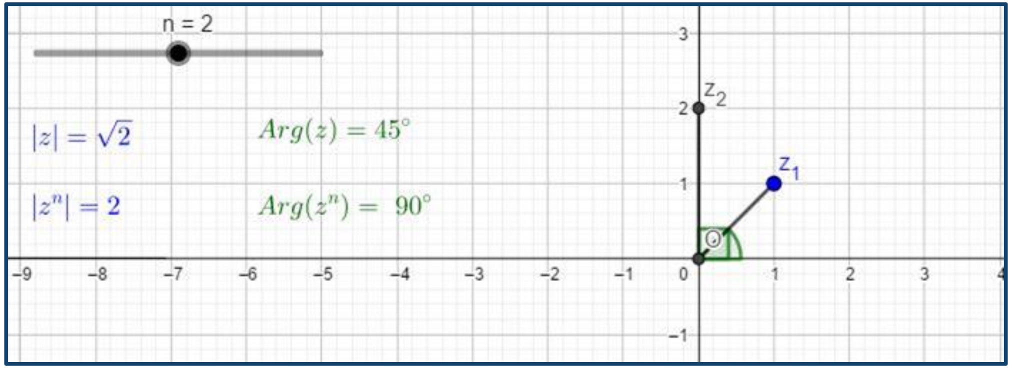
$$z = a + bi$$

$$z = rCos\theta + irSin\theta$$

$$z = r(Cos\theta + iSin\theta)$$

DeMoivre's Theorem

If z=a+bi, where a, $b \in \mathbb{R}$, what is the effect on z of raising z to the power of n (z^n), where $n \in \mathbb{Z}$?



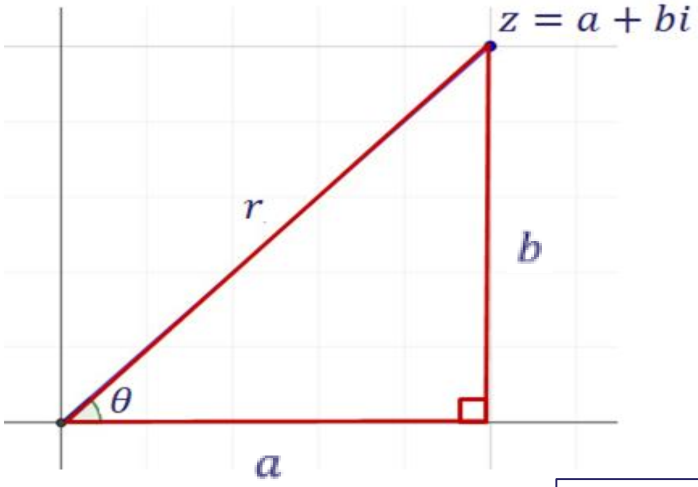
Breakout Session 3

De Moivre's Theorem

Summary of Key Learning

Multiplication	Argument (angle θ)	Modulus (r)
$z \times z = z^2$	Doubled (2θ)	Squared (r^2)
$z \times z \times z = z^3$	Tripled (3θ)	Cubed (r^3)
$z \times z \times z \times z = z^4$	Multiplied by 4 (4θ)	Multiplied by itself 4 times (r^4)
$z \times z \times z \times z \times z = z^5$	Multiplied by 5 (5θ)	Multiplied by itself 5 times (r^5)
z^n	Multiplied by n ($n\theta$)	Multiplied by itself n times (r^n)

Summary of Key Learning



$$z = a + bi$$

$$z = rCos\theta + irSin\theta$$

$$z = r(Cos\theta + iSin\theta)$$

$$\boxed{z^n = r^n(Cos\, n\theta + i\, Sin\, n\theta)}$$

Extending the Learning

Generalise your thinking:

$$z^n = r^n(Cos\, n\theta + i\, Sin\, n\theta)$$

If we were given a value for z^n, how many roots would z^n have?
How would you find these roots?

Applying the Learning

If $z = 1 + i$, find z^5 in as many ways as possible.

Which of these methods would you use to find z^{10}?

Lesson Study

IRELAND

Conclusion

Summary and Reflection

Key Messages

Understand the importance of using a geometric view to make sense of complex numbers.

Understand the importance of extending prior knowledge through using a constructivist approach.

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