



Algebra through the Lens of Functions

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Workshop Registration



<https://tinyurl.com/algebra22registration>

Overview

Workshop 1

Introduction

Exploring quadratic functions

Investigating transformations

Using manipulatives to support sense-making

Conclusion

Q&A

Introduction

Resources

<https://padlet.com/postprimarymaths/algebra2022>



Key Messages



By engaging deeply with functions at Junior Cycle, students acquire the conceptual understanding and skills to understand functions at Senior Cycle.



Multiple representations are important for sense making, for developing deep conceptual understanding and for developing adaptive thinking.

Intended Learning

Participants will understand that allowing students to engage with rich, open tasks will

- encourage them to become more confident in their mathematical ability
- develop their critical thinking skills
- provide them with the tools needed to attempt unseen problems.

Reflection

What challenges for teachers and barriers to students' learning are presented by the teaching of algebra and functions?



Sense - Making

Many students

- don't understand equality
- don't see the sense in Mathematical expressions
- don't recognise structure in expressions
- see relationships in different ways



Chief Examiners Report JC 2015

“ $2x$ was often confused with x^2 ” (JC HL)

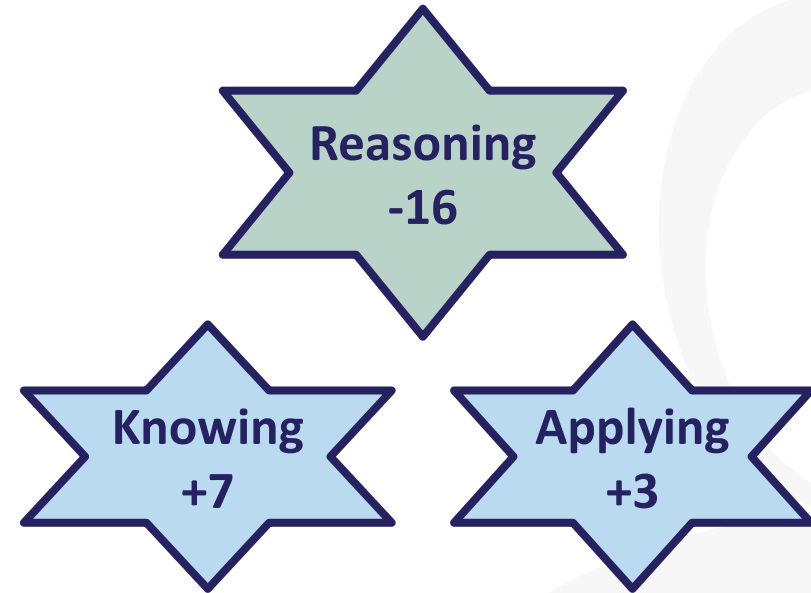
“Those who did use algebraic manipulation often displayed a general idea of what to do, but were unable to carry out the procedures accurately” (JC OL)

“candidates appeared to have difficulty understanding the significance of m and c when a line is written in the form $y = mx + c$.” (JC HL & OL)

International Research



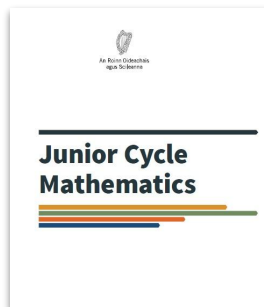
Number	+17
Data & Chance	+17
Algebra	-18
Geometry	-18



Junior Cycle Specification

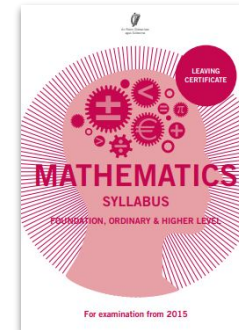
*“Learners **explore** and **analyse** the **relationships** between tables, diagrams, graphs, words, and algebraic expressions as representations of functions”*

-Page 11 Junior Certificate Specification

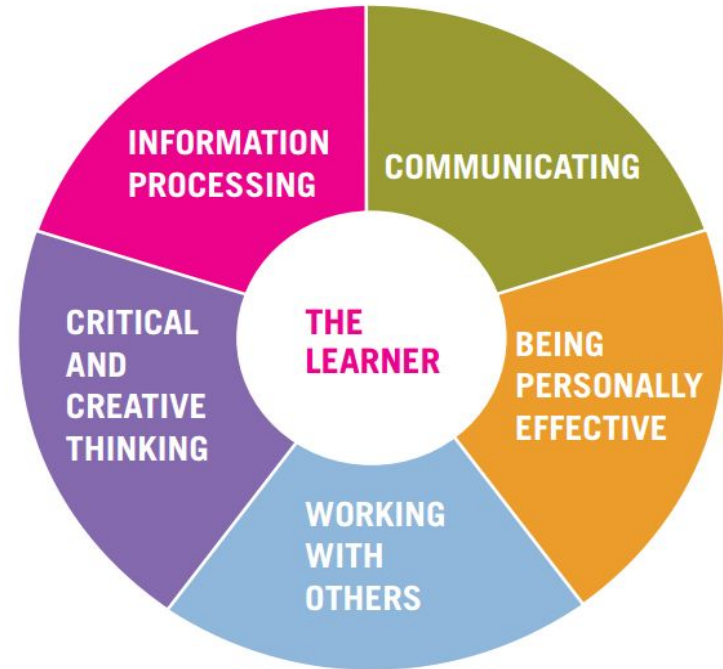


*Learners **build on** their proficiency in moving among equations, tables and graphs and become more adept at solving **real-world** problems”.*

-Page 35 Leaving Certificate Syllabus



Development of Key Skills



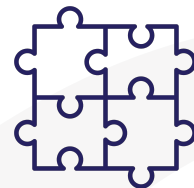
Constructivist Pedagogy

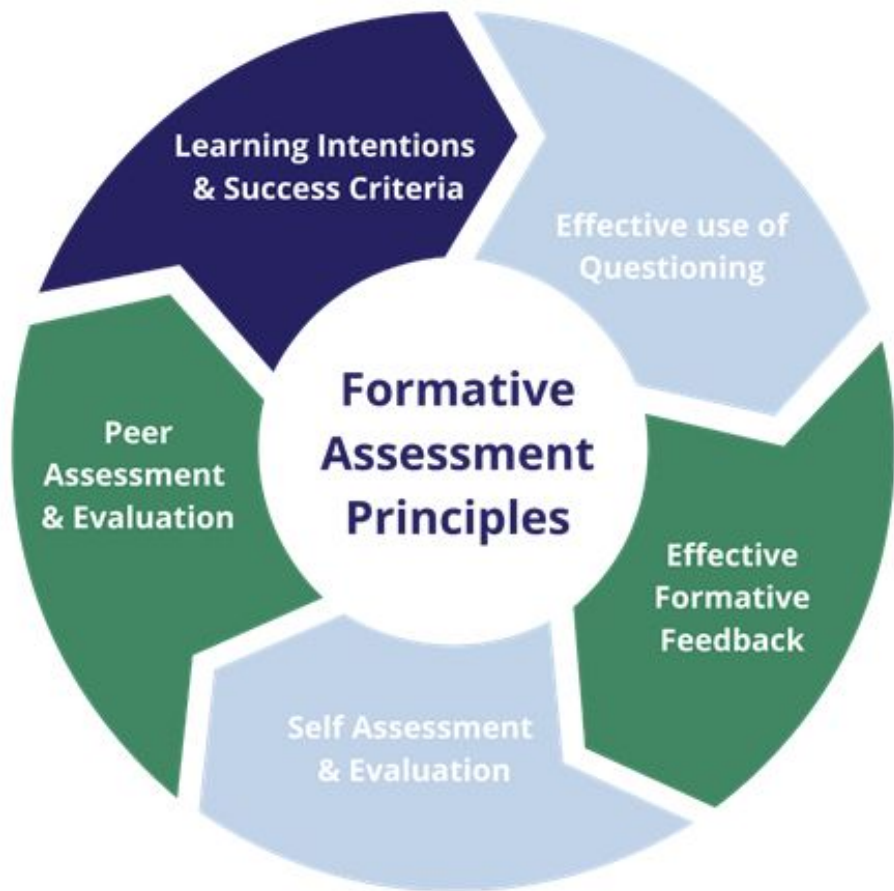
Principles of Constructivism

- The learner constructs his or her own knowledge and understanding.
- Learning does not occur when the learner passively receives information.
- Learners are the makers of meaning and knowledge, not simply the receivers.

Methodologies

- Effective Questioning
- Group Work
- Student Reflection





Problem Solving



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2017

Mathematics

Paper 1
Higher Level

Question 9

(40 marks)

The depth of water, in metres, at a certain point in a harbour varies with the tide and can be modelled by a function of the form

$$f(t) = a + b \cos ct$$

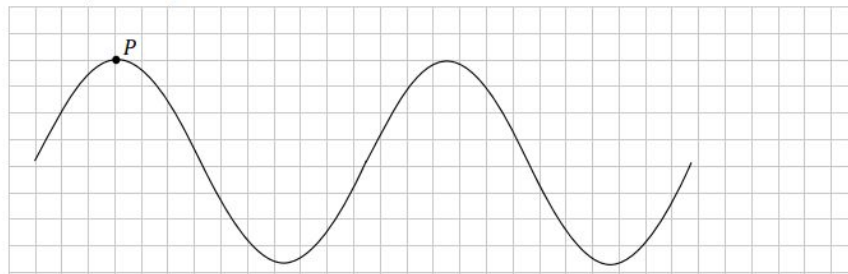
where t is the time in hours from the first high tide on a particular Saturday and a , b , and c are constants. (**Note:** ct is expressed in radians.)

On that Saturday, the following were noted:

- The depth of the water in the harbour at high tide was 5.5 m
- The depth of the water in the harbour at low tide was 1.7 m
- High tide occurred at 02:00 and again at 14:34.

- (a) Use the information you are given to add, as accurately as you can, labelled and scaled axes to the diagram below to show the graph of f over a portion of that Saturday.

The point P should represent the depth of the water in the harbour at high tide on that Saturday morning.



Introducing Quadratic Functions

Prior Content Knowledge

Exploration of linear patterns:

- Expressions
- Equivalence
- Distribution
- Multiple Representations

Creating and interpreting graphs of linear functions:

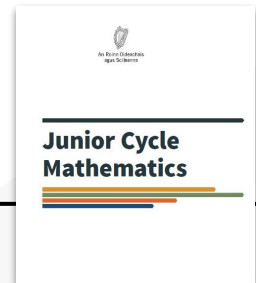
- Critical points of a linear function
- Understanding of a variable and constant
- Rate of change of a linear function

Introduction of Quadratic Functions

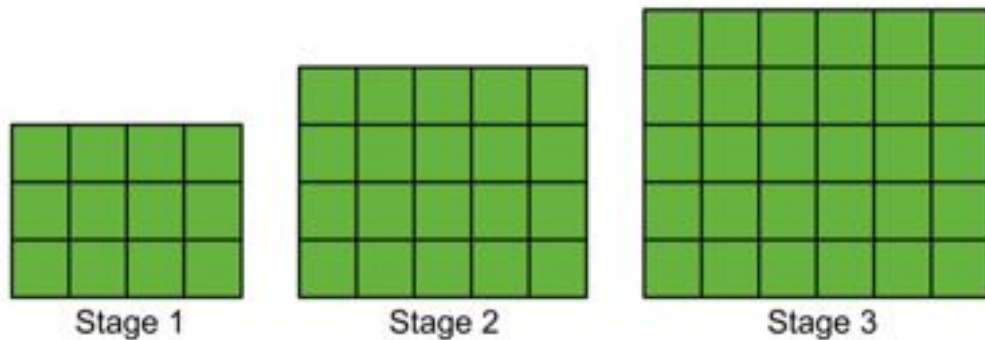
AF1: Students should **investigate** patterns and relationships:

- a. represent these patterns and relationships in tables and graphs
- b. **generate** a generalised expression in words and algebraic expressions and fluently convert between each representation
- c. **categorise patterns** as linear, non-linear, quadratic, and exponential using their defining characteristics

Junior Certificate Specification page 18

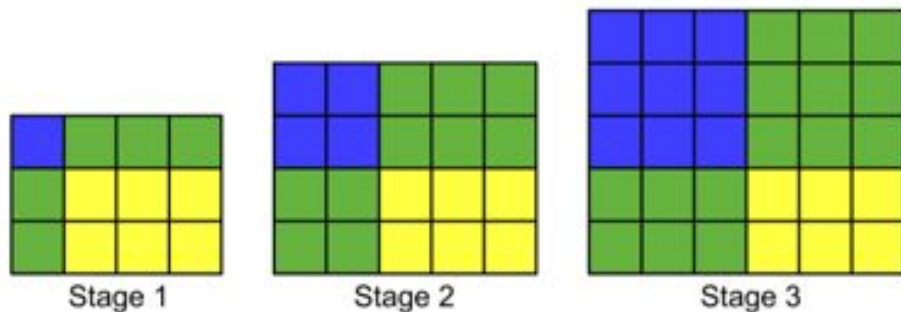


Task



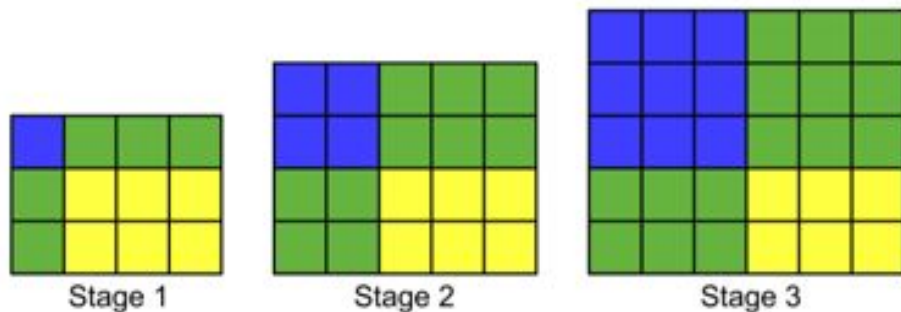
How many blocks in the 50th stage?

Task



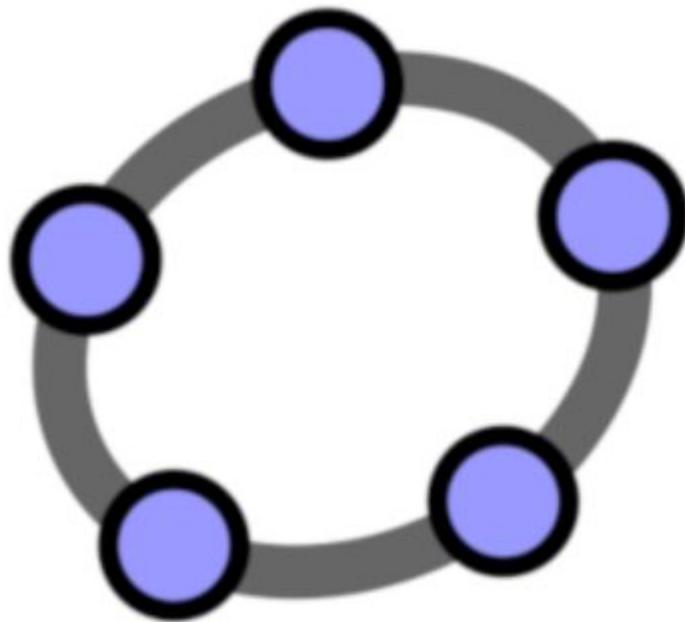
How many blocks in the 50th stage?

Task - Solutions

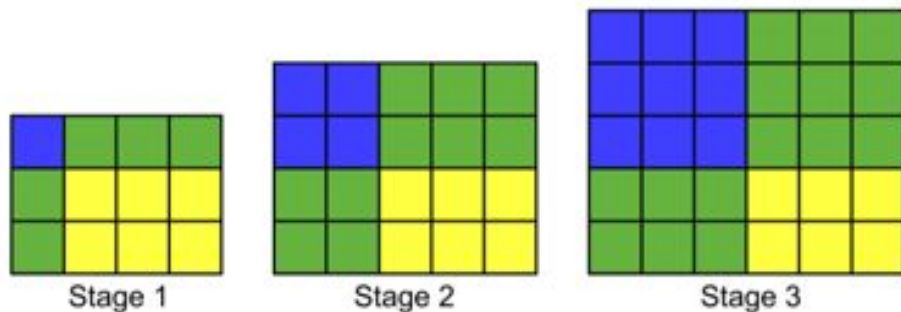


How many blocks in the 50th stage?

Multiple Representations


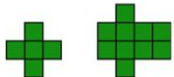
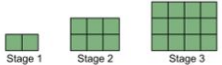

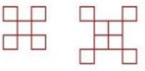
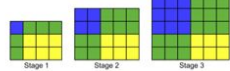

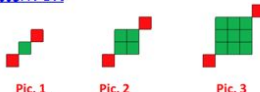
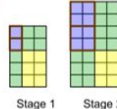

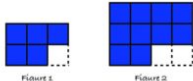
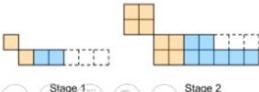
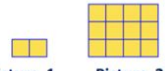
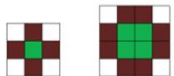


Task - Extensions



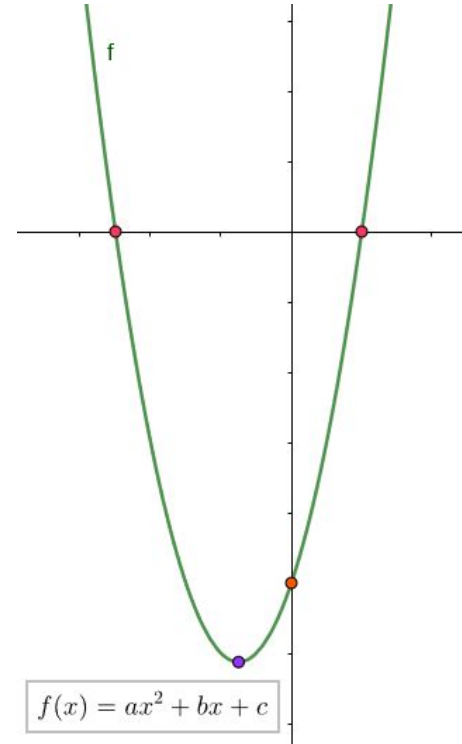
Which stage has 90 squares?

Consolidating the learning

Growing Visual Quadratic Patterns		
<p>Supporting the Student Workbook on Quadratic Patterns available on our website</p> <p>Click on the pattern to go to that page.</p>	<p>Pattern 5</p>  <p>Fig. 1 Fig. 2 Fig. 3</p>	<p>Pattern 10</p>  <p>Figure 1 Figure 2</p>
<p>Pattern 1</p>  <p>Stage 1 Stage 2 Stage 3</p>	<p>Pattern 6</p>  <p>Fig. 1 Fig. 2</p>	<p>Pattern 11</p>  <p>Step 1 Step 2</p>
<p>Pattern 2</p>  <p>Stage 1 Stage 2 Stage 3</p>	<p>Pattern 7</p>  <p>Pattern 1 Pattern 2</p>	<p>Pattern 12</p>  <p>Pic. 1 Pic. 2 Pic. 3</p>
<p>Pattern 3</p>  <p>Stage 1 Stage 2</p>	<p>Pattern 8</p>  <p>Picture 1 Picture 2</p>	<p>Pattern 13</p>  <p>Figure 1 Figure 2</p>
<p>Pattern 4</p>  <p>Stage 1 Stage 2</p>	<p>Pattern 9</p>  <p>Picture 1 Picture 2</p>	<p>Pattern 14</p> 

Next Steps: Solving Quadratic Equations

- Key Features of Quadratic Functions
- Graphical Solutions to Algebraic Inequalities
- Analysing the Table of Values of Quadratic Functions in Greater Depth



Summary

1. Students are introduced to new algebraic procedure through the exploration of patterns.
2. Students see the need for new strategies.
3. Students construct the learning.
4. Students make sense of their learning.
5. Students maintain a positive disposition to their Mathematics.



Key Features of functions

1. The domain and range.
2. Where the graph of the function meets the axes.
3. Things that remain constant and those that vary in the function.
4. The behaviour of the graph of the function.
5. The rate of change of the function.

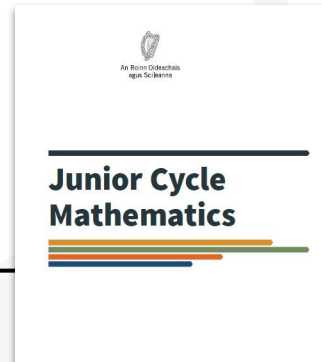
Transformations of Quadratic Functions

Transformation of Quadratic functions

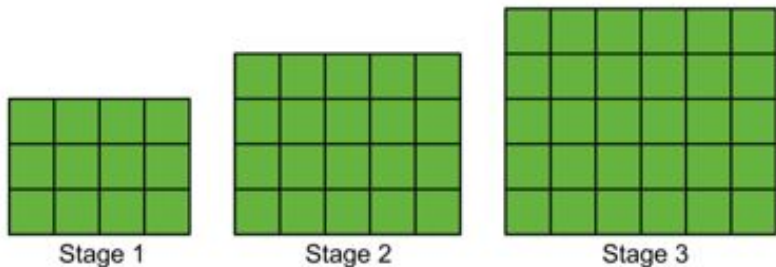
AF7: Students should *investigate* functions so that they can:

- a. demonstrate understanding of the concept of a function
- b. represent and interpret functions in different ways—**graphically, diagrammatically, in words, and algebraically**

Junior Certificate Specification page 17



Transformations



Student expressions:

$$b = s^2 + 5s + 6$$

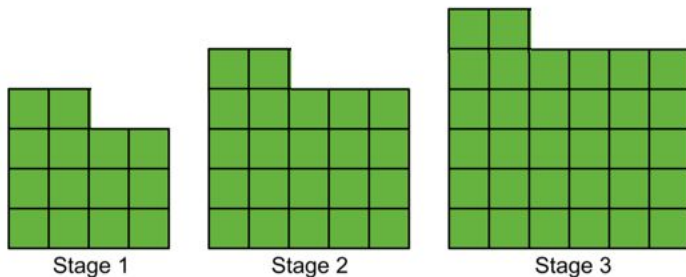
$$b = (s+2)(s+3)$$

What would happen if we:

- *add 2 blocks to each stage?*
- *double the amount of blocks in the pattern?*
- *add 2 to our stage number?*

Transformations

Adding a constant



Student original expressions:

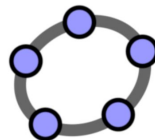
$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

What would happen if we added 2 blocks to each stage?

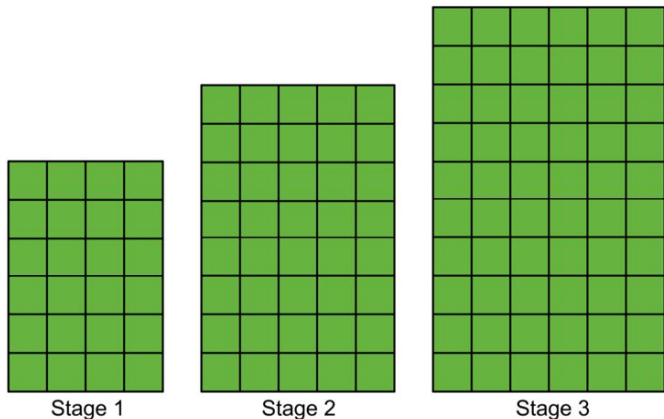
$$b = s^2 + 5s + 6 + 2$$

$$b = s^2 + 5s + 8$$



Transformations

Multiplying by a constant



Student original expressions:

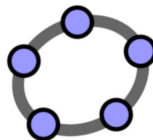
$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

What would happen if we doubled the amount of blocks in the pattern?

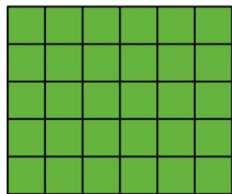
$$b = 2(s+2)(s+3)$$

$$b = 2(s^2 + 5s + 6)$$

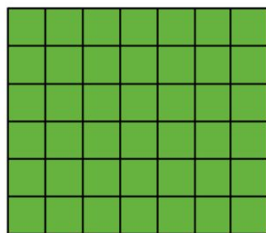


Transformations

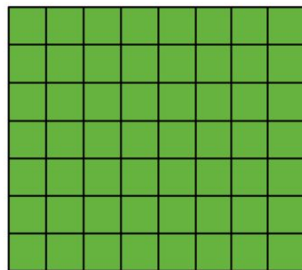
Changing the input (stage number)



Stage 1



Stage 2



Stage 3

Student original expressions:

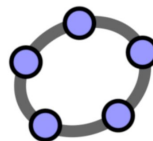
$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

What would happen if we add 2 to our stage number?

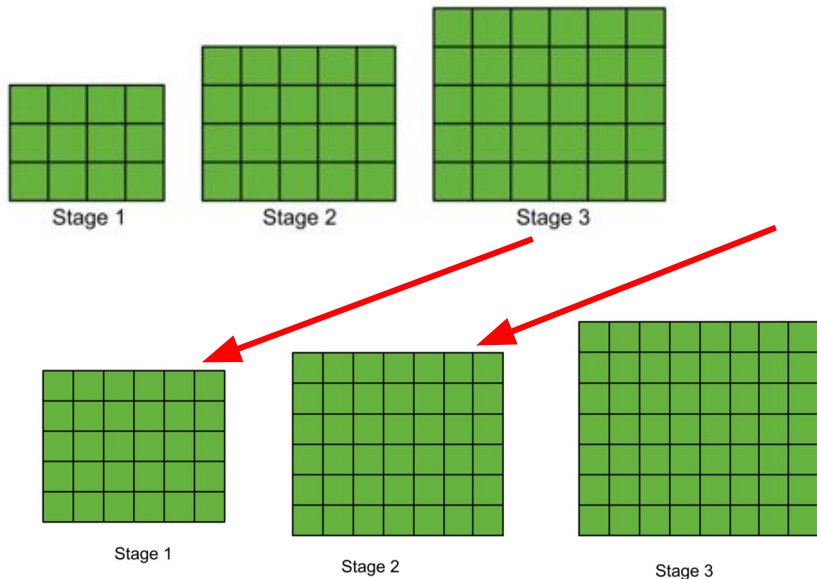
$$b = (s+4)(s+5)$$

$$b = (s+2)^2 + 5(s+2) + 6$$



Transformations

Changing the input (stage number)



Student original expressions:

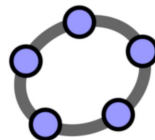
$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

What would happen if we start a new pattern at stage 3?

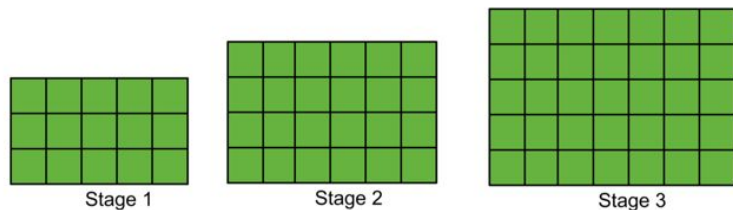
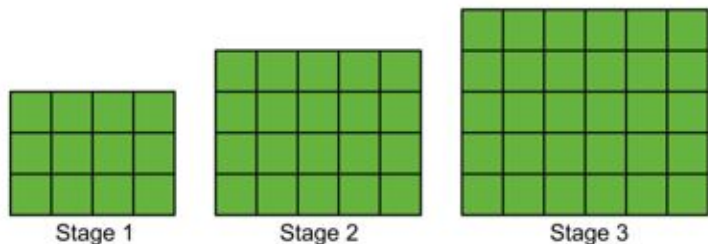
$$b = (s+4)(s+5)$$

$$b = (s+2)^2 + 5(s+2) + 6$$



Extension

Add a column



Student's original expressions:

$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

What would happen if we added an additional column?

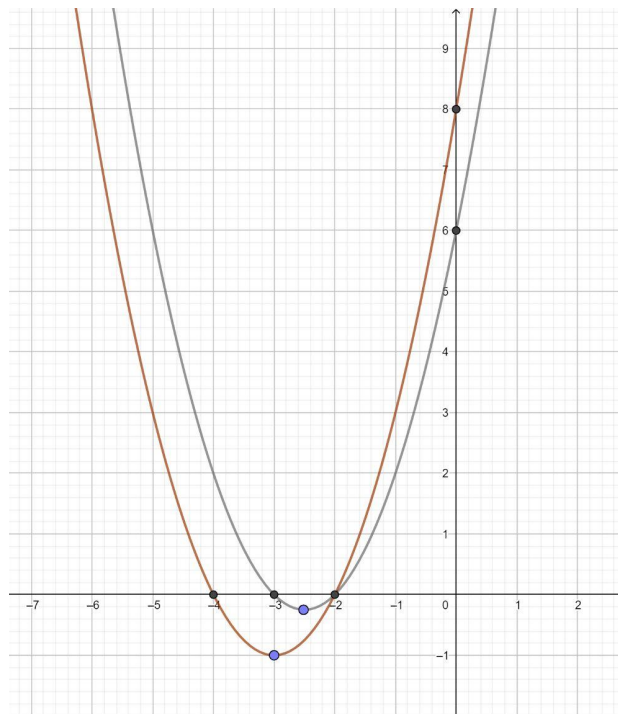
$$b = (s+2)(s+3+1)$$

$$b = (s+2)(s+4)$$

$$b = s^2 + 6s + 8$$

Extension

Add a column



Student's original expressions:

$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

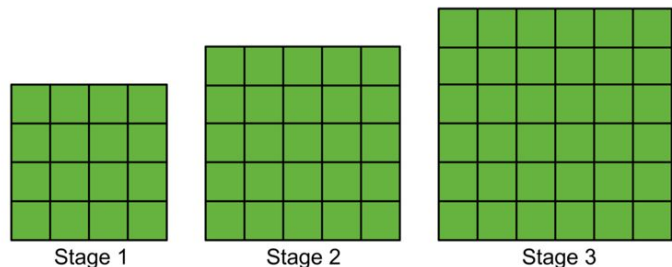
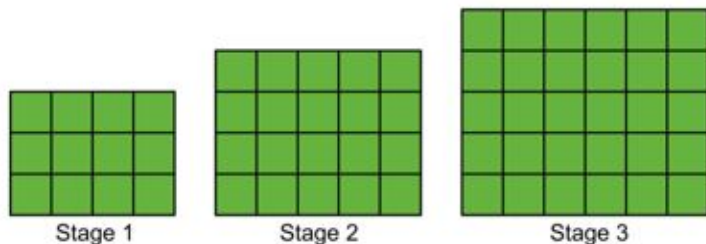
Student's new expressions:

$$b = s^2 + 6s + 8$$

$$b = (s+2)(s+4)$$

Extension

Add a row



Student's original expressions:

$$b = s^2 + 5s + 6$$

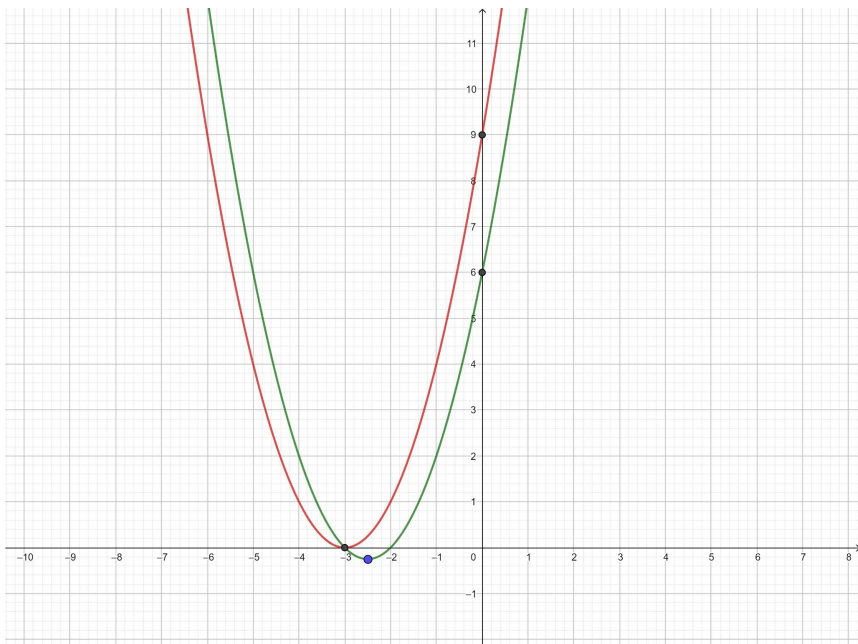
$$b = (s+2)(s+3)$$

What would happen if we add a row?

$$b = (s+3)(s+3)$$

$$b = (s+3)^2$$

Extension Add a row



Student's original expressions:

$$b = s^2 + 5s + 6$$

$$b = (s+2)(s+3)$$

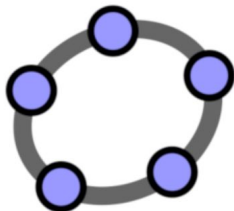
Student's new expressions:

$$b = s^2 + 6s + 9$$

$$b = (s+3)^2$$

Transformation of continuous functions

<https://www.geogebra.org/classic/egkh5rtw>

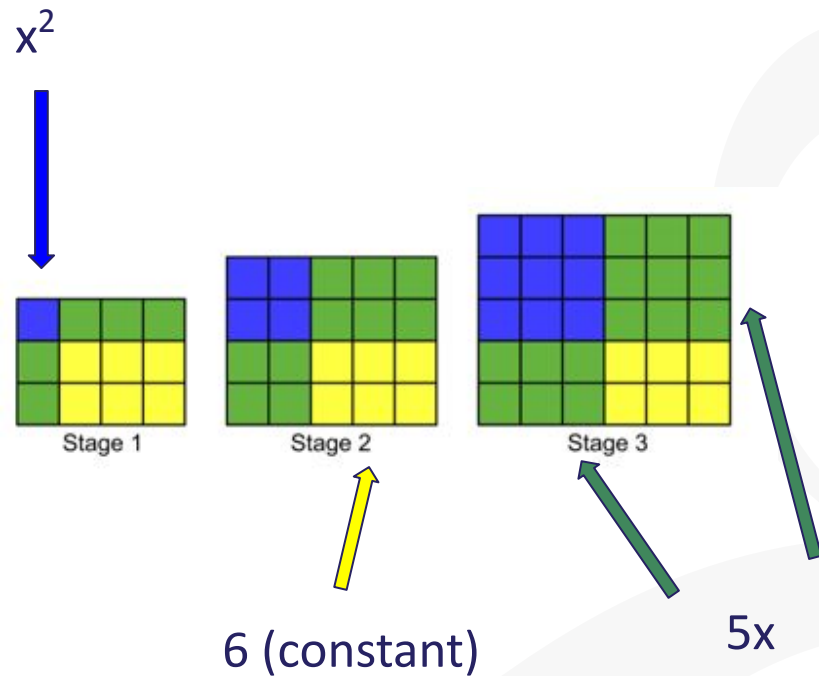
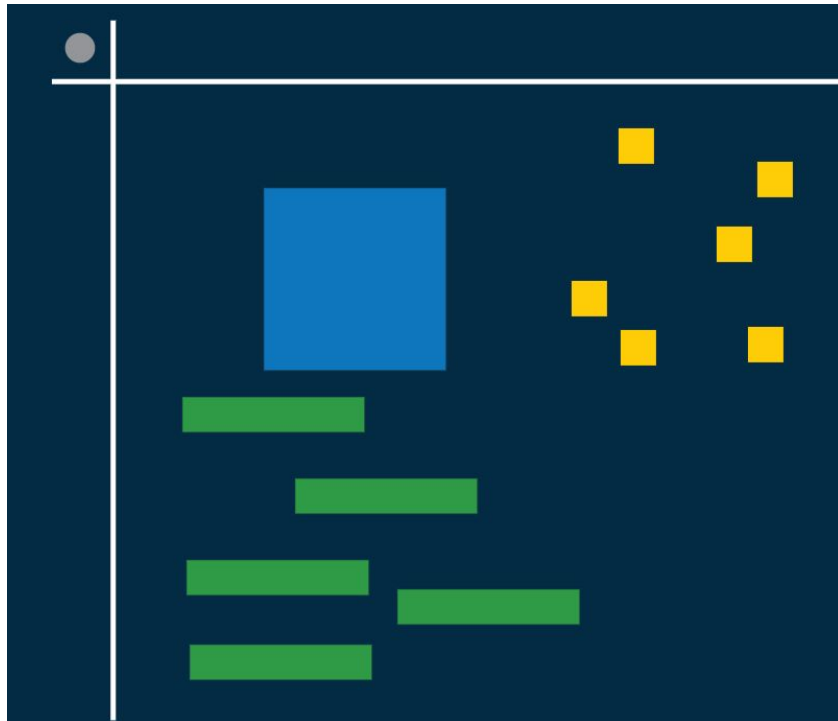


Key Features of functions

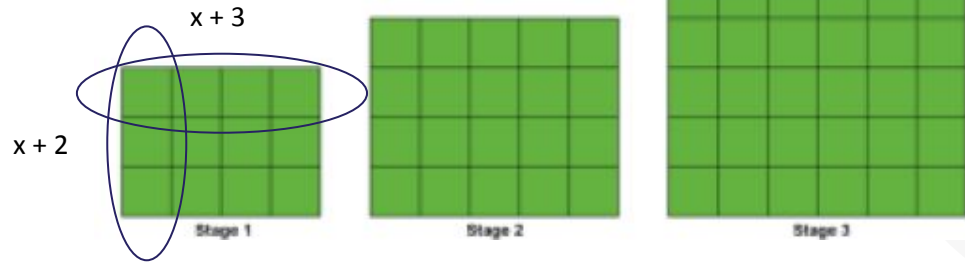
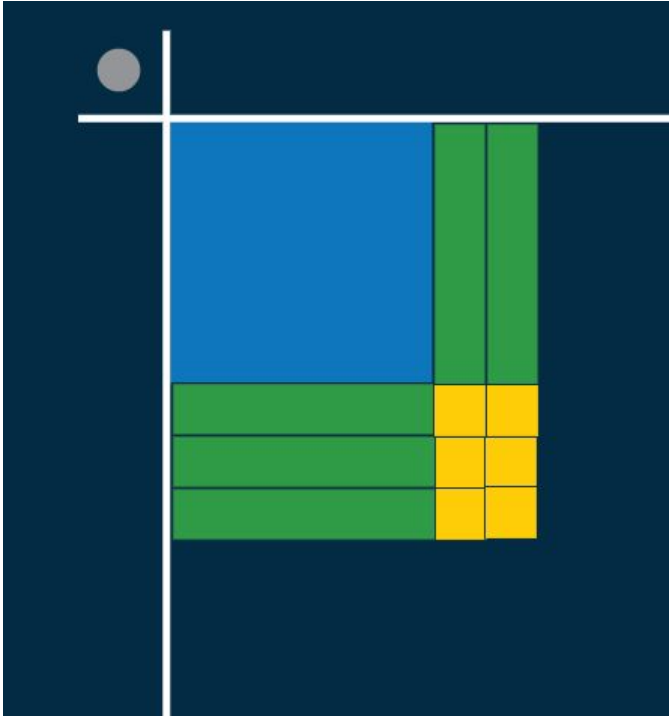
1. The domain and range.
2. Where the graph of the function meets the axes.
3. Things that remain constant and those that vary in the function.
4. The behaviour of the graph of the function.
5. The rate of change of the function.

Factorising Quadratics

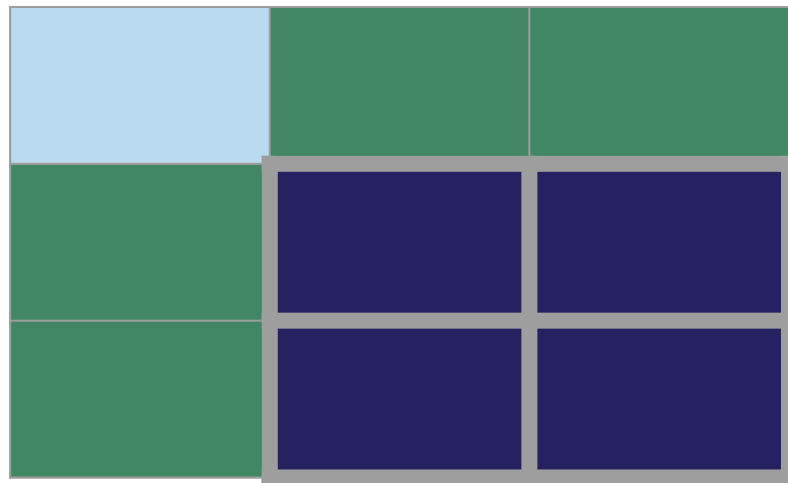
Factorising s^2+5s+6



Factorising s^2+5s+6



Summarising with a number array



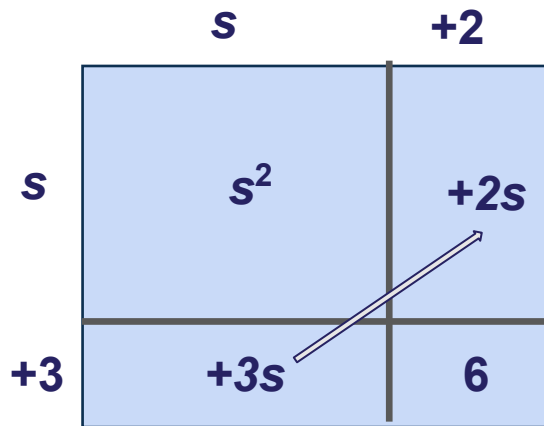
The Array Model

Multiply:

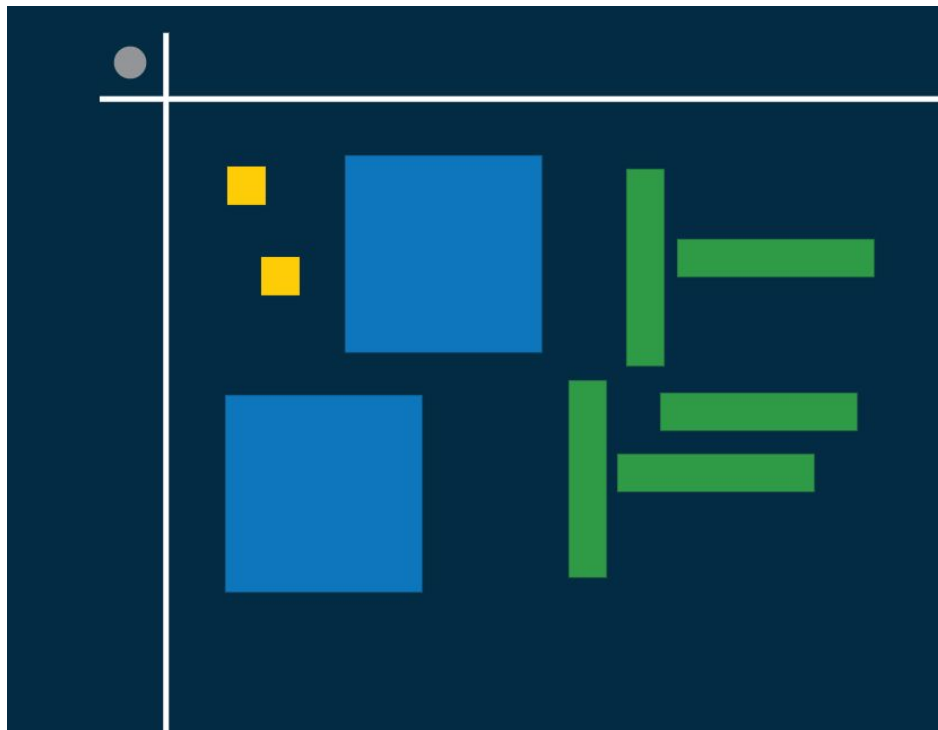
$$(s + 2)(s + 3)$$

$$= s^2 + 3s + 2s + 6$$

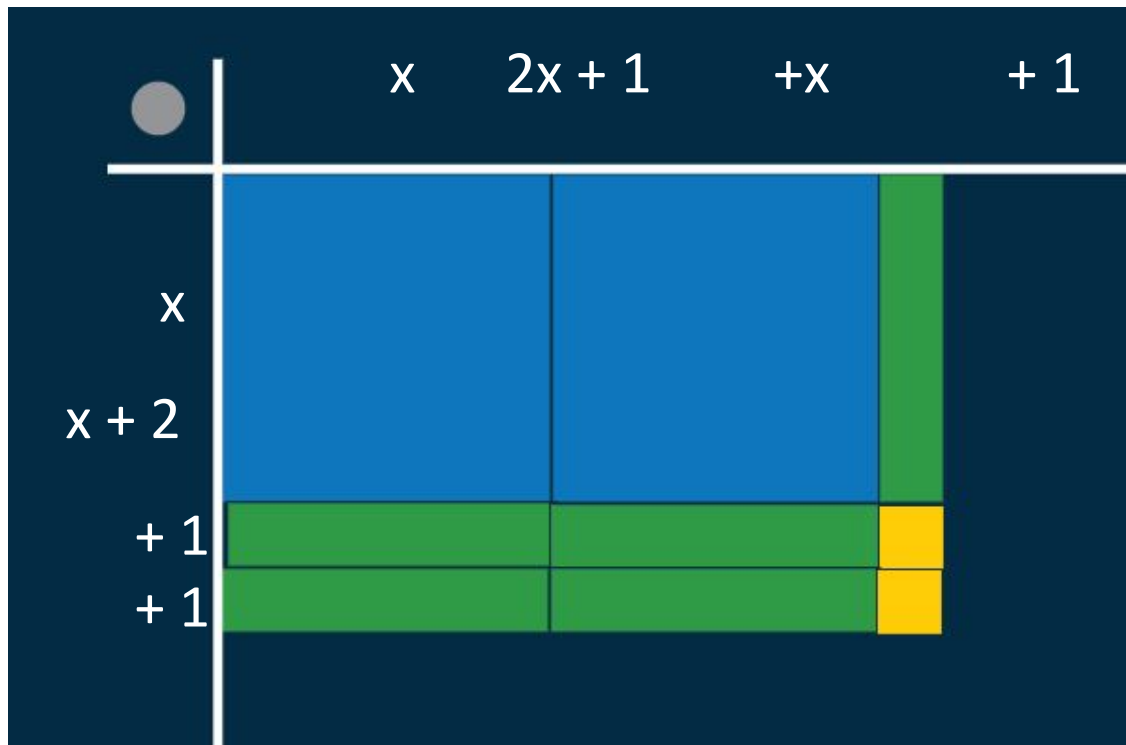
$$= s^2 + 5s + 6$$



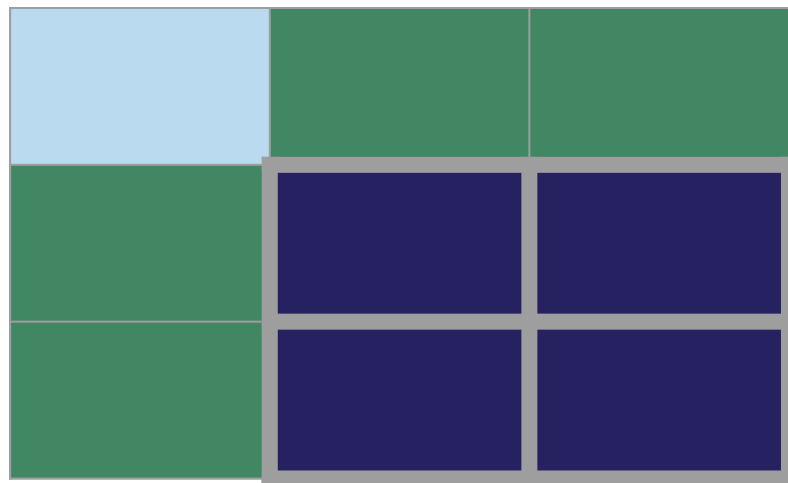
Factorising $2x^2 + 5x + 2$



Factorising

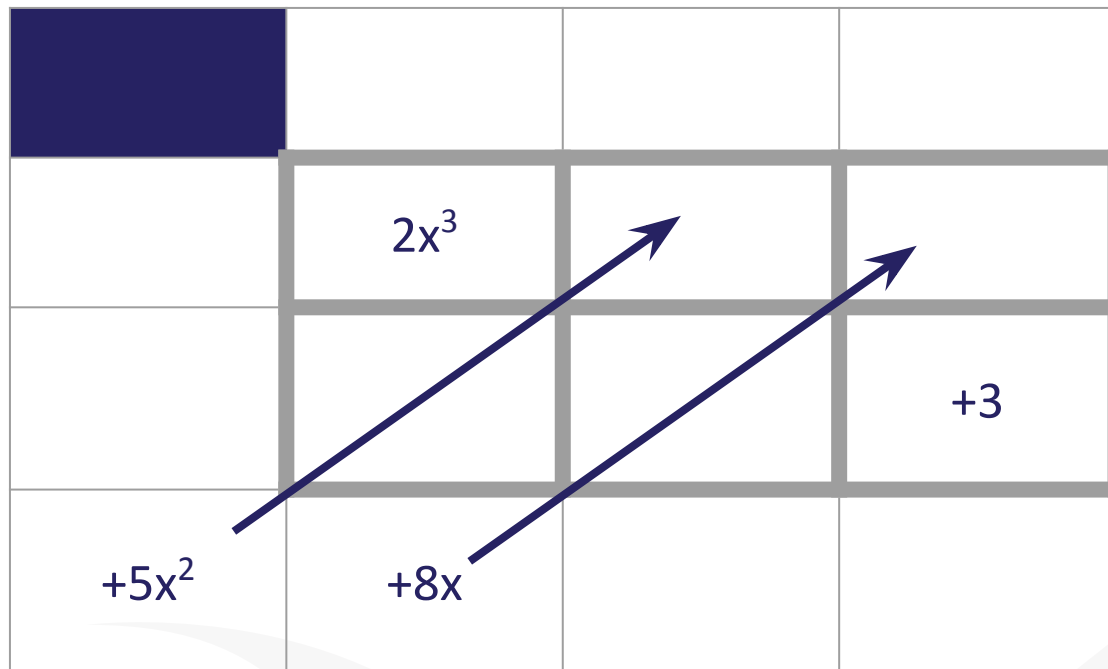


Summarising with a number array



The Cubic as a problem solving exercise

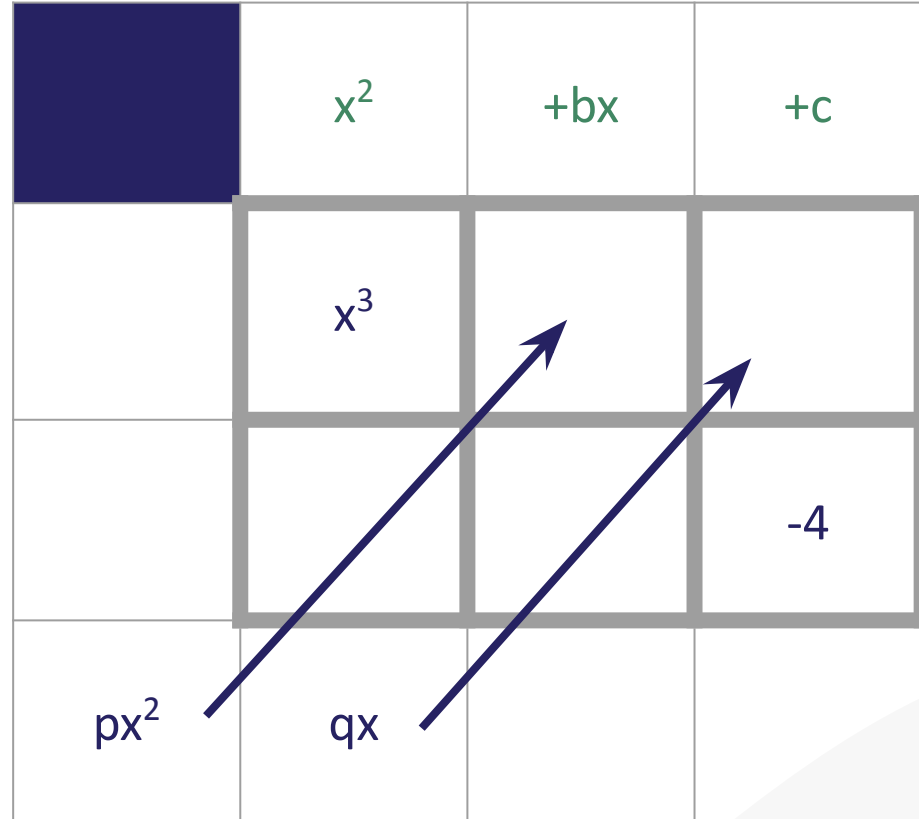
Factorising $2x^3 + 5x^2 + 8x + 3$



Future learning...

Given that $x^2 + bx + c$ is a factor of $x^3 + px^2 + qx - 4$, show that $c(p - b) = -4$.

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Reflection

Challenges of teaching Algebra & Functions

How could engaging with the approaches demonstrated in this workshop address some of the challenges mentioned earlier?

How would you bring these approaches to your classroom?



Conclusion

Key Messages



By engaging deeply with functions at Junior Cycle, students acquire the conceptual understanding and skills to understand functions at Senior Cycle.



Multiple representations are important for sense making, for developing deep conceptual understanding and for developing adaptive thinking.

Intended Learning

Participants will understand that allowing students to engage with rich, open tasks will

- encourage them to become more confident in their mathematical ability
- develop their critical thinking skills
- provide them with the tools needed to attempt unseen problems.

Evaluation

<https://tinyurl.com/algebra22evaluation>



Next Steps

Book a school visit

Contact us:

postprimarymaths@pdst.ie



Upcoming Events

Venue	Algebra Part 2
Blackrock EC	7/02/23
Cork ESC	09/02/23
Drumcondra EC	09/02/23
Galway EC	21/02/23
Limerick EC	21/02/23
Waterford	22/02/23

Keep informed of upcoming events:

<https://www.projectmaths.ie/>

School Support:

postprimarymaths@pdst.ie



Twitter:

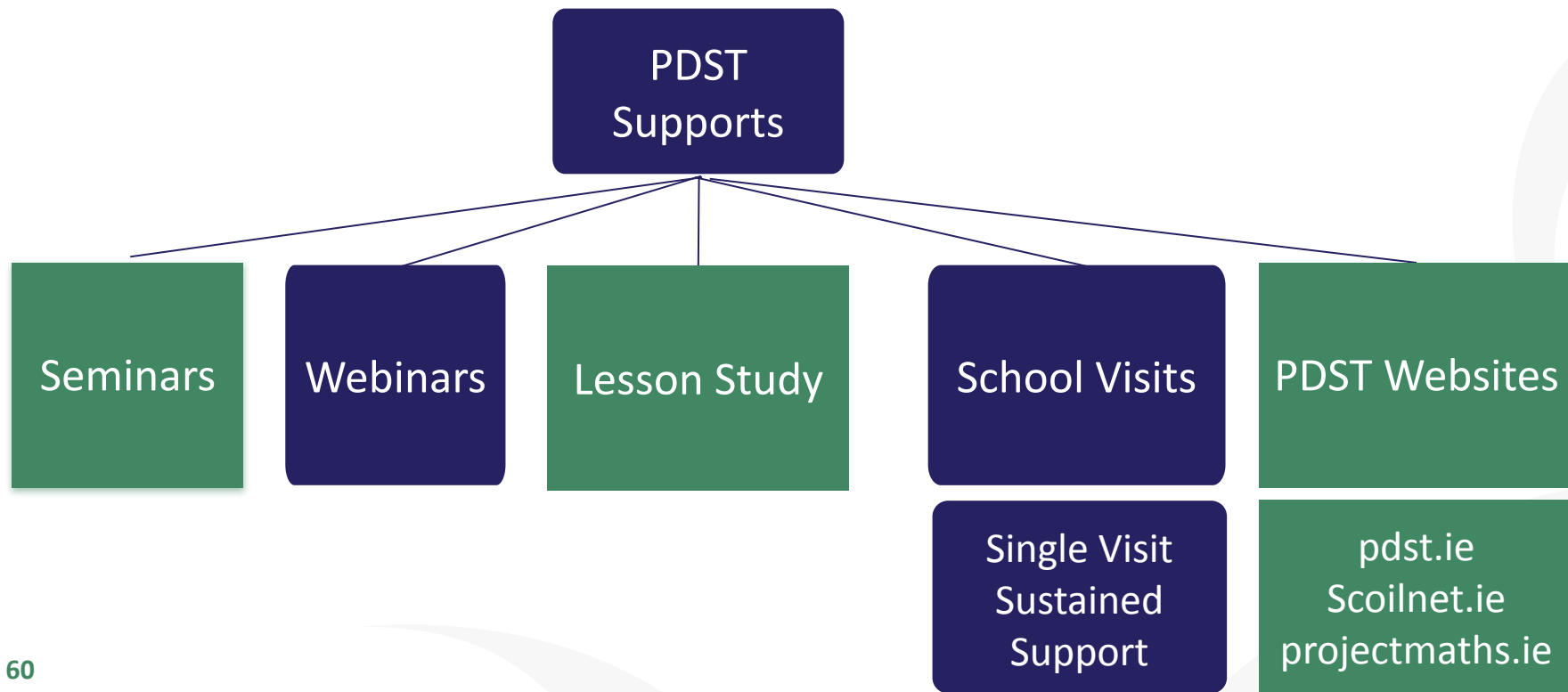
[@ProjectMaths_ie](https://twitter.com/ProjectMaths_ie)

Newsletter:

<http://eepurl.com/ghqwLD>



Supports Provided by PDST



References

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- NCCA. (2016) Junior Cycle Specification.
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- Pritchard & Woolard (2010). Psychology for the Classroom: Constructivism and Social Learning.