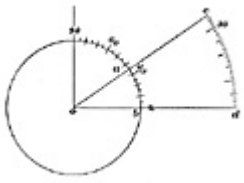


Teaching and Learning Plan
Radian Measure

Target Students:

Leaving Certificate Higher Level Students

Prior Knowledge:

Students will have studied Junior Certificate trigonometry.

Relationship to Syllabus:

Radian Measure of Angles

Duration:

In this case it will be very much dependant on the capability of the class. For some classes it may be done in one class period, for other classes it may take 2 to 3 classes.

Aims:

The class will take an explorative approach to investigate the properties of radian measure. Pupils will be provided with a range of activities and scaffolded support from the teacher to enable the pupil to generate a deep understanding of radian measure and be able to apply its use in related problems

Learning Outcomes:

At the end of using this methodology pupils will be able to:

- Work efficiently in pairs (or small groups) to investigate a new topic
- Construct an angle of size 1 radian
- Measure, approximately, the 1 radian angle in degrees
- Estimate the number radians in a semi-circle
- Estimate the number of radians in a full circle
- Demonstrate an understanding of what a radian is and how it relates to degree measure
- Be able to accurately estimate the number of radians in a given angle
- Calculate the exact number of radians in a full circle
- Derive the identity π radians = 180º
- Use the identity above to do all conversions from radians to degrees and vice versa
- Derive the formula θ (in radians) = $\dfrac{\text{Arc Length}}{\text{Radius}}$
- Use the fact that $\text{Area of Sector} = \text{Area of full circle} \times \dfrac{\text{Angle (in radians)}}{\text{Full circle(in radians)}}$

  to derive Area of Sector = $\frac{1}{2} r^2\theta$

- Use these 2 formulas in problems

Resources Required:

Full set of mathematical instruments, small piece of string, Angle Estimator (provided by teacher)

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input**:
This section focuses on teacher input
and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities** – Possible and Expected Responses: Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions**: Gives details of teacher actions designed to support and scaffold student learning.

iv. **Checking Understanding**: Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Learning Tasks: Teacher Input.	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Ask the class "How do we measure the size of an angle ?" "Why do we need a second way of measuring angles ?"	Pupils will identify degrees as a way of measuring angles. They will probably not see any need for a second way of measuring angles.	1) In mathematics, other than in practical geometry, angles are mostly measured in radians. Radians are preferred as they make calculations in trigonometry and calculus shorter and neater. 2) Later in the course we will be studying Calculus. There is a very important limit in calculus, $\lim_{x\to 0}\frac{\sin x}{x}$ which is only defined for angles expressed in radians. 3) Radians are more widely used in other branches of mathematics. In physics for example 'angular velocity' and 'angular acceleration' are typically measured in radians.	Can student verbalise the 2 ways of measuring angles and give 2 reasons why radian measure is being studied.
Write the following on the board " Angle size 5" Ask class what would they assume if this were written somewhere ?	Pupils familiar with degree measure and assume the units are in degrees.	In mathematics if no units are indicated the default metric is the radian. So " Angle size 5" actually means 5 radians. Introduce the idea that the word 'radian' is usually left out.	
What symbol is used for degrees ? What symbol is used for radians ?	Pupils familiar with degree notation i.e. 5°.	There is an international notation of superscript 'c' for radians. However it is rarely used as the default of nothing means radians. It is good practice initially to write rads(or radians or ^{c}) after the angle for radians. This can be dropped later when familiar with the metric.	Can student verbalise 3 ways that radian units can be indicated ? (4 ways if the default concept is included). Activity Sheet 1 (not to be given to class yet) has questions probing this.

Student Learning Tasks: Teacher Input.	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Task 1: Divide the class up into pairs. Give one piece of string to each pair of pupils.		Ensure that one pupil is holding the string at its full length and the other pupil is setting compass to this length, as in the picture on the left. The pieces of string will be of varying lengths. This will be re-enforced later when pupils will compare circle size and radian size in each other's work.	Various sizes of circles will be drawn by the class.
Task 2: Hand out Activity Sheet 1	Pupils must work on the circumference.	Ensure that the string is used along the circumference. The estimation of the approximate size of 1 radian (i.e. item 6 in *Activity Sheet 1 Radian Measure*) is crucial to the understanding of radians.	Pupils should have approximated the size of 1 radian to between 56° and 60°.
Item 8 Activity Sheet 1: "Where is a radian defined ?"	There is a misconception that a radian is defined from the centre, as a degree is.	Reinforce the circumference as what defines a radian. Get pupils to say what they mean in their own words when they say "An angle of 1 radian". This must be communicated by each pupil, either to each other or to the teacher. (*Item 8 Activity Sheet 1*)	Monitor the pupils' capability to **verbalise** their understanding of 1 radian. Also be able to **verbalise** the non-importance of the size of the circle to radian measure.

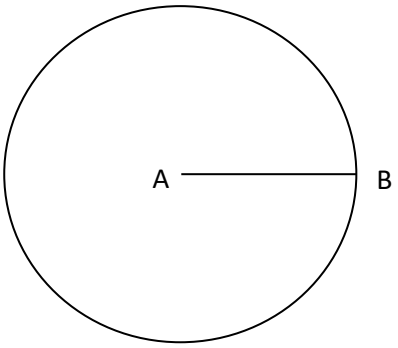
Student Learning Tasks: Teacher Input.	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Enable pupils to derive the number of radians in a circle and a semicircle.	Items 10 1nd 11 from *Activity Sheet 1.*	Monitor groups closely for this and 'scaffold' support as required. A few pupils may be able to do this without any support. Some will need substantial support. Let pupils experiment and 'problem solve' at this stage. Get pupils to focus on circumference if struggling. If necessary give the following link: **Full circle in radians = Full circle in degrees** And then work from there. Again focus on **where** a radian is defined i.e the circumference and how many degrees are in the full circle i.e. 360° .	Pupils should have evidence to show they understand this central concept. It is item 10 and 11 but they need to be able to generate and communicate this. Spend time on this, as it is important, and maybe reinforce following day (class). In class next day write the following on the board: "***In area and volume π is 22/7 but in trigonometry π is 180°***" Ask class for comments on this sentence.

<table>
<tr><th>Student Learning Tasks: Teacher Input.</th><th>Student Activities: Possible and Expected Responses</th><th>Teacher's Support and Actions</th><th>Checking Understanding</th></tr>
<tr><td>Does pupil have a deep intuitive understanding of the size of a radian ?
Give each group an Angle Estimator.</td><td>1 radian may not seem to be approximately 60° to some children. They may feel that 60° is too big for just 1 radian.</td><td>Use "Engle Estimator" to get pupils questioning each other on this. Give out Angle Estimator and get pupil to create and show various radian measures.</td><td></td></tr>
<tr><td>Item 1 on Activity Sheet 2:
Write "π radians = 180°" on board</td><td>Some conversions are easy e.g π/2 radians is 90°. However to cope with all conversions a "Teaching for Understanding" approach is recommended.</td><td><table><tr><td>Converting radians to degrees:
π radians = 180°
How would you calculate 1 radian ?
How would you the get to the required number of radians ?</td><td>Converting degrees to radians:
π radians = 180°
How would you calculate 1 degree ?
How would you the get to the required number of degrees?</td></tr></table>The approach in the table above, will re-enforce understanding, since the first line was discovered by the pupil. Recommend using this approach for as long as is necessary. If pupil 'sees' shortcuts then fine, but let that stage develop from work done, rather than being given by the teacher and 'learnt off' by pupil.</td><td>It is good practice to write radians (or rads) until pupils very familiar with the topic. May even use it all the time.
Item 1 on Activity Sheet 2 can be used to test the understanding of this.
Monitor groups and look for evidence of work shown.</td></tr>
</table>

Student Learning Tasks: Teacher Input.	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Item 4 Activity Sheet 2: Derive Area sector = ½ r^2 θ	Pupils' prior knowledge will only be using πr^2. Calculating the area of a sector using this method may need to be looked at for this section.	Again monitor groups and see what support is required. build on prior knowledge and where necessary provide this: $$\text{Area of Sector} = \text{Area of full circle} \times \frac{\text{Angle (in radians)}}{\text{Full circle(in radians)}}$$ Leave out the "in radians" clue initially, And then provide where necessary. Get groups to construct a circle and choose a sector e.g circle of 7cm and sector of angle 240°. Calculate the area of this sector both ways and use one to verify the other.	Area sector = ½ r^2 θ This is item 6 on *Activity Sheet 2.* Items 7 and 8 should help consolidate this work. Homework may be necessary.
Item 5 Activity Sheet 2: Investigate both ways of getting the area of a sector and then compare answers.	Pupils may not be aware that both sectors are the same size. See how this develops, monitor to see if more intervention is necessary.		

Radian Measure – Introduction

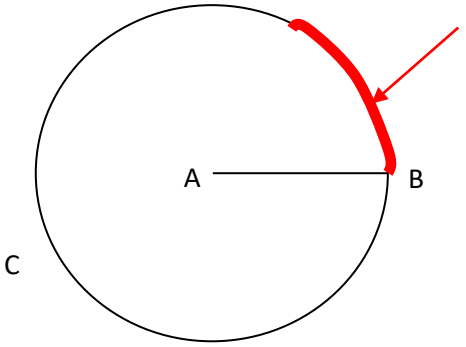
Activity Sheet 1



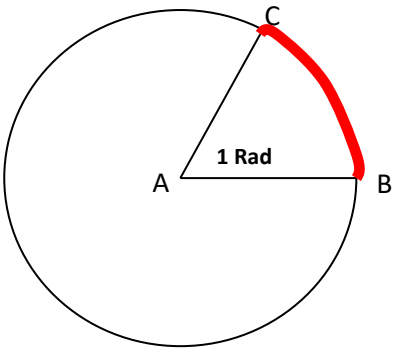
1) Construct a circle whose radius is equal to the length of the piece of string given.

2) Draw in the radius and label it AB as shown.

String



3) Using the string, place one end of it at B and lay it out on the circumference.
Mark C at the end of the string.

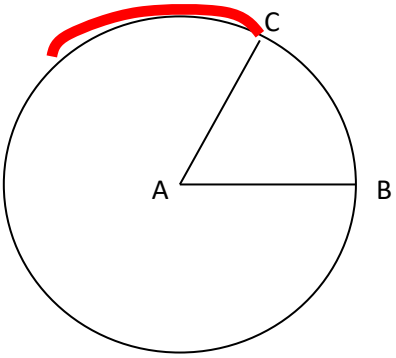


4) Join point A to point C.
The angle created in the centre of the circle is called "**1 Radian**".

NOTE: Activities 1 to 4 show how to **construct** an angle of size 1 radian

5) Using a protractor measure (and write down) the approximate size of this radian in degrees.

Approximate size of 1 radian in degrees = _____



6) Using the string **on the circumference** repeat step 4 above until the entire circumference has been covered.

7) From the work you have done on this circle answer the following :

a) Estimate how many radians are in a full circle _____

b) Estimate how many radians are in a semi-circle _____

8) Write down, in your own words, what you understand by the phrase "*An angle of size one radian*."

_____

_____

_____

9) Complete the following table, using circles drawn by the class.

Circle	**Radius**	**Estimation of 1 radian**
Circle 1 *		
Circle 2		
Circle 3		
Circle 4		

* the circle you constructed at question 1 above

Based on the table above, what affect does the size of the radius of a circle have on the size of 1 radian ?

_____

_____

_____

10)
(a) From analysing the work so far make a connection between the number of radians in a full circle and the circumference.

(b) If θ is an angle, measured in radians, derive a formula connecting θ, **(L) the length of the arc** and the **(r) radius.**

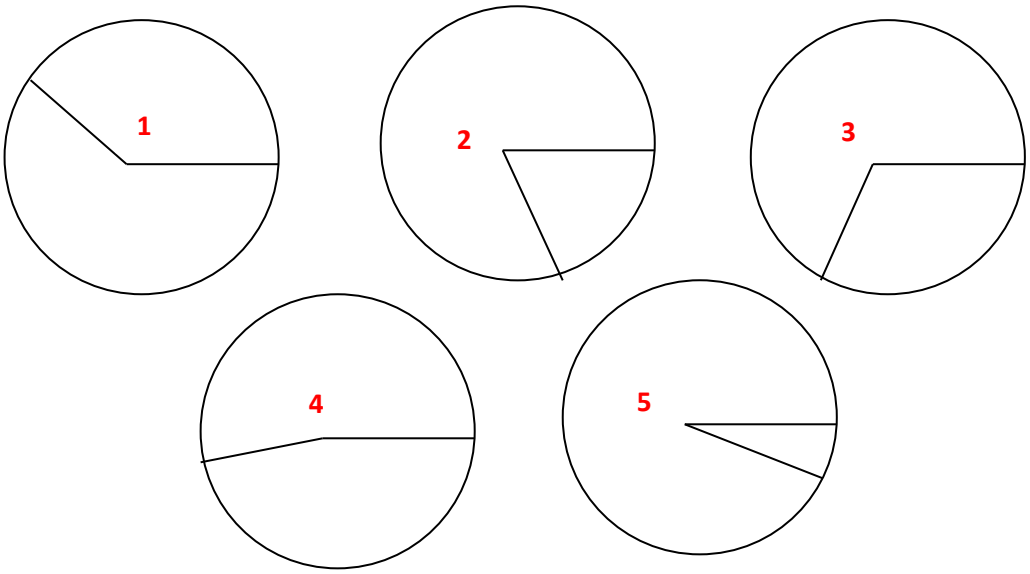
11) Using the identity generated in the previous question:
(a) How many radians are in a full circle correct to two decimal places ? _____

(b) Show that π radians = 180°

(c) How many radians are in a semi-circle , correct to two decimal places ? _____

12) Using the identity generated at question 10 above, show how to calculate the size of 1 radian, correct to one decimal place.

13) Which of the following angles 1, 2, 3, 4 or 5 represent an angle of approximately 4 radians. Explain your answer.



14) Summarise what you have learned about radians from this activity.

Radian Measure

Activity Sheet 2

1) An identity already derived (in *Q 11 on Activity Sheet 1*) was that π radians = 180º.
This identity is very useful to easily convert degrees to radians or radians to degrees.

(i) Use the identity π radians = 180º **as your starting point** show how to convert:

(a) 200° to radians (b) $\frac{7\pi}{6}$ (c) 1½ radians to degrees

(ii) Convert each of the following to degrees to radians or radians to degrees as required.

	Radians	Degrees		Radians	Degrees
1		60º	10	$\frac{\pi}{4}$	
2		30º	11	$\frac{\pi}{3}$	
3		70º	12	$\frac{2\pi}{5}$	
4		120º	13	$\frac{5\pi}{4}$	
5		80º	14	$\frac{5\pi}{6}$	
6		75º	15	4π	
7		12º	16	2	
8		300°	17	4.5	
9		720º	18	6	

2) (a) Draw any circle and **construct** an angle of 2 radians

(b) Using the identity derived at *Q10 on Activity Sheet 1* find the length of the arc of this angle algebraically.

(c) Using a protractor, measure the size of this angle in degrees _____

Comment on whether this is the degree measure you expected:

_____

_____

_____

_____

_____

_____

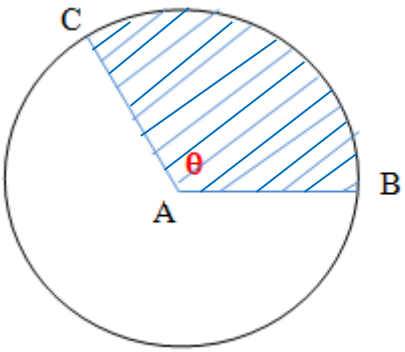
3) (a) Construct a circle of radius 5cm (b) Construct an arc of 20cm on this circle.

(c) Write down the size of the angle subtended at the centre of this circle in radians

_____

(d) Using the identity from derived at *Q10 on Activity Sheet 1* verify your answer from (c) algebraically.

4) The diagram shows a shaded sector ABC .
Show how the identity,
Area of Sector = ½ $r^2\theta$ is derived,
where r is the length of the radius.
and θ is the angle of the sector measured in radians



Hint: $$\text{Area of Sector} = \text{Area of full circle} \times \frac{\text{Angle (in radians)}}{\text{Full circle(in radians)}}$$

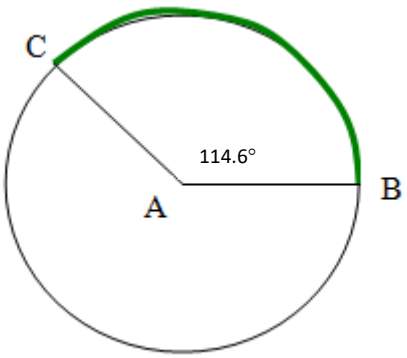
5) (a) This question looks at the area of a sector of a circle circle. It investigates using both degrees and radians to get the area of a sector and compares the answers.

Calculate the area of the sector

of angle 114.6°

in a circle of

radius 7cm.



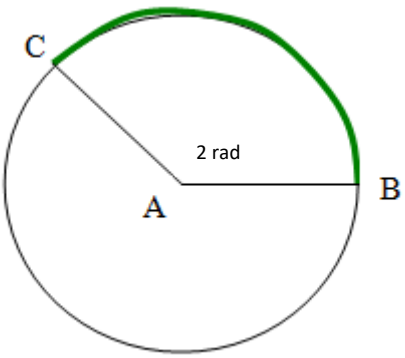
Area of sector = _____

Calculate the area of the sector

of angle 2 radians

in a circle of

radius 7cm.



Area of sector = _____

(b) Why were 114.6° and 2 radians used ?

_____

_____

(c) Comment on the two answers.

_____

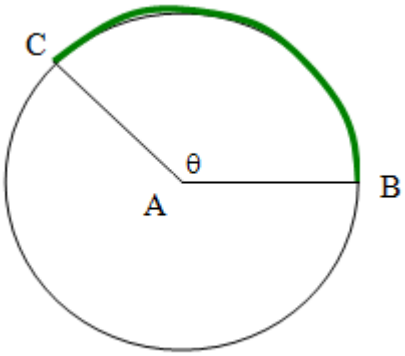
_____

_____

_____

6) The circle on the right has a minor arc length BC of 10cm.
The angle θ is 143.24°.

(a) Write down an approximation for θ in radians _____



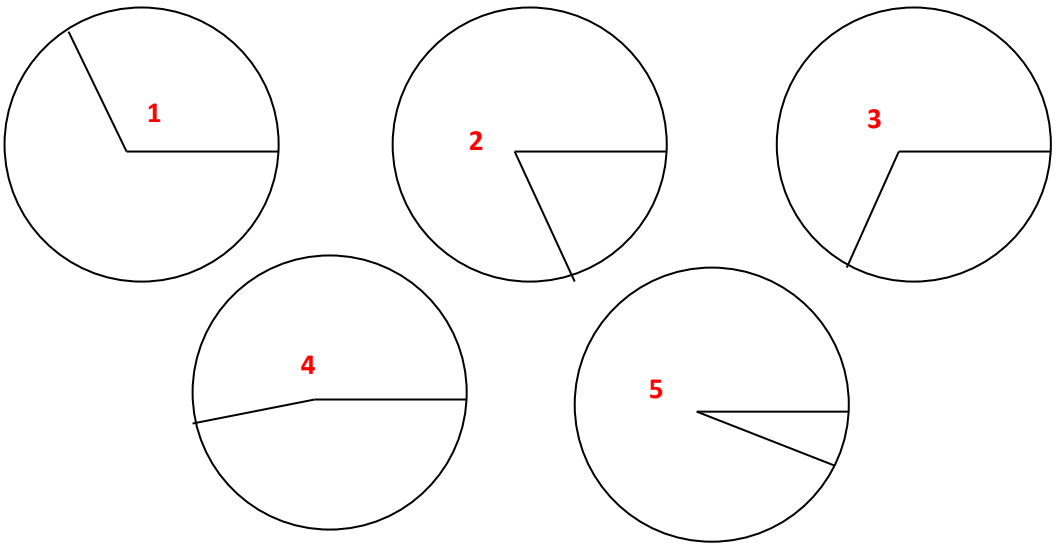
(b) Show how to calculate θ in radians (to 1 decimal place)

(c) Calculate the radius of the circle

7) Each of the circles below has a radius of 8cm.

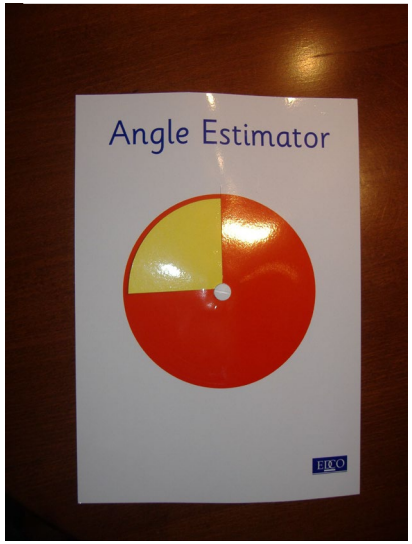
(a) Calculate the size of the angle (in radians) of a sector in a circle which has radius 8cm and area of sector of $67cm^2$ to nearest cm.

(b) Identify which of the circles below has a sector of area $67cm^2$ (to nearest cm) and explain your choice(s).



Picture 1 : Angle Estimator

Front



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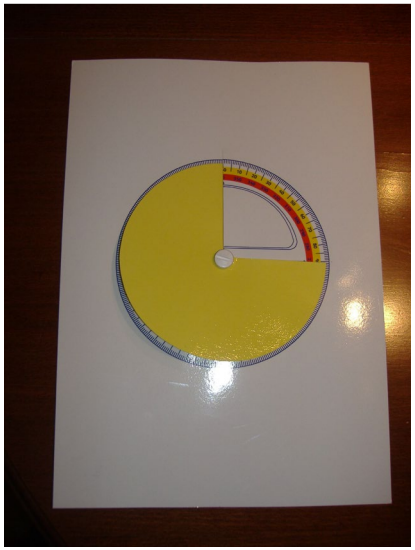

Websites on Radians:

1. Video tutorials, diagnostic tests and exercises. (Recommended)
   http://www.mathcentre.ac.uk/students.php/all_subjects/trigonometry/radian/resources/

2. 8 applications of Radian measure
   http://www.intmath.com/Trigonometric-functions/8_Applications-of-radians.php

3. Converting radians to degrees and vice versa. Examples and problems
   http://www.themathpage.com/atrig/radian-measure.htm#rad

4. Interactive unit circle and radians
   http://www.themathpage.com/atrig/radian-measure.htm#rad

5. Teachnet site on Leaving Cert Trigonometry
   http://www.teachnet.ie/hpunzet/2007/

6. BBC summary notes on trigonometry
   http://www.bbc.co.uk/scotland/education/bitesize/higher/maths/trigonometry/radian_and_equations2_rev.shtml

7. Teacher tube videos
   http://www.mathcentre.ac.uk/students.php/all_subjects/trigonometry/radian/resources/