Synthetic Geometry

Guide to Axioms and Theorems for all Levels

	Axioms and Theorems (supported by 46 definitions, 20 propositions) JC students must display understanding of the proofs of the relevant theorems. ** Formal proof required for LCHL only. ♦ These results are required as background knowledge for constructions/ applications of trigonometry.	JC OL	JC HL	LC FL	LC OL	LC HL
	Axiom 1: There is exactly one line through any two given points	✓	✓	✓♦	✓	✓
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	✓	✓	✓♦	✓	✓
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	✓	✓	✓♦	✓	✓
1	Vertically opposite angles are equal in measure.	✓	✓		✓	✓
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	✓	✓		✓	✓
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	✓	✓	✓♦	✓	✓
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	✓	✓	✓♦	✓	✓
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely**,** if two lines are parallel, then any transversal will make equal alternate angles with them.	✓	✓		✓	✓
4	The angles in any triangle add to 180^0.	✓	✓	✓♦	✓	✓
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	✓	✓		✓	✓
6	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	✓	✓		✓	✓
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.				✓	✓
8	Two sides of a triangle are together greater than the third.				✓	✓
9	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.	✓	✓		✓	✓
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.		✓			✓
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.	✓	✓		✓	✓

		JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.		✓		✓	✓
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if a line l cuts the sides AB and AC in the same ratio, then the line l is parallel to BC.		✓		✓	✓
13**	If two triangles are similar, then their sides are proportional, in order Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.	✓	✓		✓	✓
14	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.	✓	✓	✓♦	✓	✓
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.	✓	✓		✓	✓
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.	✓	✓		✓	✓
16	For a triangle, base x height does not depend on the choice of base.				✓	✓
	Definition 38: The area of a triangle is half the base by the height.				✓	✓
17	A diagonal of a parallelogram bisects the area.				✓	✓
18	The area of a parallelogram is the base x height.				✓	✓
19	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.		✓			✓
	Corollary 2: All angles at points of a circle, standing on the same arc are equal (and converse).		✓			✓
	Corollary 3: Each angle in a semi-circle is a right angle.	✓	✓		✓	✓
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.	✓	✓		✓	✓
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180^0(and converse).		✓			✓
20	(i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.				✓	✓
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.				✓	✓
21	(i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.				✓	✓