## Exploring the relationship between the Argument and Modulus when multiplying complex numbers

Follow this link to answer the questions below: https://www.geogebra.org/classic/bmedez74

How do we find the Argument of a complex number?

What do you notice about the Arguments when 2 complex numbers are multiplied?

How do we find the modulus of a complex number?

What do you notice about the moduli when 2 complex numbers are multiplied?

What do you think would happen if we were to multiply a complex number by itself?

What would happen if we were to cube that complex number?

Can you see a pattern? (click on the button to reveal exponents of z)

## The effect of multiplying a complex number by itself (complete the table):

| Multiplication of a complex number | Argument (angle θ)       | Modulus (r)                          |
|------------------------------------|--------------------------|--------------------------------------|
| $z \times z = z^2$                 | Double the argument (20) | Square the modulus (r <sup>2</sup> ) |
| z x z x z =                        |                          |                                      |
|                                    |                          |                                      |
|                                    |                          |                                      |
|                                    |                          |                                      |
| z <sup>n</sup>                     |                          |                                      |

## The effect of multiplying a complex number by itself:

| Multiplication                         | Argument (angle θ)   | Modulus (r)                                    |
|--|----------------------|--|
| z x z = z <sup>2</sup>                 | Doubled (2θ)         | Squared (r <sup>2</sup> )                      |
| z x z x z = z <sup>3</sup>             | Tripled (30)         | Cubed (r <sup>3</sup> )                        |
| $z \times z \times z \times z = z^4$   | Multiplied by 4 (40) | Multiplied by itself 4 times (r <sup>4</sup> ) |
| z x z x z x z x z x z = z <sup>5</sup> | Multiplied by 5 (50) | Multiplied by itself 5 times (r <sup>5</sup> ) |
| z <sup>n</sup>                         | Multiplied by n (nθ) | Multiplied by itself n times (r <sup>n</sup> ) |

De Moivre's theorem:  $z^n = r^n (\cos n\theta + i \sin n\theta)$