



Challenging All Students' Mathematical Thinking

Advisor Name
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Timetable

Session 1 9:15 - 10:30	Introduction Rich Tasks
Break (15 mins)	
Session 2 10:45 - 12:00	Formative Assessment
Break (15 mins)	
Session 3 12:15 - 13:15	Good Practice - Activities and Resources Conclusion and Reflection

Key Messages

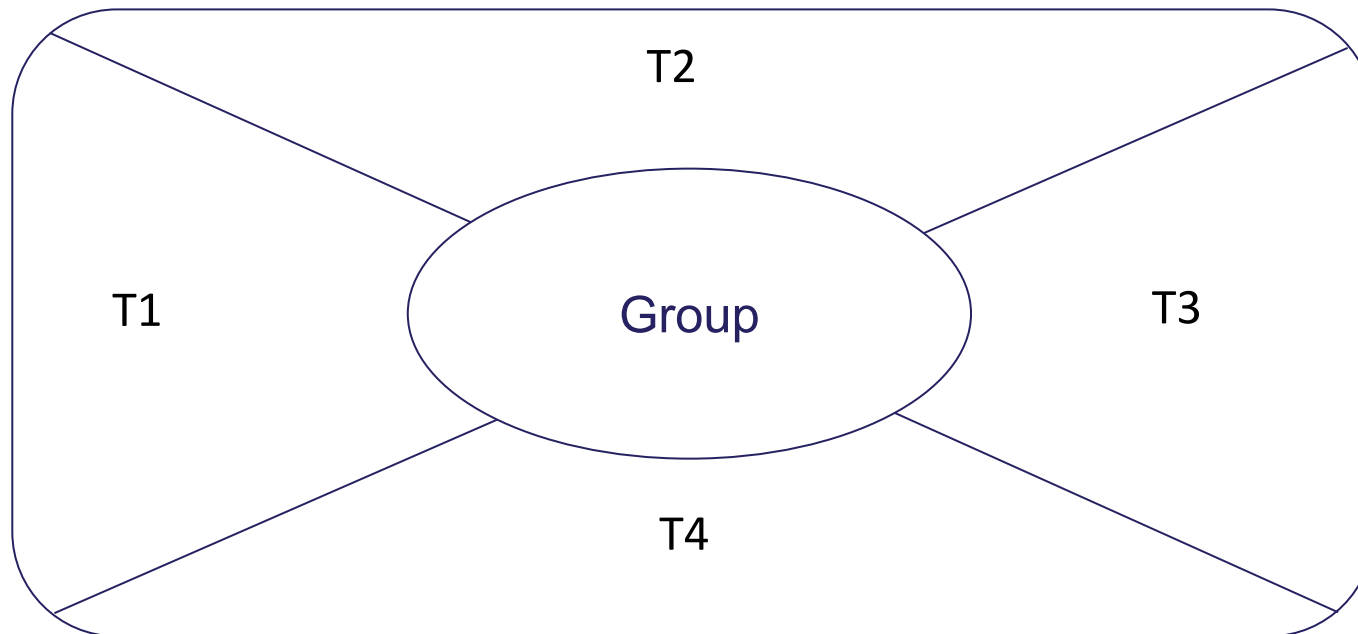
- It is important that all our maths students are challenged to continually develop their Mathematical thinking.
- The use of rich tasks combined with personalised formative feedback can be used to support all students in developing their mathematical thinking and proficiency.
- Utilising a constructivist approach and building upon prior knowledge enables students to extend their learning.

Padlet

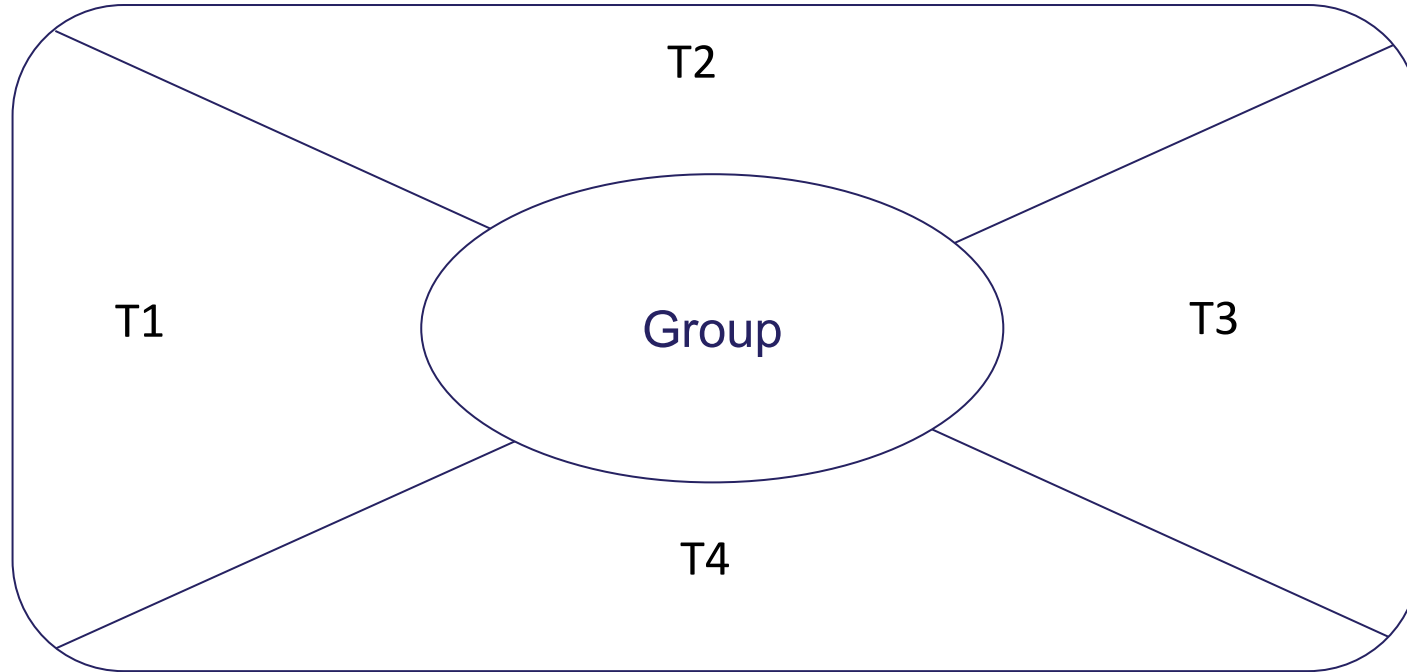
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What are the main Challenges for teachers in facilitating effective learning for the wide range of ability levels in their maths class?



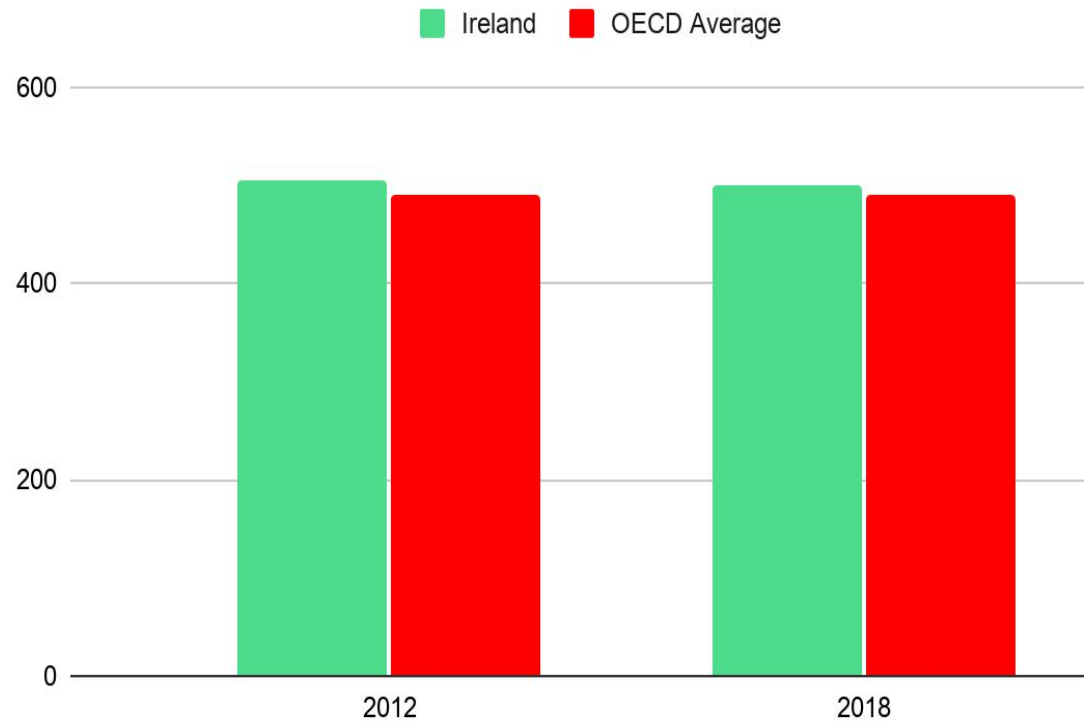
What are the main Opportunities for teachers in facilitating effective learning for the wide range of ability levels in their maths class?



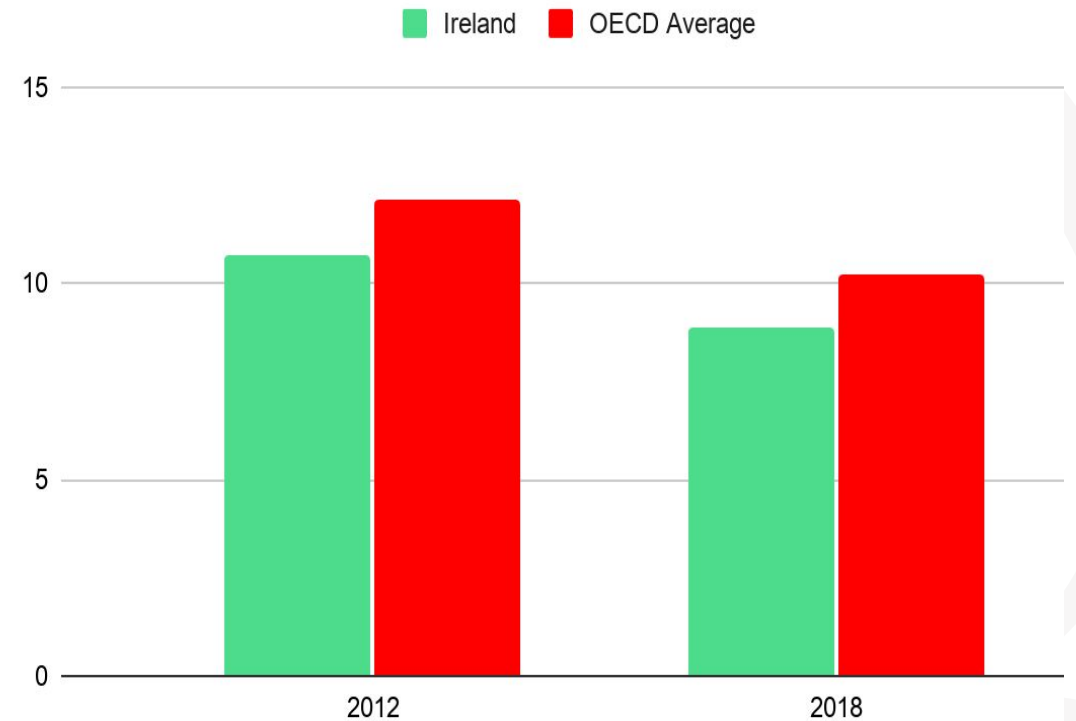
PISA: Higher Achieving Students



PISA Average



% of Higher-Achieving Students in PISA



TIMSS: Student Engagement in Maths Class



IEA

TIMSS & PIRLS

International Study Center
Lynch School of Education, Boston College

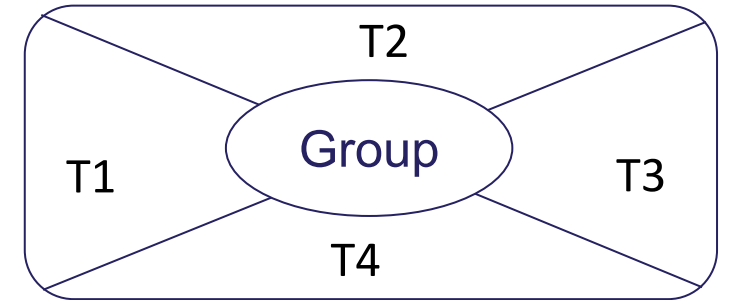


In Ireland, Fourth Class students were generally positive about their Mathematics lessons, and were more likely to find their lessons very engaging than their international peers (73% vs. 68%)

Second Year students in Ireland were less likely to report finding their lessons very engaging than on average across all TIMSS countries (37% vs. 43%), and more likely to indicate that their Mathematics lessons were less than engaging (22% vs. 17%)”.

(Students’ perspectives on learning mathematics and science: Results from TIMSS 2015 in Ireland.)

What are the main Challenges & Opportunities for teachers in facilitating effective learning for the wide range of ability levels in their maths class?



How would a young Ramanujan get on in your school?

Looking At Our School 2016

Statements of Highly Effective Practice

Under the domain of “learner outcomes”

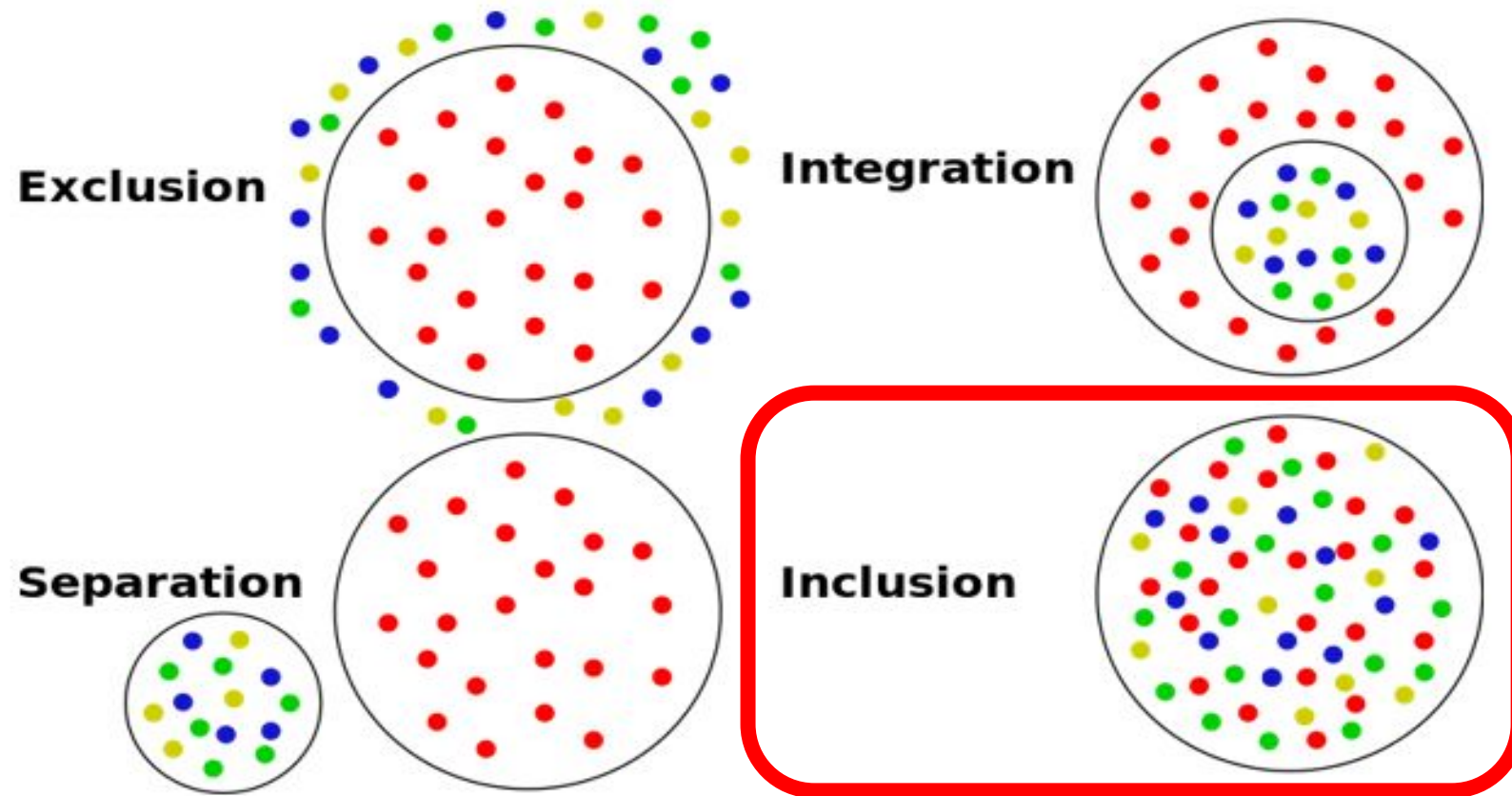
Under the domain of “teacher’s individual practice”:



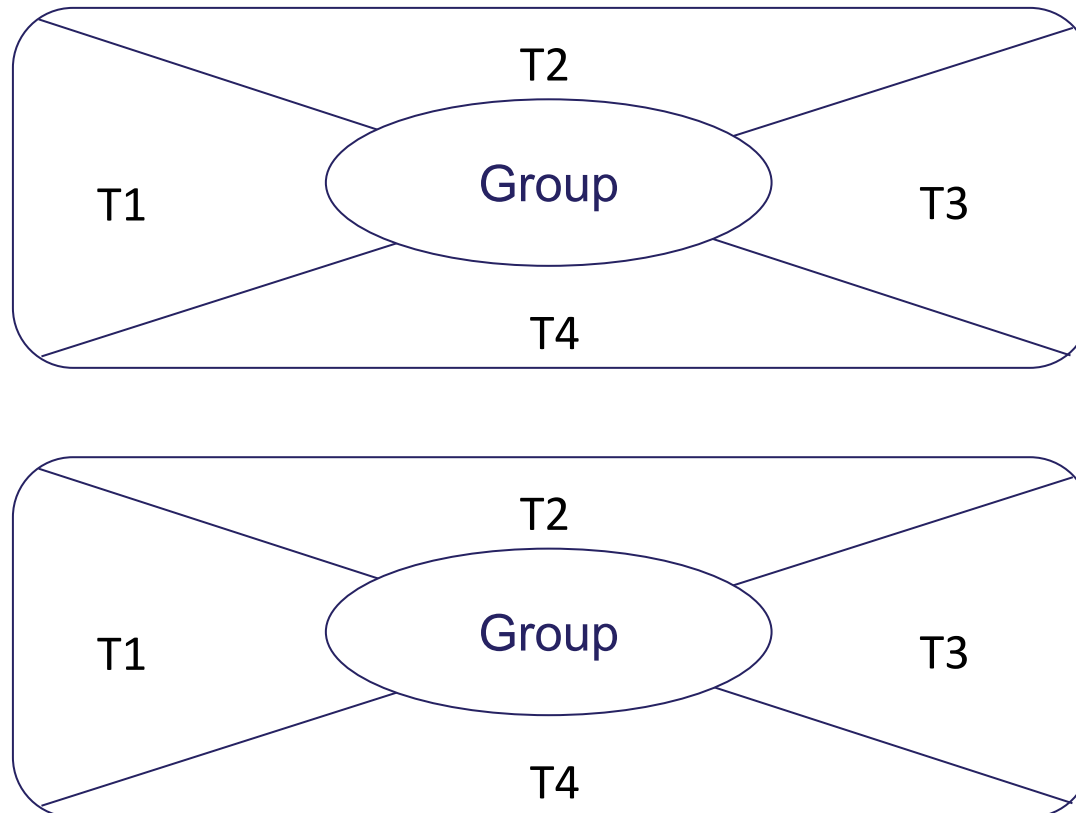
Students are motivated to learn **through** having a clear sense of attainable **and** **challenging** learning outcomes.

Teachers identify and **thoroughly** prepare in advance resources **tailored to match** the specific learning intentions of each lesson, or series of lessons, and **individual students’ learning needs**.

What does Inclusion currently look Like In Your School?



Reflect on what the information presented means for your practice



What are the main Opportunities & Challenges for teachers in facilitating effective learning for the wide range of ability levels in their maths class?



Rich Tasks

Task intended Learning outcome.

Participants will understand that a rich task, designed to include formative feedback, can challenge a range of students' mathematical thinking and ability.

Rich Task

What are the key features of rich tasks?

- Accessible to all learners
- Multiple approaches and representations
- Collaboration and discussion
- Engagement, curiosity and creativity
- Cross-topic connections
- Opportunities for extension

Leaving Certificate Syllabus

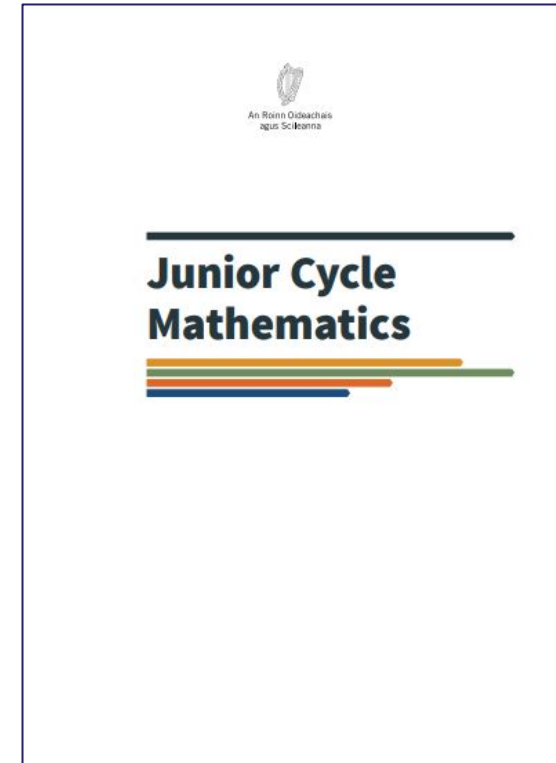
Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.3 Outcomes of random processes	<ul style="list-style-type: none"> – find the probability that two independent events both occur – apply an understanding of Bernoulli trials* – solve problems involving up to 3 Bernoulli 	<ul style="list-style-type: none"> – solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required)
1.2 Concepts of probability	<ul style="list-style-type: none"> – use set theory to discuss experiments, outcomes, sample spaces – discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams 	<ul style="list-style-type: none"> – extend their understanding of the basic rules of probability (AND/OR, mutually
1.7 Analysing, interpreting and drawing inferences from data	<ul style="list-style-type: none"> – recognise the concept of a hypothesis test – recognise the role of expected value in decision making and explore the issue of fair games 	<ul style="list-style-type: none"> – $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ – Multiplication Rule (Independent Events): $P(A \cap B) = P(A) \times P(B)$ – Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B A)$ – solve problems involving sampling, with or without replacement

Students Prior Knowledge

Statistics and probability strand

SP.2 investigate the outcomes of experiments so that they can:

- generate a sample space for the probability in a systematic way, including tree diagrams for successive events and two-way tables for independent events
- use the principle that, in the case of equally likely outcomes, the probability of an event is given by the number of outcomes of interest divided by the total number of outcomes
- use relative frequency as an estimate of the probability of an event, given experimental data, and recognise that increasing the number of times an experiment is repeated generally leads to progressively better estimates of its theoretical probability



Section 1 - Exploration of Rich Tasks

Psychic Powers Test.

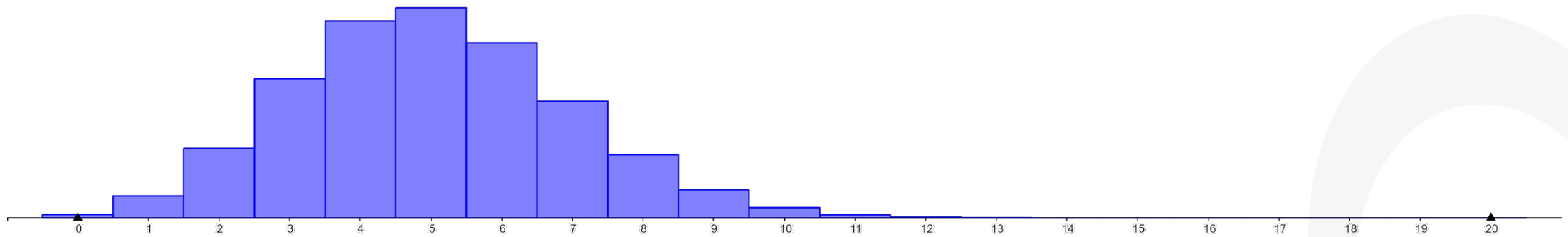
A student claims he has psychic powers. To test this claim another student will pick a card from a deck of 52 cards. This card will be noted and then replaced into the deck.

At the same time the person being tested will write down the suit of the card.

The test will be repeated 20 times



Not for presentation to All: Effective use of ICT to enhance the task



Prior knowledge

SP.1 Investigate the outcomes of experiments so that they can:

1. a. generate a sample space for an experiment in a systematic way, including tree diagrams for successive events and two-way tables for independent events

2. b. use the fundamental principle of counting to solve authentic problems

SP.2 Investigate random events so that they can:

1. a. demonstrate understanding that probability is a measure on a scale of 0-1 of how likely an event (including an everyday event) is to occur
2. b. use the principle that, in the case of equally likely outcomes, the probability of an event is given by the number of outcomes of interest divided by the total number of outcomes
3. c. use relative frequency as an estimate of the probability of an event, given experimental data, and recognise that increasing the number of times an experiment is repeated generally leads to progressively better estimates of its theoretical probability

Learning Outcomes (Syllabus content.)

LCOL Syllabus, LCHL Syllabus (in Bold)

1.1 Counting

- count the number of ways of selecting r objects from n distinct objects
- **count the number of ways of selecting r objects from n distinct objects**
- **compute binomial coefficients**

1.2 Concepts of probability

- solve problems involving sampling, with or without replacement

1.3 Outcomes of random processes

- find the probability that two independent events both occur
- apply an understanding of Bernoulli trials
- solve problems involving up to 3 Bernoulli trials
- calculate the probability that the k th success occurs on the n th Bernoulli trial where n is specified
- solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required)
- calculate the probability that the k th success occurs on the n th Bernoulli trial

1.7 Analysing, interpreting and drawing inferences from data

- recognise the concept of a hypothesis test

Formative Assessment Notes:

Open Questions (Rather than Closed):

Why/How/Could and Would

Formative Feedback:

John Hattie's priorities for effective teaching and learning

Transparent goals.

The more transparent the teacher makes the learning goals, then the more likely the student is to engage in the work needed to meet the goal.

Success Criteria.

The more the student is aware of the criteria of success, then the more the student can see the specific action that is needed to attain these criteria.

Rapid formative feedback.

The more there is feedback about progress from prior to desired outcomes the more positive attributes to learning are developed.

Learning Outcome, Learning Intention, Success Criteria.

Permutations
& Combination
LO 1.1

Break down to 3
experiments. Are any
outcomes repeated
Investigate

Sampling
LO 1.2

Break problem
down and repeat
without replacement

Independent not Independent
LO 1.3

Card Task

Bernoulli trials.
LO 1.3

Break Problem down
to 3/4 experiments
Draw tree diagram.

Inferential Stats
LO 1.7

Draw histogram.

Apply the Binomial theorem
LO 4.1

Reflection:

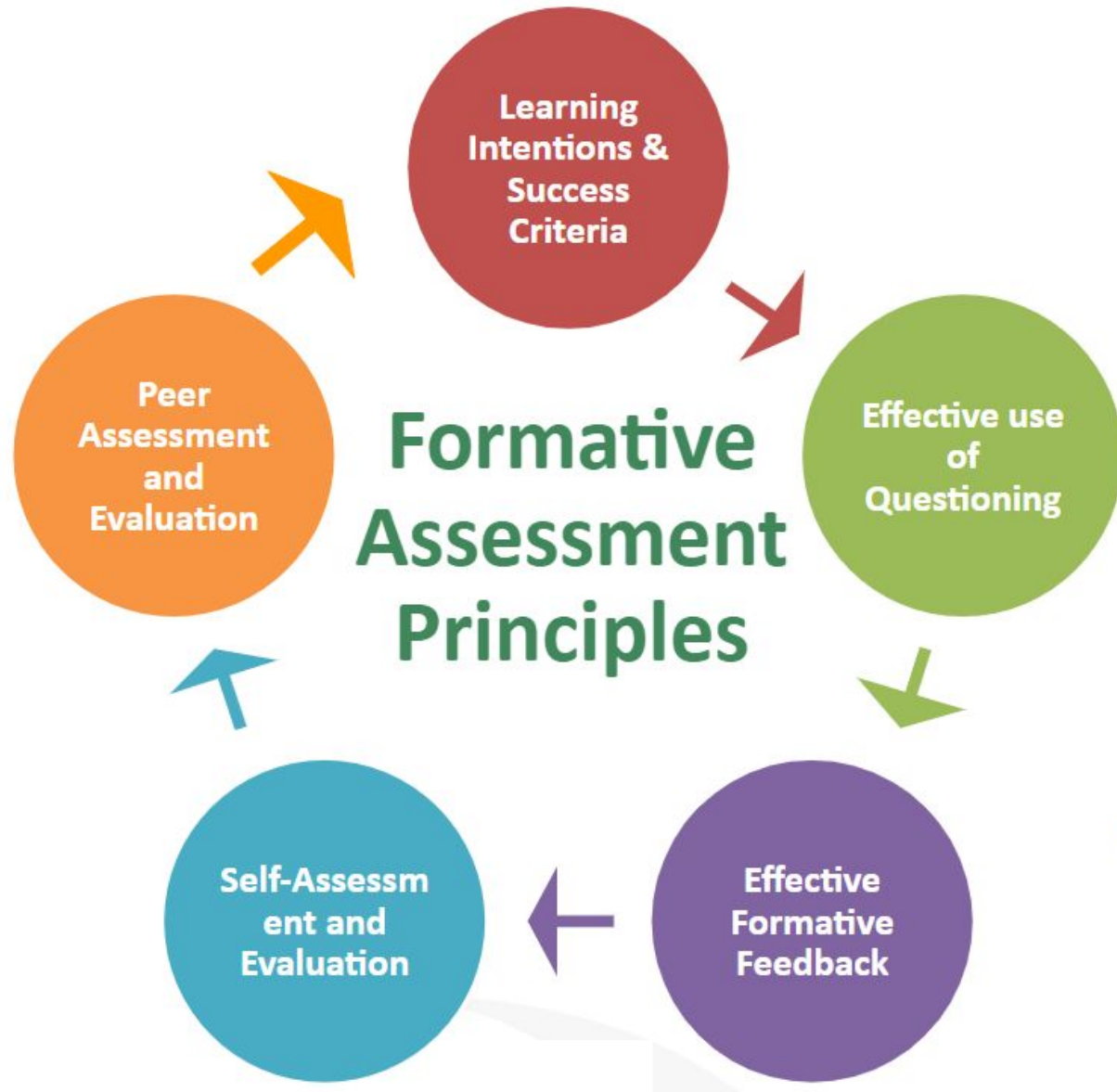
- Providing motivation and challenge to higher-achieving students while being accessible to all the students in the class
- promoting the development of key skills (communication, working with others, critical and creative thinking, information processing) and mathematical competency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition)
- reinforcing mathematical learning previously seen by students



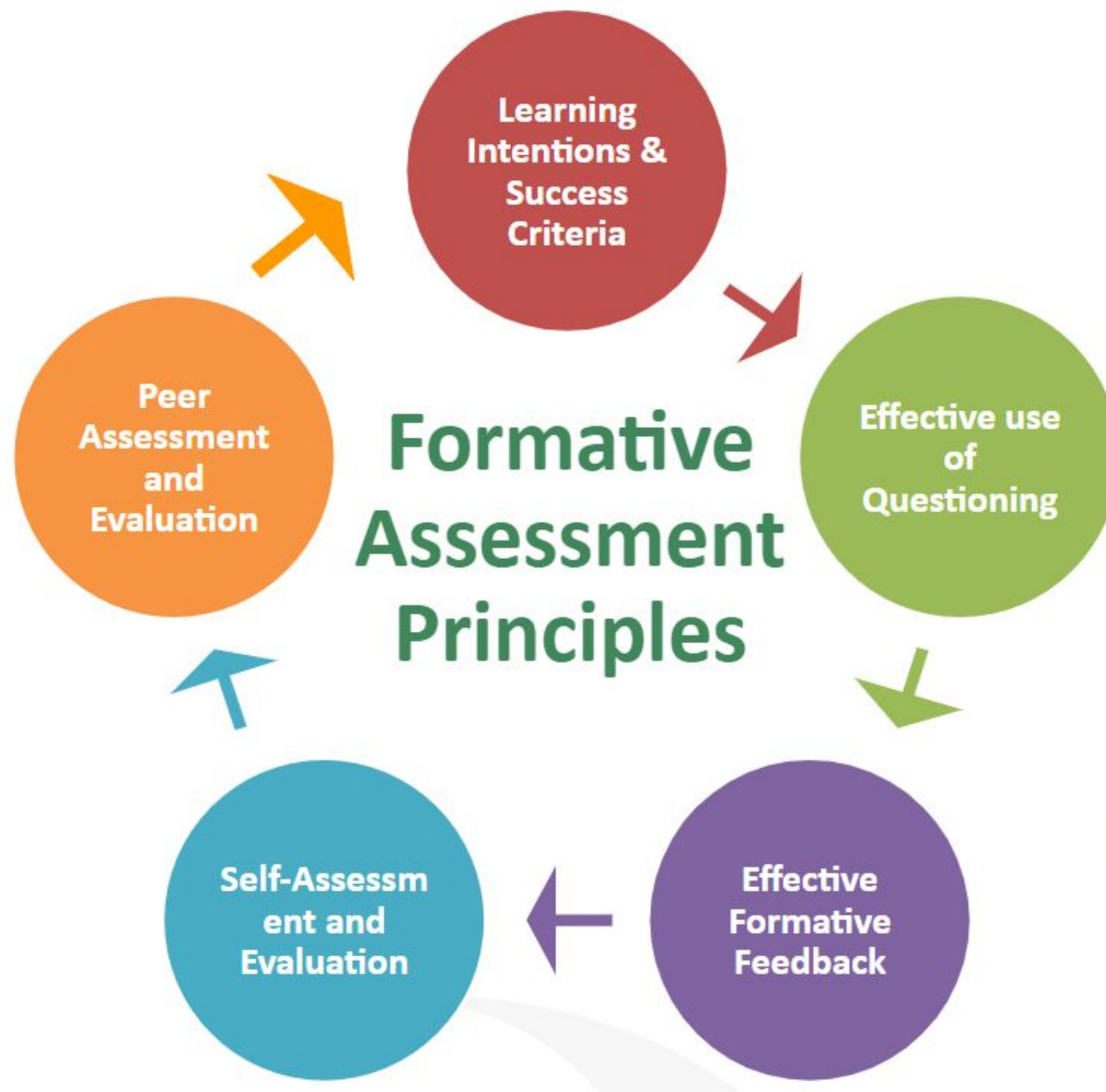


Formative Assessment

Formative Assessment



Adapted from William & Leahy, (2015) and Wylie et al (2008)



Adapted from William & Leahy, (2015) and Wylie et al (2008)

Formative Assessment



What is the purpose of asking questions?

Assessment

Assessment is the process of generating, gathering, recording, interpreting, using and reporting evidence of learning in individuals, groups or systems. Educational assessment provides information about progress in learning, and achievement in developing skills, knowledge, behaviours and attitudes (NCCA, 2015)

Dylan Wiliam discusses learning intentions



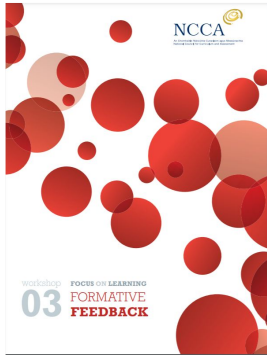
Formative Feedback statements



Activity 1
The sorting sheet

Yes	No

Exercise adapted from the NCCA
Focus on Learning Formative
Feedback (NCCA, 2015)



Activity 2
Classification worksheet

Reminder prompt	Scaffold prompt	Example prompt

01 Tell us more. What is the slope? What does this mean?

02 Gold star

03 Explain why you think this

04 How can we tell where a point lies in relation to a circle. Is the distance from the centre to the point shorter than the radius? Longer than the radius? The same as the radius? What does this mean?

05 Beautiful, neat work

06 Well done!!

07 You have given one root of the equation. Are there any others?

08 Develop these ideas further

09 What type of graph would be most suitable to present these data?

10 Good, but not as good as your brother's!

11 How do you know...?

12 You must try harder

13 First place in the class

14 Try one of these or one of your own instead of a bar chart - stem and leaf chart, histogram

15 Lovely diagram

16 You're the best

17 10/10

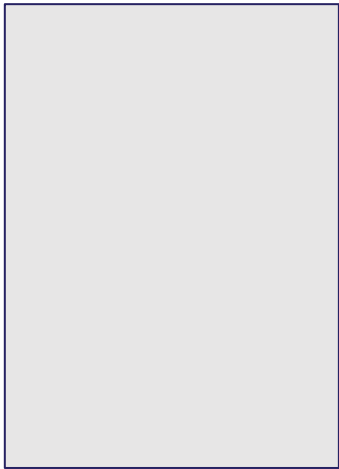
18 The slope being zero at this point means... (finish this sentence)

19 You gave very good reasons why the triangles are congruent. To improve your work, you need to include all of the reasons. Look back and check to see which ones you have left out.

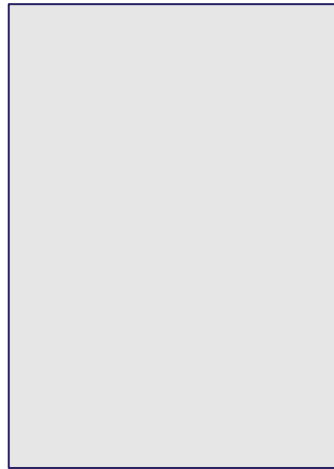
20 What justification can you give for this being a right-angled triangle?

Applying Formative Assessment to Student Work

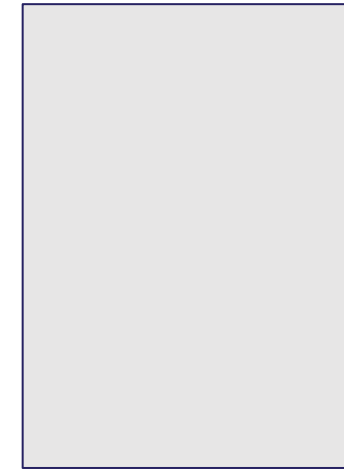
Student A



Student B



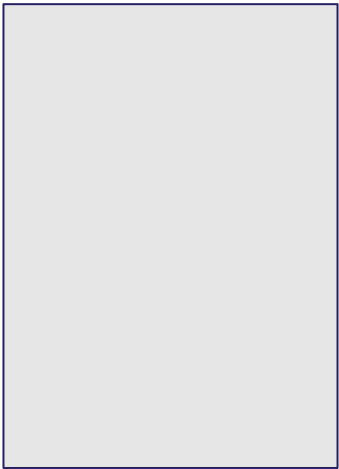
Student C



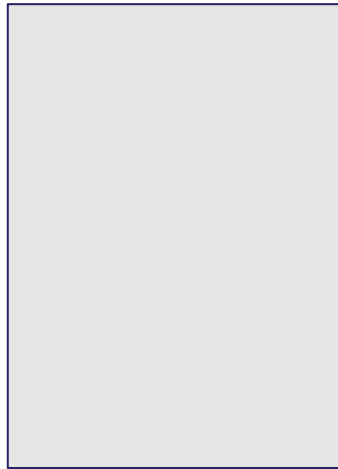


Reflection on Application of Formative Feedback

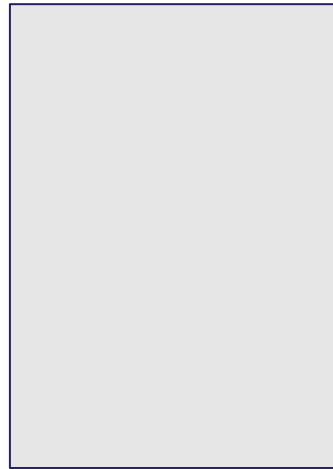
Student A



Student B



Student C

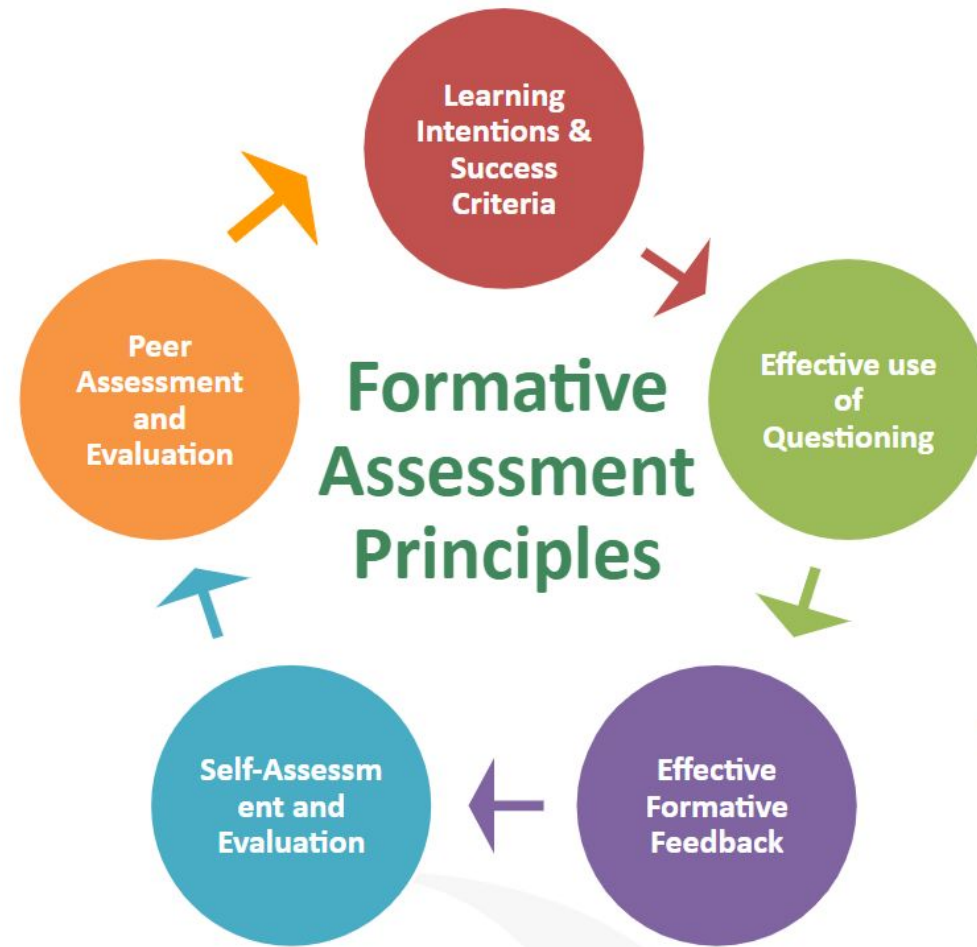


How will you implement effective formative feedback in your maths classes?



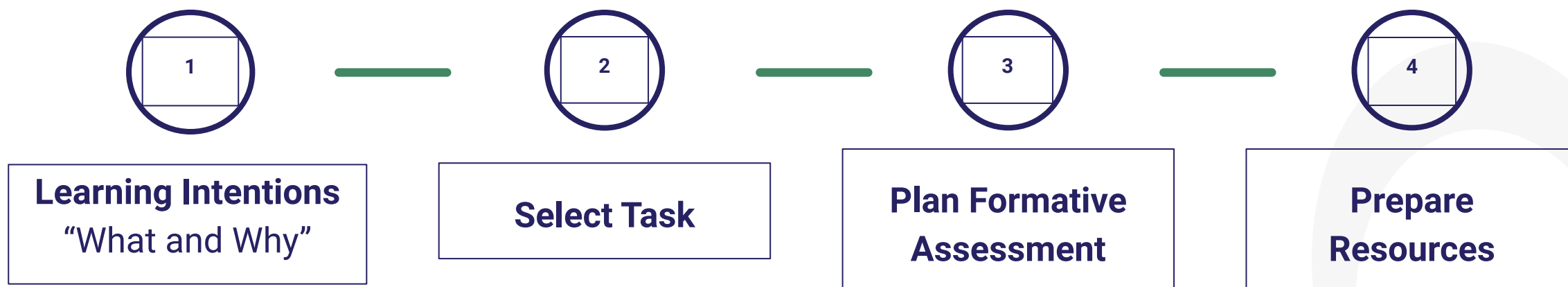
Reflection on Formative Feedback

How will you implement effective formative feedback in your maths classes?



Adapted from William & Leahy, (2015) and Wylie et al (2008)

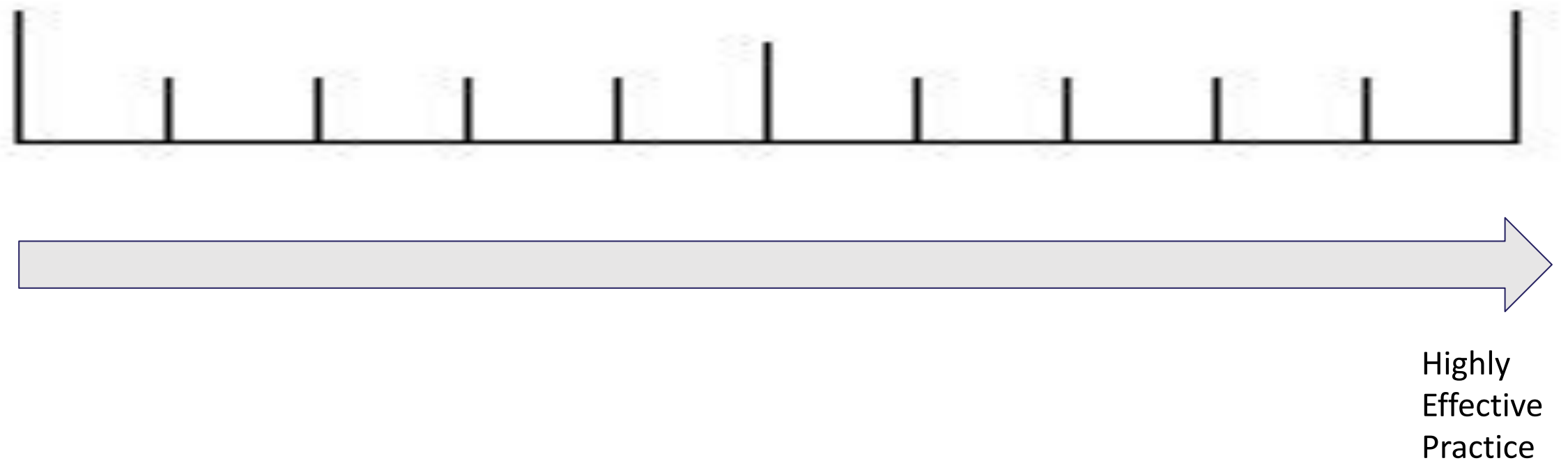
Rich Task Design





Criteria For Good Practice

What makes for good practice?



What makes for good practice?

A) I give my best students different work to do so that they are all challenged.

B) Students set questions for each other to challenge each other.

C) Students regularly present their work and ideas to the class

D) Anyone who finishes early gets an extra worksheet so they have more to do.

E) Students tackle open ended tasks that have no defined correct answer or method. They can use the maths that they know to present solutions.

F) Students take part in Mathsweek activities.

G) Students get to pick their own questions from the textbook. They can attempt the questions appropriate to their level.

H) We have a problem solving class every Friday. Students get to choose their problems from a box of problems.

I) Our best students compete in the IMTA maths competition.

J) Students are encouraged to come up with multiple ways to solve the same problem so they are always busy.

K) The better students in my class help those who need it.

L) Students correct each other's work.

Activities in Your School and Further Programmes and Resources

Here are some external programmes and competitions. Considering what we've just discussed, how can they be used to best support your classroom learning?

- Maths olympiad & local 3rd level programmes
- IMTA and IAMTA competitions
- Mathsweek
- Maths eyes competition
- CSO-John Hooper competition
- Problem solving websites: nrich, Uni. of Waterloo,



Reflection & Close

Learning Outcomes

We have 3 Learning Outcomes on the next slide but we would like you to think about:

- what **you** have gained from this evening
- how **you** will bring the learning from this workshop back to your school

Learning Outcomes

By the end of this seminar, participants will:

- Recognise their responsibility to challenge and stretch all students in the Mathematics classroom.
- Have a clear understanding of what is meant by a rich task and how they can be used to progress all students' learning.
- Understand how effective use of formative assessment can give all students a sense of ownership and motivation and direction to progress their learning.

WS Resources on Padlet

The screenshot shows a Padlet board titled "Challenging All Students' Mathematical Thinking" by Michael Walsh. The board is organized into three columns: "Workshop Material", "Workshop Resources", and "Key Documents".

- Workshop Resources Column:**
 - NCCA Formative Feedback PDF document
 - NCCA Effective Questioning PDF document
- Key Documents Column:**
 - looking-at-our-school-2016 PDF document
 - SC_Maths_syllabus_2015 PDF document
 - Digital Learning Framework for Post-Primary PDF document
 - PISA in Classrooms: (partially visible)

The right side of the board features a large PDST logo and the text: "Professional Development Service for Teachers" and "An tSeirbhís um Fhorbairt Ghairmiúil do Mhúinteoirí".

Evaluation





PDST Post Primary Maths Team

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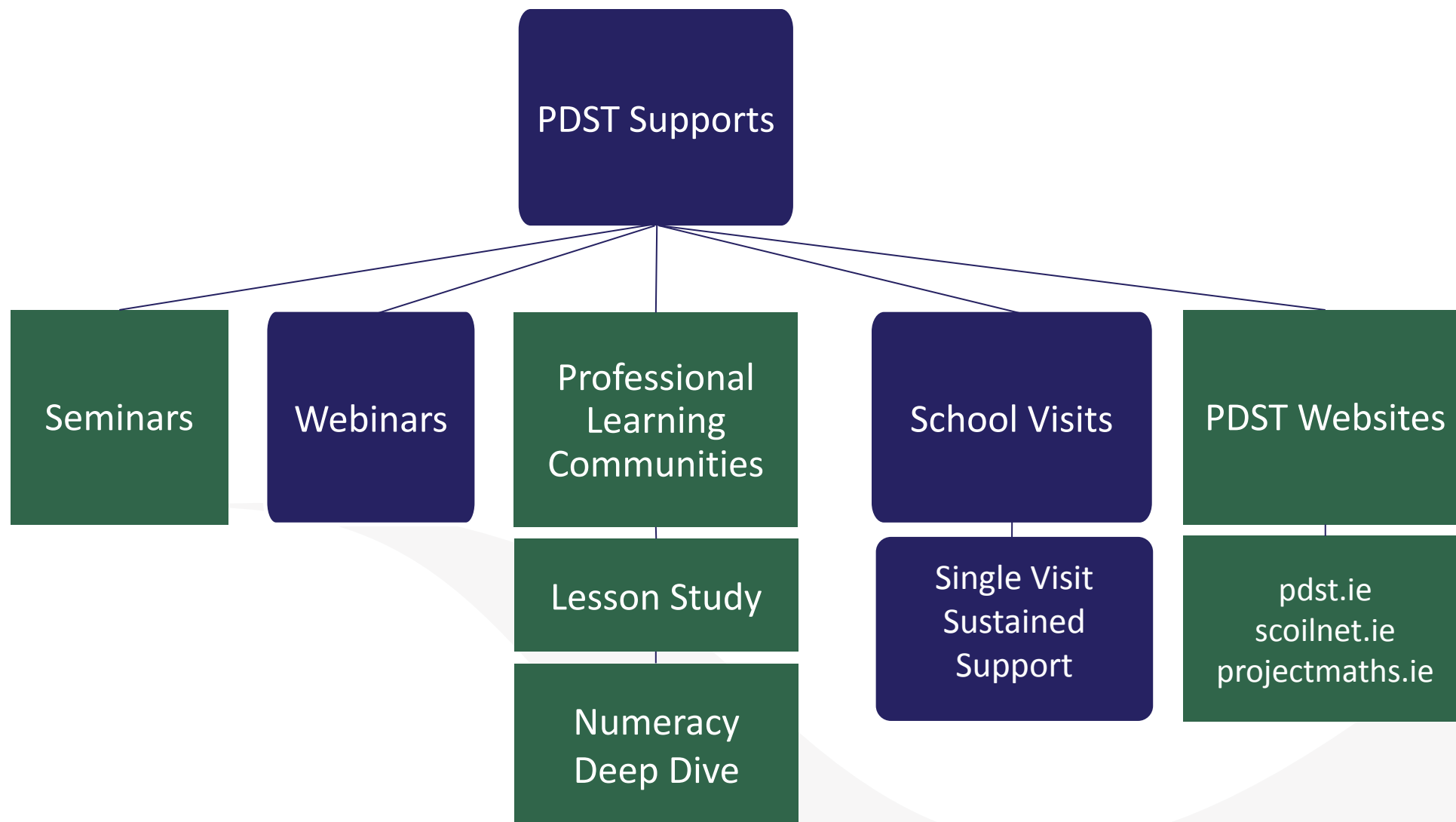
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Darren Murphy
Enda Donnelly
Warren McIntyre
Michael Walsh



@ProjectMaths_ie

PDST Supports



School Support

Book a school visit:
<https://www.projectmaths.ie>

Contact us:
postprimarymaths@pdst.ie

Reading List

Florian, L. and Spratt, J. (2013) Enacting Inclusion: A Framework for Interrogating Inclusive Practice. *European Journal of Special Needs Education*, 28, 119-135.

<https://doi.org/10.1080/08856257.2013.778111>

Perkins, R. and Shiel, G. (2016) PISA in Classrooms: Implications for the teaching and learning of mathematics in Ireland

Perkins, R., Clerkin, A and Chubb, E. (2020) Students's perspectives on learning mathematics and science: Results from TIMSS 2015 in Ireland

Boaler J. (2017) Positive Norms

<https://www.youcubed.org/wp-content/uploads/2017/08/2017-Positive-Norms-DeFacilitation Notesion-Paper-1.pdf>

O'Meara, N., Prendergast, M. & Treacy, P. (2020). What's the point? Impact of Ireland's bonus points initiative on student profile in mathematics classrooms. *Issues in Educational Research*, 30(4), 1418-1441.

<http://www.iier.org.au/iier30/omeara.pdf>

Dylan Wiliam, LSI: Learning Sciences International, Strategy 1: Clarifying, Sharing, and Understanding Learning Intentions,

<https://youtu.be/fC29lyqPVr0>, retrieved 9 March 2021

Vygotsky on Zone of Proximal Development, <https://www.simplypsychology.org/Zone-of-Proximal-Development.html>

NCCA, Focus on Learning Formative Feedback, 2015

Hattie, J., & Clarke, S. (2019). *Visible learning: Feedback*.

Boaler, J. Wiliam, D. & Brown, M. (2000). Students' experiences of ability grouping: Disaffection, polarisation and the construction of failure. *British Educational Research Journal*, 26(5), 631-649.