The 5 Levels of Geometric Thought with examples

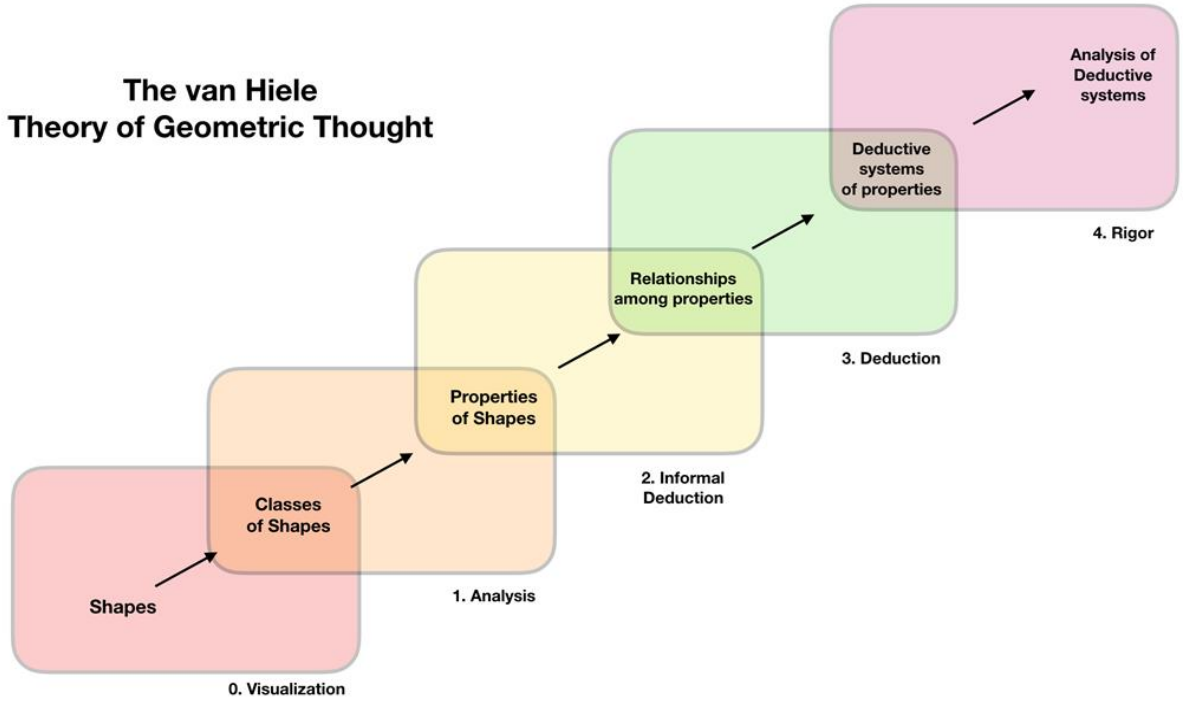

Table 1 The van Hiele model of geometric understanding describes a progression that is independent of age or grade level.

Level	Name	Description	Example	Teacher Activity
0	Visualization	See geometric shapes as a whole; do not focus on their particular attributes.	A student would identify a square but would be unable to articulate that it has four congruent sides with right angles.	Reinforce this level by encouraging students to group shapes according to their similarities.
1	Analysis	Recognize that each shape has different properties; identify the shape by that property.	A student is able to identify that a parallelogram has two pairs of parallel sides, and that if a quadrilateral has two pairs of parallel sides it is identified as a parallelogram.	Play the game "guess my rule," in which shapes that "fit" the rule are placed inside the circle and those that do not are outside the circle (see Russell and Economopoulos 2008).
2	Informal deduction	See the interrelationships between figures.	Given the definition of a rectangle as a quadrilateral with right angles, a student could identify a square as a rectangle.	Create hierarchies (i.e., organizational charts of the relationships) or Venn diagrams of quadrilaterals to show how the attributes of one shape imply or are related to the attributes of others.
3	Formal deduction	Construct proofs rather than just memorize them; see the possibility of developing a proof in more than one way.	Given three properties about a quadrilateral, a student could logically deduce which statement implies which about the quadrilateral (see **fig. 1**).	Provide situations in which students could use a variety of different angles depending on what was given (e.g., alternate interior or corresponding angles being congruent, or same-side interior angles being supplementary).
4	Rigor	Learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems.	Students should understand that other geometries exist and that what is important is the structure of axioms, postulates, and theorems.	Study non-Euclidean geometries such as Taxi Cab geometry (Krause 1987).

Fig. 1 Students should be able to answer this question using formal deduction.

Three Properties of a Quadrilateral
Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?
a. D implies S, which implies R.
b. D implies R, which implies S.
c. S implies R, which implies D.
d. R implies D, which implies S.
e. R implies S, which implies D.

Source: Usiskin (1992)

The 5 phases of Learning in a Van Hiele Theory of Geometric Thought Classroom